Exponentiated Weibull inverted exponential distribution: Model, properties and applications

Arun Kumar Chaudhary
Department of Management Science,
Nepal Commerce Campus, Tribhuvan University
Email: akchaudhary1@yahoo.com

Lal Babu Sah Telee
Corresponding Author
Department of Management Science,
Nepal Commerce Campus, Tribhuvan University
Email: lalbabu3131@gmail.com

Dhirendra Kumar Yadav
Department of Mathematics,
Ramsworup Ramsagar Multiple Campus, Janakpur
Email: yadav.dhirendrakumar@gmail.com

Abstract

This study is based on formulation of a new probability model having four parameters. Model parameters estimated via Maximum Likelihood, Least Squares, and Cramer–Von Mises methods are utilized. Some statistical properties like reliability function, hazard rate function, quantile functions are studied. Applicability of the model is tested using a real data set. Box plot and TTT plots are used to explain the nature of the data. For model validation, Q-Q plot, P-P plots as well as information criteria values such as Akaike Information criteria, Bayesian Information criteria, Corrected Akaike information criteria and Hannan-Quinn information criterion values are obtained. For testing the goodness of fit of the model and the model taken for comparison, Kolmogorov- Smirnov, Cramer Von-Mises, and Anderson darling test are applied. To study of the performance of MLEs, Monte-Carlo simulation is presented. All the calculations are performed using R programming language.

Keywords: Parameters, Maximum likelihood, Information criteria, Bayesian information, Goodness of fit

Cite this paper

Chaudhary, A.K., Sah Telee, L.B., & Yadav, D.K. (2021). Exponentiated Weibull inverted exponential distribution: Model, properties and applications. *NCC Journal, 6*(1), 1-10. https://doi.org/10.3126/nccj.v6i1.57785

Introduction

Research needs data and analysis of the data used. Probability distribution is one of the tools for analyzing the data. There are many data in study that cannot be clearly explained using the classical probability models. Over the past decades, numerous new probability models have been introduced in literature for enhanced precision. In literature we can find numerous techniques of getting new probability model. Some techniques are using family of distributions, adding some extra parameters and modifying the existing probability models. There are various exponentiated models such as the exponentiated generalized class of distributions (Cordeiro & Ortega, 2013), exponentiated Weibull distribution (Nadarajah et al., 2013) and Exponentiated distributions (Al-Hussaini & Ahsanullah, 2015) etc. A modified Weibull distribution by (Lai, et al., 2003), Beta modified distribution given by (Silva et al., 2010) and a new modified Weibull distribution by (Almalki & Yuan, 2013) are modified probability models. Weibull-H class (Cordeiro et al., 2017) and the exponential model's extension (Nadarajah & Haghighi, 2011) are used for formulating many new probability models.

In recent years, generalization is one of the important techniques of formulating new probability models. Weibull generalized family of distributions given by (Bourguignon et al., 2014) is used for formulation of probability models. The CDF and PDF of the Weibull generalized family of distribution are given below:

$$G(x:\alpha,\beta)=1-\exp\left[-\alpha\left(\frac{H(x;\xi)}{\overline{H}(x;\xi)}\right)^{\beta}\right] \tag{1.1}$$

$$g(x;\alpha,\beta)=\alpha\beta h(x:\xi)\left(\frac{H(x;\xi)^{\beta-1}}{\overline{H}(x,\xi)^{\beta+1}}\right)\exp\left[-\alpha\left(\frac{H(x;\xi)}{\overline{H}(x,\xi)}\right)^{\beta}\right] \tag{1.2}$$

Where, $H(x;\xi)$, $\overline{H}(x,\xi)^{\beta+1}$ and $h(x:\xi)$ are the CDF, reliability function and PDF of the base line distribution respectively for $x\geq0$, (β, θ)>0. Taking inverted exponential or inverse exponential (IE), as base distribution Oguntunde (2017, July) formulated the Weibull- inverted exponential distribution.

$$H(x;\theta)=e^{-\theta/x} \tag{1.3}$$

We can define $\overline{H}(x;\theta)$ as,

$$\overline{H}(x;\theta)=1-e^{-\theta/x} \tag{1.4}$$

Also define function,

$$\left[\frac{H(x;\theta)}{\overline{H}(x;\theta)}\right]^{\beta}=\left[\frac{e^{-\theta/x}}{1-e^{-\theta/x}}\right]^{\beta}=(e^{\theta/x}-1)^{-\beta} \tag{1.5}$$

Substituting equation (1.5) in equation (1.1) and (1.2), we can get the CDF and PDF of the Weibull inverted exponential distribution as,

$$G(x;\theta,\alpha,\beta)=1-\exp\left\{-\alpha\left(e^{\theta/x}-1\right)^{-\beta}\right\};x\geq0,\quad(\theta,\alpha,\beta)>0 \tag{1.6}$$

$$g(x;\theta,\alpha,\beta)=\left(\frac{\alpha\theta\beta}{x^2}\right)e^{\theta/x}\left(e^{\theta/x}-1\right)^{-(1+\beta)}\exp\left\{-\alpha\left(e^{\theta/x}-1\right)^{-\beta}\right\};x\geq0,\quad(\theta,\alpha,\beta)>0 \tag{1.7}$$

In this article, an extra non negative parameter is taken as exponent of the Weibull inverted exponential distribution to formulate a new distribution Exponentiated Weibull Inverted Exponential (EWIE) distribution. Let λ is non- negative constant. Taking of exponentiation to the CDF $G(x;\theta,\alpha,\beta)$ of the Weibull inverted exponential distribution results proposed model as the Exponentiated Weibull inverted exponential distribution with CDF and PDF defined by,

$$F(x;\theta,\alpha,\beta,\lambda)=\left[1-\exp\left\{-\alpha\left(e^{\theta/x}-1\right)^{-\beta}\right\}\right]^{\lambda};x\geq0,\quad(\theta,\alpha,\beta,\lambda)>0 \tag{1.8}$$

$$f(x;\theta,\alpha,\beta,\lambda)=\left(\frac{\theta\alpha\beta\lambda}{x^2}\right)e^{\theta/x}\left(e^{\theta/x}-1\right)^{-(1+\beta)}\exp\left\{-\alpha\left(e^{\theta/x}-1\right)^{-\beta}\right\}\left[1-\exp\left\{-\alpha\left(e^{\theta/x}-1\right)^{-\beta}\right\}\right]^{\lambda-1};x\geq0,\quad(\theta,\alpha,\beta,\lambda)>0 \tag{1.9}$$

Properties of model

Reliability function

Reliability function of the defined model is given as,

$$R(x;\theta,\alpha,\beta,\lambda)=1-\left[1-\exp\left\{-\alpha\left(e^{\theta/x}-1\right)^{-\beta}\right\}\right]^{\lambda};x\geq0,\quad(\theta,\alpha,\beta,\lambda)>0 \tag{2.1}$$

Hazard rate function

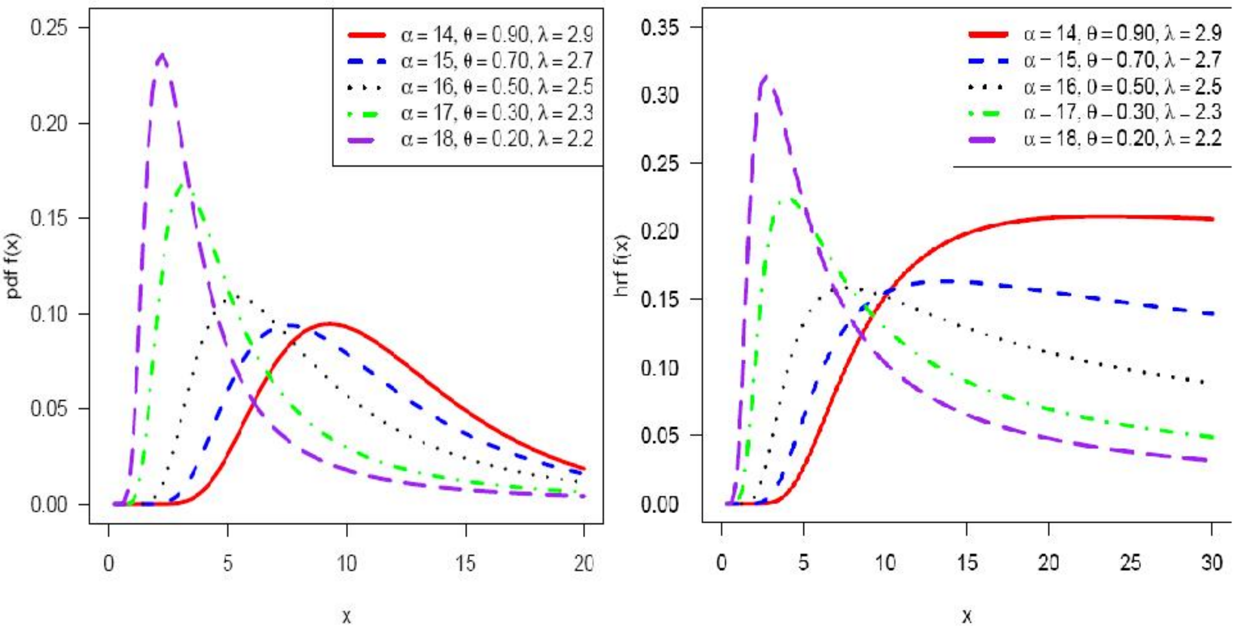
Hazard rate function of the proposed model is defined by:

$$h(x) = \left(\frac{\sim\ ^{-}\text{a}}{x^{2}}\right) e^{\sim/x}\left(e^{\sim/x}-1\right)^{-(1+^{-})} \exp\left\{-\ \left(e^{\sim/x}-1\right)^{-}\right\} \quad (2.2)$$

$$\left[1-\exp\left\{-\ \left(e^{\sim/x}-1\right)^{-}\right\}\right]^{\text{a}-1}\left[1-\left[1-\exp\left\{-\ \left(e^{\sim/x}-1\right)^{-}\right\}\right]^{\text{a}}\right]^{-1} \quad x>0$$

The probability density function and the hazard rate function of the proposed model EWIE is displayed in Figure 1.

Figure 1
Probability density function (left) and hazard rate function (right) of the EWIE



Density curve is of different shape for different values of the parameters showing that proposed model is flexible. The hazard rate curve is increasing – decreasing and of inverted bathtub shape.

Cumulative hazard rate function

The cumulative hazard rate function H(*x*) is given as

$$H(x) = -\ln R(x) = -\ln\left\{1-\left[1-\exp\left\{-\ \left(e^{\sim/x}-1\right)^{-}\right\}\right]^{\text{a}}\right\} \quad (2.3)$$

Quantile function

$$Q(u) = \sim\left[\ln\left[1+\left\{\left(-1/\ \ \right)\ln\left(1-u^{(1/\text{a})}\right)\right\}^{(-1/^{-})}\right]\right] \quad , \quad 0 < u < 1 \quad (2.4)$$

Putting u = ½, we can get the median of the proposed model as:

$$\text{Median} = \sim\left[\ln\left[1+\left\{\left(-1/\quad\right)\ln\left(1-\left(0.5\right)^{\left(1/ª\right)}\right)\right\}^{\left(-1/¯\right)}\right]\right] \tag{2.5}$$

Random deviate generation

Random deviate generation of the model is given by:

$$x = \sim\left[\ln\left[1+\left\{\left(-1/\quad\right)\ln\left(1-u^{1/ª}\right)\right\}^{\left(-1/¯\right)}\right]\right]^{-1};\quad 0 < u < 1 \tag{2.6}$$

Asymptotic properties of the Model

We check $\lim_{x\to 0} f(x) = \lim_{x\to\infty} f(x)$ to ascertain density function's asymptotic properties. If model satisfies the asymptotic properties, then there will be unique modal value. Taking limiting at end points,

$$\lim_{x\to 0} f(x) = \lim_{x\to 0}\left(\frac{\sim\ ¯ª}{x^2}\right)e^{\sim/x}\left(e^{\sim/x}-1\right)^{-(1+¯)}\exp\left\{-\ \left(e^{\sim/x}-1\right)^{¯}\right\}\left[1-\exp\left\{-\ \left(e^{\sim/x}-1\right)^{¯}\right\}\right]^{ª-1} = 0 \tag{2.7}$$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty}\left(\frac{\sim\ ¯ª}{x^2}\right)e^{\sim/x}\left(e^{\sim/x}-1\right)^{-(1+¯)}\exp\left\{-\ \left(e^{\sim/x}-1\right)^{¯}\right\}\left[1-\exp\left\{-\ \left(e^{\sim/x}-1\right)^{¯}\right\}\right]^{ª-1} = 0 \tag{2.8}$$

Here, $\lim_{x\to 0} f(x) = \lim_{x\to\infty} f(x)$ so modal value of the proposed model will exist.

Skewness and kurtosis

Skewness pertains to data consistency. We applied Bowley's skewness coefficient (Al-saiary et al., 2019) using quantiles for analysis as:

$$\text{SK}(\text{B}) = \frac{\text{Q}(0.25)+\text{Q}(0.75)-2\times\text{Q}(0.50)}{\text{Q}(0.75)-\text{Q}(0.25)}$$

The calculation of Octiles Kurtosis coefficients from (Moors, 1998) and (Al-saiary et al., 2019) involves the following relation.

$$\text{K}_\text{u} = \frac{\text{Q}(0.875)-\text{Q}(0.625)+\text{Q}(0.375)-\text{Q}(0.125)}{\text{Q}(0.75)-\text{Q}(0.25)}$$

Parameter estimation techniques

Parameters can be estimated applying different methods. We have applied following methods.

Estimation using maximum likelihood (MLE)

Defining the log likelihood function for the proposed model in (1.9). Let $\underline{x} = (x_1,\ldots,x_n)$ be a random sample of size 'n' from MEIE then the log likelihood function can be written as:

$$l(\sim,\ ,¯,ª\mid\underline{x}) = n\ln(\sim\ ¯ª) - 2\sum_{i=1}^{n}x_i - \ \sum_{i=1}^{n}(e^{\sim/x_i}-1)^{¯} + \sim\sum_{i=1}^{n}\left(\frac{1}{x_i}\right) - (1+¯)\sum_{i=1}^{n}\ln(e^{\sim/x_i}-1)^{¯} +$$
$$(ª-1)\sum_{i=1}^{n}\ln\left(1-\exp\left(-\ (e^{\sim/x_i}-1)^{¯}\right)\right) \tag{3.1}$$

After differentiating (3.1) with respect to α, β, θ and λ, we can get the first order and second order partial derivatives of log likelihood function.

Solving above first order derivatives to zero, parameters of the proposed model can be estimated. Solution of above equation is not possible so computer programming can be used. Let $\hat{\Theta} = (\hat{\alpha}, \hat{\beta}, \hat{\theta}, \hat{\lambda})$ and $\Theta = (\alpha, \beta, \theta, \lambda)$, are estimated constants and parameter vector respectively then resulting asymptotic normality will be, $\left(\hat{\Theta} - \Theta\right) \rightarrow N_3\left[0, \left(I(\Theta)\right)^{-1}\right]$. The Fisher's information matrix $I(\Theta)$ can be given by:

$$I(\Theta) = -\begin{pmatrix} E\left(\frac{\partial^2 l}{\partial \alpha^2}\right) & E\left(\frac{\partial^2 l}{\partial \alpha \partial \beta}\right) & E\left(\frac{\partial^2 l}{\partial \alpha \partial \theta}\right) & E\left(\frac{\partial^2 l}{\partial \alpha \partial \lambda}\right) \\ E\left(\frac{\partial^2 l}{\partial \beta . \partial \alpha}\right) & E\left(\frac{\partial^2 l}{\partial \beta^2}\right) & E\left(\frac{\partial^2 l}{\partial \beta \partial \theta}\right) & E\left(\frac{\partial^2 l}{\partial \beta . \partial \lambda}\right) \\ E\left(\frac{\partial^2 l}{\partial \theta . \partial \alpha}\right) & E\left(\frac{\partial^2 l}{\partial \theta \partial \beta}\right) & E\left(\frac{\partial^2 l}{\partial \theta^2}\right) & E\left(\frac{\partial^2 l}{\partial \theta \partial \lambda}\right) \\ E\left(\frac{\partial^2 l}{\partial \lambda . \partial \alpha}\right) & E\left(\frac{\partial^2 l}{\partial \lambda \partial \beta}\right) & E\left(\frac{\partial^2 l}{\partial \lambda \partial \theta}\right) & E\left(\frac{\partial^2 l}{\partial \lambda^2}\right) \end{pmatrix}$$

Asymptotic variance $\left(I(\Theta)\right)^{-1}$ of MLE is worthless because Θ cannot be obtained. Let $O(\hat{\Theta})$ be the observed fisher information matrix. Estimate $O(\hat{\Theta})$ of $I(\underline{\Theta})$ hessian matrix H can be obtained as:

$$O(\hat{\Theta}) = -\begin{pmatrix} \left(\frac{\partial^2 l}{\partial \alpha^2}\right) & \left(\frac{\partial^2 l}{\partial \alpha \partial \beta}\right) & \left(\frac{\partial^2 l}{\partial \alpha \partial \theta}\right) & \left(\frac{\partial^2 l}{\partial \alpha \partial \lambda}\right) \\ \left(\frac{\partial^2 l}{\partial \beta . \partial \alpha}\right) & \left(\frac{\partial^2 l}{\partial \beta^2}\right) & \left(\frac{\partial^2 l}{\partial \beta \partial \theta}\right) & \left(\frac{\partial^2 l}{\partial \beta . \partial \lambda}\right) \\ \left(\frac{\partial^2 l}{\partial \theta . \partial \alpha}\right) & \left(\frac{\partial^2 l}{\partial \theta \partial \beta}\right) & \left(\frac{\partial^2 l}{\partial \theta^2}\right) & \left(\frac{\partial^2 l}{\partial \theta \partial \lambda}\right) \\ \left(\frac{\partial^2 l}{\partial \lambda . \partial \alpha}\right) & \left(\frac{\partial^2 l}{\partial \lambda \partial \beta}\right) & \left(\frac{\partial^2 l}{\partial \lambda \partial \theta}\right) & \left(\frac{\partial^2 l}{\partial \lambda^2}\right) \end{pmatrix} = -H(\Theta)\Big|_{\left(\Theta = \hat{\Theta}\right)} \qquad (3.2)$$

Variance covariance matrix is:

$$\left[-H(\Theta)\Big|_{(\Theta=\hat{\Theta})}\right]^{-1} = \begin{pmatrix} Var(\hat{\alpha}) & Cov(\hat{\alpha}, \hat{\beta}) & Cov(\hat{\alpha}, \hat{\theta}) & Cov(\hat{\alpha}, \hat{\lambda}) \\ Cov(\hat{\beta}, \hat{\alpha}) & Var(\hat{\beta}) & Cov(\hat{\beta}, \hat{\theta}) & Cov(\hat{\beta}, \hat{\lambda}) \\ Cov(\hat{\theta}, \hat{\alpha}) & Cov(\hat{\theta}, \hat{\beta}) & Var(\hat{\theta}) & Cov(\hat{\theta}, \hat{\lambda}) \\ Cov(\hat{\lambda}, \hat{\alpha}) & Cov(\hat{\lambda}, \hat{\beta}) & Cov(\hat{\lambda}, \hat{\theta}) & Var(\hat{\lambda}) \end{pmatrix} \qquad (3.3)$$

Here,100(1-γ) % C.I. for α, β ,θ and λ are:

$$\hat{\alpha} \pm Z_{\gamma/2}\sqrt{Var(\hat{\alpha})}\,,\ \hat{\beta} \pm Z_{\gamma/2}\sqrt{Var(\hat{\beta})}, \hat{\theta} \pm Z_{\gamma/2}\sqrt{Var(\hat{\theta})} \text{ and } \hat{\lambda} \pm Z_{\gamma/2}\sqrt{Var(\hat{\lambda})}$$

Estimation using least-square (LSE)

Let $X_{(1)} < X_{(2)} < \ldots < X_{(n)}$ is ordered random variables and a random sample $\{X_1, X_2, \ldots, X_n\}$ of size n is taken from a distribution function F (.). We define a function A using $F(X_{(i)})$ as CDF of ordered statistics by equation (3.4)

$$A(x; \alpha, \beta, \theta, \lambda) = \sum_{i=1}^{n}\left[F(X_{(i)}) - \frac{i}{n+1}\right]^2 \qquad (3.4)$$

Minimizing function (3.4), parameters of proposed model EWIE can be obtained. For minimization of (3.4) first order and second order partial derivatives can be obtained by differentiating function A with respect to unknown parameters.

Parameters can be also obtained by weighted LSE minimizing the function D in (3.5)

$$D(X; \alpha, \beta, \theta, \lambda) = \sum_{i=1}^{n} w_i\left[F(X_{(i)}) - \frac{i}{n+1}\right] \quad \text{Where, } w_i = \frac{1}{Var(X_{(i)})} = \frac{(n+1)^2(n+2)}{i(n-i+1)}$$

Using the CDF of the order statistics and weight w_i in above expression with respect to α, β, θ and λ

$$D(X;\alpha,\beta,\theta,\lambda)=\sum_{i=1}^{n}\frac{(n+1)^2(n+2)}{i(n-i+1)}\left[F(X_{(i)})-\frac{i}{n+1}\right]^2 \quad (3.5)$$

Cramer-Von-Mises (CVM) method

Using this method, parameters α, β, θ and λ can be estimated by minimizing the function (3.6)

$$Z(X;\alpha,\beta,\theta,\lambda)=\frac{1}{12n}+\sum_{i=1}^{n}\left[F(x_{i:n}\mid\alpha,\beta,\theta,\lambda)-\frac{2i-1}{2n}\right]^2 \quad (3.6)$$

Differentiating (3.5) with respect to α, β, θ and λ, we can get the first and second order partial derivatives of function Z and solving $\frac{\partial Z}{\partial\alpha}=0, \frac{\partial Z}{\partial\beta}=0, \frac{\partial Z}{\partial\theta}=0$ and $\frac{\partial Z}{\partial\lambda}=0$, CVM estimates can be obtained.

Applications

Data set

The data set is number of major earthquakes (7.0+) from 1900 to 2018 are provided below (https://earthquake.usgs.gov/).

13, 14, 8, 10, 16, 26, 32, 27, 18, 32, 36, 24, 22, 23, 22, 18,25, 21, 21, 14,8,11,14, 23, 18, 17, 19, 20, 22, 19, 13, 26, 13, 14, 22, 24, 21, 22, 26, 21, 23, 24, 27, 41, 31, 27, 35, 26, 28, 36, 39, 21, 17, 22, 17, 19, 15, 34, 10, 15, 22, 18, 15, 20, 15, 22, 19, 16, 30, 27, 29, 23, 20, 16, 21, 21, 25, 16, 18, 15, 18, 14, 10, 15, 8,15, 6, 11, 8, 7, 18, 17, 13, 12, 13, 20, 15, 16, 12, 18, 15, 16, 13, 15, 16, 11, 11, 18, 12, 17, 24, 20, 16, 19, 12, 19, 16, 7, 17

Exploratory data analysis

Exploratory data analysis reveals inherent patterns and extracts key data variables. Figure 2 illustrates the box plot and Total Time Test (TTT) plot for the provided data. TTT validates the data's suitability for a particular probability model. The subsequent expression represents the empirical TTT plot.

$$T\left(\frac{r}{n}\right)=\sum_{i=1}^{n}y_{(i:n)}+(n-r)y_{i:n}\left(\sum_{i=1}^{n}y_{(i:n)}\right)^{-1}$$

Where, r = 1, 2... n and $y_{(i:n)}(i=1,2,\ldots,r)$ be sample order statistics . Since TTT plot of data is concave indicating that increasing the hazard rate shape of the proposed distribution.

Figure 2

Boxplot (left panel) and TTT plot (right Plot)

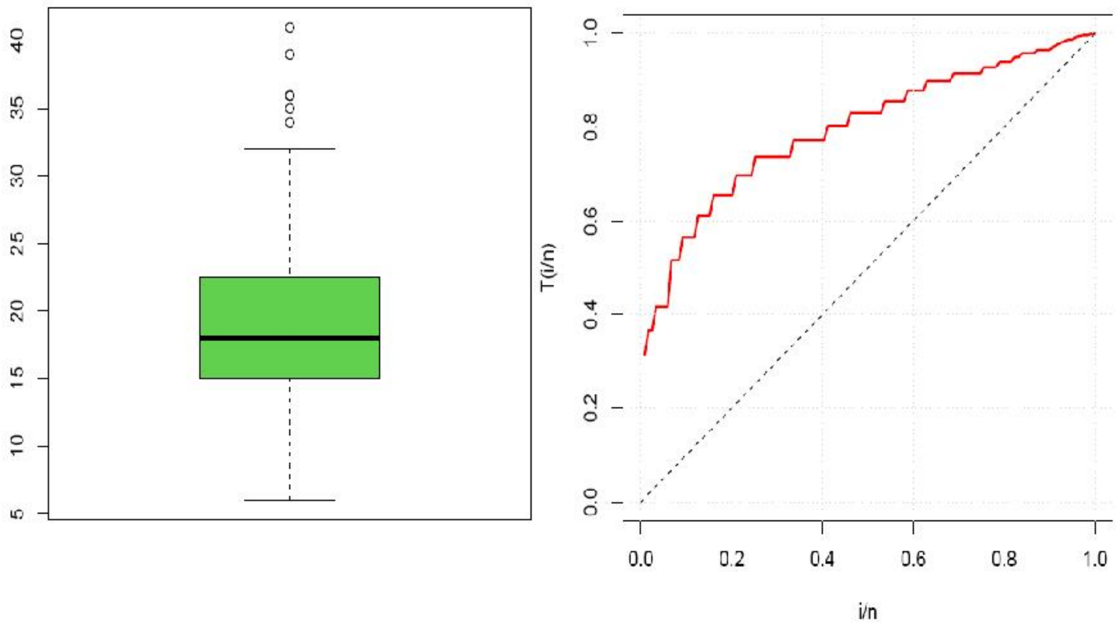


Table 1
Summary Statistics

Minimum	Q_1	Median	Mean	Q_3	SD	Skewness	Kurtosis	Max.
6.00	15.00	18.00	19.08	22.50	6.993	0.728	3.570	41

The dataset exhibits positive skewness and non-normal shape.

Parameter estimation

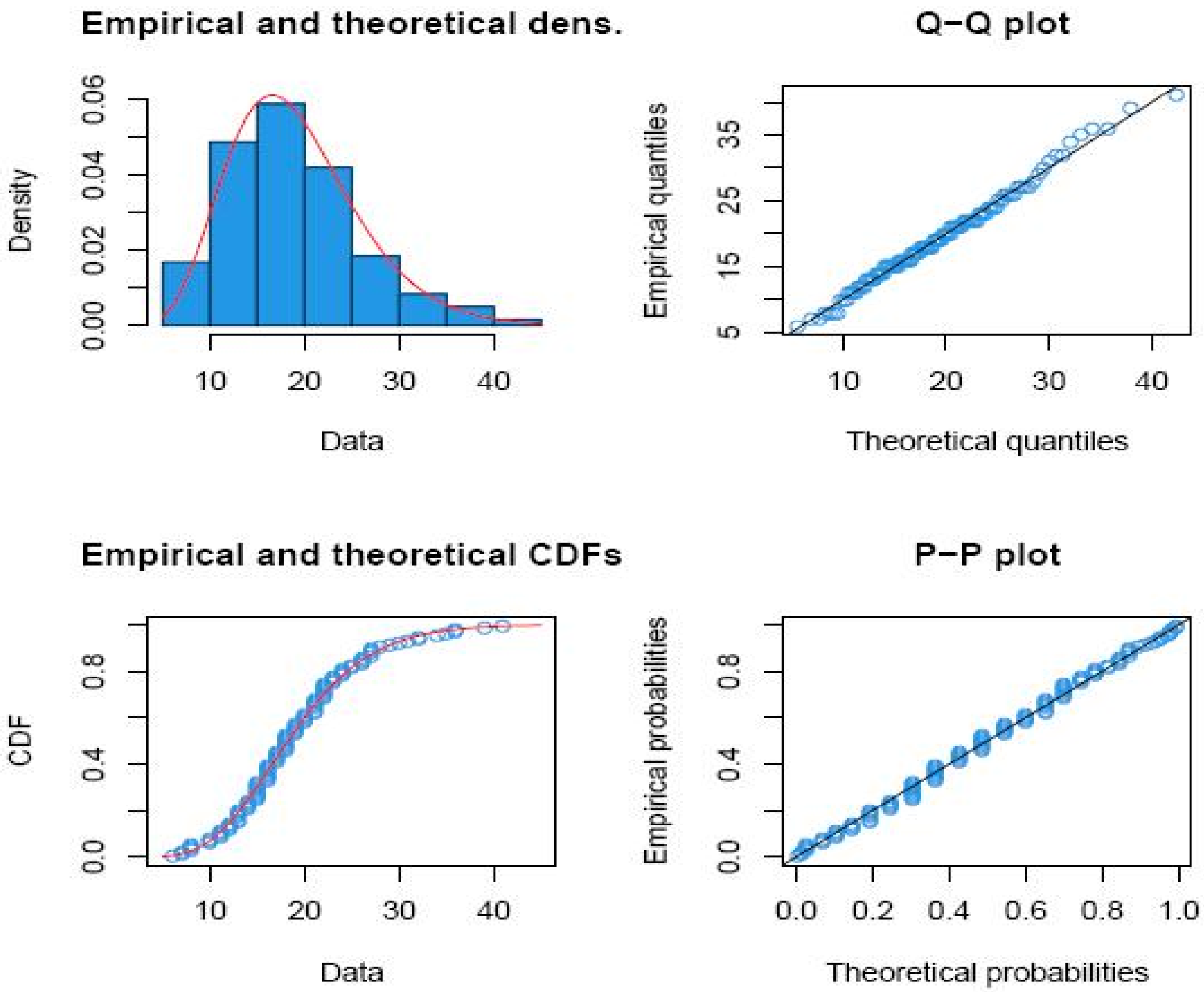
Due to the nonlinearity of partial derivatives, parameter estimation via analytical methods is infeasible. Instead, the R software's optim() function is employed for estimation (R Core Team, 2020). Table 2 presents MLE, along with standard error of estimate (SE), for the parameters.

Table 2
Estimated Parameters Using MLE, LSE and, CVME

Parameters	MLE	LSE	CVM
Alpha	0.8917	1.7057	0.5423
Beta	0.0159	0.0722	0.0060
Theta	1.5723	1.4302	1.6256
Lambda	3.6520	4.7294	3.8285

Different graphical plots like histogram and the fitted density curve, Q-Q plot, empirical versus theoretical cdf and P-P plot of the proposed model EWIE is displayed in Figure 3.

Figure 3
Histogram vs fitted pdf, Q-Q plot, empirical versus theoretical cdf and P-P plot of EWIE



For testing the applicability, three previously defined probability models are considered. Model considered are: Logistic inverse exponential (LIE) distribution (Chaudhary et al., 2020), Modified Weibull (MW) distribution (Lai et al., 2003), and Weibull Extension Model (Tang et al., 2003).

Table 3 displays the parameter estimate by MLE method for models taken in consideration.

Table 3
Estimated parameters by using MLE

Model	Alpha	Beta	Theta	Lambda
EWIE	0.8917(0.8896)	0.0159(0.0088)	1.5723(0.4117)	3.6520(2.4549)
LIE	3.3079(0.2560)	-	-	12.3560(0.4217)
WE	115.9452(89.7898)	2.8848(0.1999)	-	1.1221(1.7360)
MW	0.0022(0.0002)	2.0001(0.1040)	-	0.0029(0.0114)

For testing the validity of the model different information criteria such as Akaike information criterion (AIC), Corrected Akaike information criterion (CAIC), Bayesian information criterion (BIC) and Hannan-Quinn information criterion (HQIC), are computed. The findings are displayed in Table 4. Finding shows that the proposed model EWIE fits data better compared to the competing models.

Table 4
Log-likelihood (LL), AIC, BIC, CAIC, and HQIC.

Model	LL	AIC	BIC	CAIC	HQIC
EWIE	-394.2792	796.5584	807.6749	796.9092	801.0724
LIE	-397.9190	799.8379	805.3962	799.9414	802.0949
WE	-398.3256	802.6512	810.9886	802.8599	806.0368
MW	-409.0469	824.0938	832.4311	824.3025	827.4793

Kolmogorov-Smirnov (KS), Cramer-Von Mises (W) statistics and, Anderson-Darling (A^2) for the proposed model as well as the competing models are tabulated in Table 5.

Table 5
KS, W and A^2 for goodness of fit test

Model	KS	W	A^2
EWIE	0.0607(0.7738)	0.0450(0.9066)	0.3037(0.9352)
LIE	0.0633(0.7262)	0.0655(0.7803)	0.6534(0.5986)
WE	0.0872(0.3260)	0.1646(0.3484)	1.0410(0.3363)
MW	0.1640(0.0033)	0.7805(0.0080)	4.7274(0.0039)

Lower values of test statistics and greater values of p show that the model fits well to the considered data set compared to the competing models.

Simulation study

To study of the performance of MLEs, Monte-Carlo simulation is presented. For study tool bias is used. Here, 1000 times repetition is done for generating 20 samples of size n= (50, 100, 150, 200, 250, 300, 350, 400, 450, 500, 550, 600, 650, 700, 750, 800, 850, 900, 950, 1000) taking the parameter set as $\alpha=5$, $\beta = 4$, $\theta= 2$, $\lambda=3$. Table 6 contains the average values, and biases.

Table 6
Mean estimates and mean bias

n	Estimates				Bias			
	α=5	β = 4	θ= 2	λ=3	α=5	β = 4	θ= 2	λ=3
50	5.6506	6.5240	2.2922	6.2217	0.6506	2.5240	0.2922	3.2217
100	5.3035	6.4036	2.1134	4.1239	0.3035	2.4036	0.1134	1.1239
150	5.1741	6.1207	2.0585	3.7674	0.1741	2.1207	0.0585	0.7674
200	5.2838	6.3612	2.0327	3.5307	0.2838	2.3612	0.0327	0.5307
250	5.3008	6.3070	2.0384	3.3662	0.3008	2.3070	0.0384	0.3662
300	5.3034	6.1041	2.0213	3.3295	0.3034	2.1041	0.0213	0.3295
350	5.2752	5.9664	2.0146	3.2787	0.2752	1.9664	0.0146	0.2787
400	5.1974	5.5796	2.0145	3.2485	0.1974	1.5796	0.0145	0.2485
450	5.2586	5.5844	2.0084	3.1960	0.0259	1.5844	0.0084	0.1960
500	5.2177	5.5249	2.0071	3.1865	0.2177	1.5249	0.0071	0.1865
550	5.2139	5.4466	2.0013	3.1607	0.2139	1.4466	0.0012	0.1607
600	5.2308	5.3867	1.9958	3.1588	0.2308	1.3867	-0.0042	0.1587
650	5.2402	5.3339	1.9920	3.1512	0.2402	0.0080	1.3339	0.1512
700	5.1796	5.1304	1.9954	3.1467	0.1796	0.0046	1.1304	0.1467
750	5.1035	4.8023	2.0022	3.1476	0.1035	0.8023	0.0022	0.1476
800	5.1682	4.9904	1.9968	3.1125	0.1682	0.0032	0.9904	0.1125
850	5.1859	4.9546	1.9956	3.0959	0.1859	0.0044	0.9546	0.0959
900	5.0965	4.6786	1.9995	3.1168	0.09645	0.0005	0.6786	0.1168
950	5.1554	4.8857	1.9960	3.0938	0.1554	0.0040	0.8857	0.0938
1000	5.1296	4.7216	1.9995	3.0882	0.1296	0.0004.81	0.7216	0.0882

Conclusion

In this study a four parameters continuous probability model called Exponentiated Weibull Inverted exponential distribution is formulated. Different statistical properties such as hazard rate function, random deviate generation and quantile functions are studied. Parameters of the model are estimated using three methods, MLE, CVM and LSE. Applicability of the model is tested by taking a real data set. For validity testing, different information criteria like AIC, BIC, CAIC and HQIC are obtained. Goodness of fit of the model is tested by finding Kolmogrov-Smirnov, Cramer- von Mises and Anderson-Darling test statistics values along with corresponding p-values. To study of the performance of MLEs, Monte-Carlo simulation is presented.

References

Al-Hussaini, E. K., & Ahsanullah, M. (2015). *Exponentiated distributions*. *Atlantis Studies in Probability and Statistics*, Paris: Atlantis Press.

Almalki, S. J., & Yuan, J. (2013). A new modified Weibull distribution. *Reliability Engineering & System Safety*, *111*, 164-170.

Al-saiary, Z. A., Bakoban, R. A., & Al-zahrani, A. A. (2019). Characterizations of the beta kumaraswamy exponential distribution. *Mathematics*, *8*(1), 23.

Bourguignon, M., Silva, R. B., & Cordeiro, G. M. (2014). The Weibull-G family of probability distributions. *Journal of data science*, 12(1), 53-68.

Chaudhary, A. K., & Kumar, V. (2020). Logistic Inverse exponential distribution with properties and applications. *International Journal of Mathematics Trends and Technology (IJMTT)*, *66*(10), 151-162.

Cordeiro, G. M., Afify, A. Z., Yousof, H. M., Pescim, R. R., & Aryal, G. R. (2017). The exponentiated Weibull-H family of distributions: Theory and Applications. *Mediterranean Journal of Mathematics*, *14*, 1-22.

Cordeiro, G. M., Ortega, E. M., & da Cunha, D. C. (2013). The exponentiated generalized class of distributions. *Journal of data science*, 11(1), 1-27.

Lai, C. D., Xie, M., & Murthy, D. N. P. (2003). A modified Weibull distribution. *IEEE Transactions on reliability*, 52(1), 33-37.

Moors, J. J. A. (1988). A quantile alternative for kurtosis. *Journal of the Royal Statistical Society: Series D (The Statistician)*, *37*(1), 25-32.

Nadarajah, S., & Haghighi, F. (2011). An extension of the exponential distribution. *Statistics*, *45*(6), 543-558.

Nadarajah, S., Cordeiro, G. M., & Ortega, E. M. (2013). The exponentiated Weibull distribution: a survey. *Statistical Papers*, *54*, 839-877.

Oguntunde, P. E., Adejumo, A. O., & Owoloko, E. A. (2017, July). The Weibull-inverted exponential distribution: A generalization of the inverse exponential distribution. *World Congress on Engineering.*

Silva, G. O., Edwin, M., Ortega, M., & Cordeiro, G. M. (2010). The beta modified Weibull distribution. *Lifetime data analysis*, *16*(3), 409.

Tang, Y., Xie, M., & Goh, T. N. (2003). Statistical analysis of a Weibull extension model. *Communications in Statistics-Theory and Methods*, *32*(5), 913-928.

Team, R. D. C. (2020). A language and environment for statistical computing. *http://www. R-project. org*.

USGS (1900-2018). World-M7+ (1900-2018). The USGS Earthquake hazards program. https://earthquake.usgs.gov/earthquakes/browse/m7-world.php?year=1900-2018