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ON BANACH SPACE VALUED ORLICZ FUNCTION SPACE $\ell(X, U, \|.\|, \xi)$ AND ITS GENERALIZED FORM $\ell(X, U, \|.\|, \xi, \lambda, p)$

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ABSTRACT
This paper attempts to introduce and study a new class ℓ (X, U, $\|.\|$, ξ) and its generalized form ℓ (X, U, $\|.\|$, ξ, λ, p) of Normed space valued function space using Orlicz function ξ as the generalizations of the basic sequence space ℓ. Besides the investigation of linear topological structures of the class ℓ (X, U, $\|.\|$, ξ) when topologized it with suitable natural norm, our primarily interest is to explore the conditions pertaining to the containment relation between the classes ℓ (X, U, $\|.\|$, ξ, λ, p) in terms of different values of λ and p.

Key words: Orlicz function, Orlicz Space, Normal space, GK-, GFK-, GAK-, GAD- and GC-function space.

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1. INTRODUCTION AND PRELIMINARIES

We begin with recalling some requisites that are used in this paper.

Definition 1.1: An Orlicz function is a function $\xi:[0,\infty) \to [0,\infty)$ which is continuous, non decreasing and convex with $\xi(0) = 0$, $\xi(u) > 0$ for $u > 0$, and $\xi(u) \to \infty$ as $u \to \infty$. Obviously, Orlicz function generalizes the function

$$\xi(u) = u^p, (1 \leq p < \infty).$$

An Orlicz function ξ can be represented in the following integral form

$$\xi(u) = \int_0^u q(t)\, dt$$

where q, known as the kernel of ξ, is right-differentiable for $t \geq 0$, $q(0) = 0$, $q(t) > 0$ for $t > 0$, q is non decreasing, and $q(t) \to \infty$ as $t \to \infty$,(see, [9]).

Definition 1.2: An Orlicz function ξ is said to satisfy Δ_2 - condition for all values of u, if there exists a constant $K > 0$ such that

$$\xi(2u) \leq K\, \xi(u) \text{ , for all } u \geq 0.$$

The Δ_2-condition is equivalent to the satisfaction of the inequality

$$\xi(Lu) \leq K\, L\, \xi(u)$$

for all values of u for which $L > 1$, (see, [9]).

Definition 1.3: J. Lindenstrauss & L. Tzafriri [10] used the idea of Orlicz function to construct the sequence space

$$\ell_\xi = \inf\left\{ \bar{u} = (u_k): \sum_{k=1}^{\infty} \xi\left(\frac{|u_k|}{\rho}\right) < \infty \text{ for some } \rho > 0 \right\}$$

of scalars (u_k) ,which forms a Banach space with Luxemburg norm defined by

$$\|\bar{u}\|_\xi = \inf\left\{ \rho > 0: \sum_{k=1}^{\infty} \xi\left(\frac{|u_k|}{\rho}\right) \leq 1 \right\}.$$

The space ℓ_ξ is called an Orlicz sequence space and is closely related to the space ℓ_p with

$$\xi(u) = u^p, (1 \leq p < \infty).$$

They have very rich topological and geometrical properties that do not occur in ordinary ℓ_p space.

Subsequently, M.Basariv & S. Altundag [1] , V.N. Bhardwaj & I. Bala [2], S.T. Chen [3], D. Ghosh & P.D. Srivastava [4], P.K. Kamthan & M. Gupta [6],V.A. Khan [7], E. Kolk [8], N.P.Pahari [12] , S.D. Parashar & B. Choudhary [13], K.C. Rao & N. Subremanina [14], M.M. Rao & Z.D. Ren [15], E. Savas & S.Patterson [16] , J.K. Srivastava & N.P.Pahari [18,19] and many others have been introduced and studied the algebraic and topological properties of various sequence and function spaces using Orlicz function as the generalizations of various well known sequence spaces and function spaces.

Definition 1.4: A normed space $(X, \|.\|)$ is a linear space X together with the mapping $\|.\| : X \to \boldsymbol{R}_+$ (called *norm* on X) such that for all $x, y \in X$ and $\alpha \in \boldsymbol{C}$, we have

N_1: $\|x\| \geq 0$ and $\|x\| = 0$ if and only if $x = \theta$;

N_2: $\|\alpha x\| = |\alpha| \|x\|$; and

N_3: $\|x + y\| \leq \|x\| + \|y\|$.

Clearly by N_1 and N_2, algebraic operations of addition and scalar multiplication in the normed space X are continuous i.e., if (x_n) and (y_n) are sequences in the normed space X with $x, y \in X$ such that $x_n \to x$, $y_n \to y$ in X, and (α_n) a sequence of scalars with $\alpha \in C$ such that $\alpha_n \to \alpha$ in $\boldsymbol{C}$ then $x_n + y_n \to x + y$ and $\alpha_n x_n \to \alpha x$ in X.

In fact, I. J. Maddox [11], B. K. Srivastava [17], J. K. Srivastava and N.P.Pahari [18,19], R. K. Tiwari and J. K. Srivastava [20, 21], and many others have been introduced and studied the algebraic and topological properties of various function spaces in normed space. All these function spaces generalize and unify various existing basic sequence spaces studied in Functional Analysis.

Definition 1.5: Let U be a normed space and $S(U) =\{ \phi : X \to U\}$ be the classes of U-valued functions. Then $S(U)$ is called *solid* **(***or normal***)** if $\phi \in S(U)$ and scalars $\alpha(x), x \in X$ such that $|\alpha(x)| \leq 1$, $x \in X$ implies

$$\alpha(x)\, \phi(x) \in S(U).$$

Corresponding to *K*–, *AK*–, *AD*– properties of scalar sequence spaces (see, [5] Kamthan and Gupta ,1981), for Banach space valued GK–, *GAK*–, *GAD*– ,*GC*– and *GFK*- function spaces (see, Gupta and Patterson,1982) , we have defined as follows:

Definition 1.6: Let U be a normed space and $S(U) =\{ \phi : X \to U\}$ be the class of U-valued functions. Then $S(U)$ is called a GK– function space if $P_x : S(U) \to U$, where $P_x(\phi) = \phi(x)$, is continuous for each $x \in X$. A *GK*– function space is called

(i) a GAD- function space if $\Phi(U)$ is dense in $S(U)$, where

$$\Phi(U) = \{\phi: X \to U \text{ such that } \phi(x) = \theta \text{ , for all but finitely many } x \in X \},$$

(ii) a *G AK*- function space if for each $\phi \in S(U)$ the directed system $(\phi_J)_{J \in F(X)}$, of j^{th} sections of ϕ with set theoretic inclusion converges to ϕ , where

$$s_J(\phi)(x) = \begin{cases} \phi(x) \text{ , if } x \in J \text{ and} \\ 0, \text{ if } x \in X / J. \end{cases}$$

(iii) a *GC*- function space if for each $x \in X$, the mapping $R_x : U \to S(U)$ is continuous , where $R_x(u) = \delta_x^u$ and $\delta_x^u : X \to U$ is defined by

$$\delta_x^u(y) = \begin{cases} u \text{ , if } x = y \text{ and} \\ \theta \text{ ,when } y \neq x. \end{cases}$$

(iv) a *GFK*- function space if $(S(U), \Im)$ is complete linear metric space.

2. THE CLASSES $\ell\,(X, U, \|\,.\,\|, \xi)$ AND $\ell\,(X, U, \|\,.\,\|, \xi, \lambda, p)$

Let X be an arbitrary non empty set (not necessarily countable) and $\mathcal{F}(X)$ be the collection of all finite subsets of X directed by inclusion relation. Let $(U, \|\,.\,\|)$ be a Banach space over the field of complex number **C.** Let p, q denotes the functions on $X \to \boldsymbol{R}^+$, the set of positive real numbers, and $\ell_\infty(X, \boldsymbol{R}^+) = \{ p : X \to \boldsymbol{R}^+$ such that $\sup_x p(x) < \infty\}$. Further we shall write λ, μ for functions on $X \to \boldsymbol{C} \setminus \{0\}$ and the collection of all such functions will be denoted by $s(X, \boldsymbol{C} \setminus \{0\})$.

We now introduce the following new classes of Banach space U – valued functions using Orlicz function ξ.

(i) $\ell\,(X, U, \|\,.\,\|, \xi) = \{ \phi : X \to U : \sum_{x \in X} \xi\left(\frac{\|\phi(x)\|}{\rho}\right) < \infty$ for some $\rho > 0$. …(2.1)

and

(ii) $\ell(X,U, \|\,.\,\|, \xi, \lambda, p) = \{\phi : X \to U$: for some $\rho > 0$, $\sum_{x \in X} \xi\left(\frac{\|\lambda(x)\, \phi(x)\|^{p(x)}}{\rho}\right)$ is summable $\}$. …(2.2)

Further when $\lambda : X \to \boldsymbol{C} \setminus \{0\}$ is a function such that $\lambda(x) = 1$ for all x, then $\ell(X, U, \|\,.\,\|, \xi, \lambda, p)$ will be denoted by $\ell(X, U, \|\,.\,\|, \xi, p)$ and when $p : X \to \boldsymbol{R}^+$ is a function such that $p(x) = 1$ for all x, then $\ell(X, U, \|\,.\,\|, \xi, \lambda, p)$ will be denoted by $\ell(X, U, \|\,.\,\|, \xi, \lambda)$.

If in the definition of ℓ $(X, U, \|\,.\,\|, \xi)$ the phrase 'for some $\rho > 0$' is replaced by 'for every $\rho > 0$' then we denote this subclass by $\overline{\ell}$ $(X, U, \|\,.\,\|, \xi)$. Thus

$$\overline{\ell}\,(X, U, \|\,.\,\|, \xi) = \{\, \phi : X \to U : \sum_{x \in X} \xi\left(\frac{\|\phi(x)\|}{\rho}\right) < \infty \text{ for every } \rho > 0\} \quad \ldots(2.3)$$

Similarly, we define the subclass $\overline{\ell}$ $(X, U, \|\,.\,\|, \xi, \lambda, p)$ of ℓ $(X, U, \|\,.\,\|, \xi, \lambda, p)$ as follows:

$$\overline{\ell}\,(X, U, \|\,.\,\|, \xi, \lambda, p) = \{\phi : X \to U : \text{for every } \rho > 0, \sum_{x \in X} \xi\left(\frac{\|\lambda(x)\,\phi(x)\|^{p(x)}}{\rho}\right) \text{ is summable}\}. \quad \ldots(2.4)$$

3. TOPOLOGICAL STRUCTURES OF ℓ $(X, U, \|\,.\,\|, \xi)$

As far as linear space structure of the class $\ell(X, U, \|\,.\,\|, \xi)$ over the field **C**, are concerned throughout we shall take point-wise vector operations,

i.e., $(\phi + \psi) = \phi(x) + g(x),\ x \in X$

and

$$(\alpha\,\phi)(x) = \alpha\phi(x),\ x \in X,\ \alpha \in \mathrm{C}.$$

Moreover, we shall denote zero element of these spaces by θ by which we mean the function $\theta : X \to U$ such that $\theta(x) = 0$ for all $x \in X$.

Theorem 3.1: $\ell(X, U, \|\,.\,\|, \xi)$ **is a linear space over the field *C* with respect to the pointwise vector operations.**

Proof:

Let $\phi,\ \psi \in \ell(X, U, \|\,.\,\|, \xi)$ and $\alpha, \beta \in \boldsymbol{C}$. So there exist $\rho_1 > 0$ and $\rho_2 > 0$ such that

$$\sum_{x \in X} \xi\left(\frac{\|\phi(x)\|}{\rho_1}\right) < \infty \text{ and } \sum_{x \in X} \xi\left(\frac{\|\psi(x)\|}{\rho_2}\right) < \infty.$$

Let $\rho_3 = \max(2|\alpha|\,\rho_1, 2|\beta|\rho_2)$. Since ξ is non-decreasing and convex. So

$$\xi\left(\frac{\|\alpha\,\phi(x) + \beta\,\psi(x)\|}{\rho_3}\right) \le \xi\left(\frac{\|\alpha\,\phi(x)\|}{\rho_3} + \frac{\beta\,\psi(x)\|}{\rho_3}\right)$$

$$\le \frac{1}{2}\,\xi\left(\frac{\|\phi(x)\|}{\rho_1}\right) + \frac{1}{2}\,\xi\left(\frac{\|\psi(x)\|}{\rho_2}\right) < \infty.$$

This shows that $\alpha\,\phi + \beta\psi \in \ell\,(X, U, \|\,.\,\|, \xi)$.

Hence, $\ell(X, U, \|\,.\,\|, \xi)$ is a linear space. This completes the proof.

Theorem 3.2: If ξ satisfies the Δ_2 – condition then $\overline{\ell}$ $(X, U, \|\,.\,\|, \xi) = \ell$ $(X, U, \|\,.\,\|, \xi)$.

Proof:

Here we prove that $\ell(X, U, \|\,.\,\|, \xi) \subset \overline{\ell}$ $(X, U, \|\,.\,\|, \xi)$ since the reverse inclusion is always true.

Let $\phi \in \ell(X, U, \|\,.\,\|, \xi)$. Then for some $\rho > 0$,

$$\sum_{x \in X} \xi\left(\frac{\|\phi(x)\|}{\rho}\right) < \infty$$

so there exists $J \in F(X)$ such that

$$\xi\left(\frac{\|\phi(x)\|}{\rho}\right) \le 1, \text{ for each } X \in X / J. \quad \ldots(3.1)$$

Choose an arbitrary $\eta > 0$. If $\rho \le \eta$, then $\sum_{x \in X} \xi\left(\frac{\|\phi(x)\|}{\rho}\right) < \infty$ follows easily.

Now suppose, $\eta < \rho$ and put $K = \frac{\rho}{\eta}$. One can determine $R \equiv R_K > 0$ and $r \equiv r_K > 0$ with $\xi(Kt) \le R\,\xi(t)$ for all t in $(0, r]$. Now by (3.1) we easily get

$$\xi\left(\frac{\|\phi(x)\|}{\rho}\right) < \frac{1}{2}\,rq\left(\frac{r}{2}\right), \text{ for all } x \in X / J_1, J_1 \in F(X), J_1 \cap J = \phi \quad \ldots(3.2)$$

where q is the Kernel associated with ξ and the last inequalities yields

$$\frac{\|\phi(x)\|}{\rho} \le r, \text{ for all } x \in X / J_1. \quad \ldots(3.3)$$

For otherwise, we can find $J_2 \supset J_1$ with $\frac{\|\phi(x)\|}{\rho} > r$, and thus

$$\xi\left(\frac{\|\phi(x)\|}{\rho}\right) \ge \int_{r/2}^{\|\phi(x)\| / \rho} q(t)\,dt > \frac{1}{2}\,rq\left(\frac{r}{2}\right),$$

which contradicts (3.2). Using (3.3) one finds that

$$\sum_{x \in X/J} \xi\left(\frac{\| \phi(x) \|}{\eta}\right) \leq \sum_{x \in X/J} \xi\left(\frac{\| \phi(x) \|}{\rho}\right)$$

and hence $\sum_{x \in X} \xi\left(\frac{\| \phi(x) \|}{\eta}\right) < \infty$ for every $\eta > 0$ i.e.; $f \in \bar{\ell}\ (X, U, \| \cdot \|, \xi)$.

This completes the proof.

The following Theorem is the immediate consequence of Theorem 3.1, proved by Srivastava and Pahari (2011).

Theorem 3.3: If U is a Banach Space, then $\ell(X, U, \| \cdot \|, \xi)$ is a complete normed space with respect to the norm

$$\| f \|_1 = \inf\left\{\rho > 0: \sum_{x \in X} \xi\left(\frac{\| f(x) \|}{\rho}\right) \leq 1\right\}. \qquad \ldots(3.4)$$

Theorem 3.4: If U is a Banach space, then $\bar{\ell}\ (X, U, \| \cdot \|, \xi)$ is a GK - function space.

Proof:

Let $f \in \bar{\ell}\ (X, U, \| \cdot \|, \xi)$. Then from the definition of norm, we have

$$\sum_{x \in X}^{\infty} \xi\left(\frac{\| \phi(x) \|}{\| \phi \|_1}\right) \leq 1$$

$$\text{i.e.,} \quad \xi\left(\frac{\| \phi(x) \|}{\| \phi \|_1}\right) \leq 1, \text{ for each } x \in X.$$

One can find fixed real numbers $r > 1$ and $t_0 > 0$ such that

$$r\left(\frac{t_0}{2}\right) q\left(\frac{t_0}{2}\right) \geq 1$$

where, q is the kernel associated with ξ. Hence for each $x \in X$

$$\xi\left(\frac{\| \phi(x) \|}{\| \phi \|_1}\right) \leq r\left(\frac{t_0}{2}\right) q\left(\frac{t_0}{2}\right)$$

which by integral representation gives

$$\frac{\| \phi\,(x) \|}{\| \phi \|_1} \leq rt_0$$

$$\text{i.e.,} \quad \| \phi(x) \| \leq rt_0 \| \phi \|_1.$$

Hence the continuity of projection map $P_x : \bar{\ell}\ (X, U, \| \cdot \|, M) \to U$, where $P_x(\phi) = \phi(x)$, follows from

$$\| P_x(\phi) \| = \| \phi(x) \| \leq rt_0 \| \phi \|_1.$$

Thus, $\bar{\ell}\ (X, U, \| \cdot \|, M)$ is a GK- function space.

Theorem 3.5: If U is a Banach space, then $\bar{\ell}\ (X, U, \| \cdot \|, \xi)$ is a GFK - function space.

Proof:

Since $\ell(X, U, \| \cdot \|, \xi)$ is a complete normed space under the norm given by

$$\| f \|_1 = \inf\left\{\rho > 0: \sum_{x \in X} \xi\left(\frac{\| f(x) \|}{\rho}\right) \leq 1\right\},$$

and $\bar{\ell}\ (X, U, \| \cdot \|, \xi)$ is a subspace of $\ell(X, U, \| \cdot \|, \xi)$, so to show that $\bar{\ell}\ (X, U, \| \cdot \|, \xi)$ is complete under the norm $\| \cdot \|_1$, it is sufficient to show that it is closed. For this let us consider (ϕ_n) sequence in $\bar{\ell}\ (X, U, \| \cdot \|, \xi)$ such that $\| \phi_n - \phi \|_1 \to 0$, $n \to \infty$, where $\phi \in \ell(X, U, \| \cdot \|, \xi)$. So for given $\rho > 0$, we can choose an integer n_0 such that,

$$\| \phi_n - \phi \|_1 < \frac{\rho}{2}, \text{ for all } n > n_0.$$

Consider,

$$\xi\left(\frac{\| \phi(x) \|}{\rho}\right) \leq \frac{1}{2}\xi\left(\frac{2 \| \phi_n(x) - \phi(x) \|}{\rho}\right) + \frac{1}{2}\xi\left(\frac{2 \| \phi_n(x) \|}{\rho}\right)$$

$$\leq \frac{1}{2}\xi\left(\frac{\| \phi_n(x) - \phi(x) \|}{\| \phi_n - \phi \|_1}\right) + \frac{1}{2}\xi\left(\frac{2 \| \phi(x) \|}{\rho}\right)$$

Since, $\sum_{x \in X} \xi\left(\frac{\| \phi_n(x) - \phi(x) \|}{\| \phi_n - \phi \|_1}\right) \leq 1$ and $\sum_{x \in X} \xi\left(\frac{2 \| \phi_n(x) \|}{\rho}\right) < \infty$, so

$$\sum_{x \in X} \xi\left(\frac{\| \phi_n(x) \|}{\rho}\right) < \infty.$$

This implies $f \in \bar{\ell}\ (X, U, \| \cdot \|, \xi)$. Hence $\bar{\ell}\ (X, U, \| \cdot \|, \xi)$ is complete.

Theorem 3.6: If U is a Banach space, then $\bar{\ell}\ (X, U, \| \cdot \|, \xi)$ is a AK –function space.

Proof:

Let $\phi \in \overline{\ell}\ (X, U, \xi)$. For each $\varepsilon > 0$, one can find a $J \in F(X)$, such that

$$\sum_{x \in X/J} \xi\left(\frac{\|\phi(x)\|}{\varepsilon}\right) \le 1.$$

Hence for $J_1 \supset J$,

$$\|\phi - s_{J1}(\phi)\|_1 \le \inf\{\rho > 0: \sum_{x\in X/J_1} \xi\left(\frac{\|\phi(x)\|}{\rho}\right) \le 1\}$$

i.e., $\|\phi - s_{J1}(\phi)\|_1 < \varepsilon$

where, $$s_J(\phi)(x) = \begin{cases} \phi(x), & \text{if } x \in J \text{ and} \\ 0, & \text{if } x \in X/J. \end{cases}$$

Thus, $(s_J(\phi))_{J\in F(X)}$ converge to f. Hence $\overline{\ell}\ (X, U, \|\cdot\|, \xi)$ is an AK - function space.

Theorem 3.7: If U is a Banach space, then $\overline{\ell}\ (X, U, \|\cdot\|, \xi)$ is a GAD -function space.
Proof:

Let $\phi \in \overline{\ell}\ (X, U, \|\cdot\|, \xi)$. Then for every $\varepsilon > 0$,

$$\sum_{x\in X} \xi\left(\frac{\|\phi(x)\|}{\rho}\right) \le 1.$$

Now for the function $\phi : X \to U$ such that $\phi(x) = \theta$, for all but finitely many $x \in X$ belonging to $\Phi(U)$

$$\|\phi\|_1 = \inf\{\rho > 0 : \sum_{x\in X} \xi\left(\frac{\|\phi(x)\|}{\rho}\right) \le 1\} < \varepsilon.$$

Hence, $\Phi(U)$ is dense in $\overline{\ell}\ (X, U, \|\cdot\|, \xi)$. Thus, $\overline{\ell}\ (X, U, \|\cdot\|, \xi)$ being a GK – space is a GAD – space.

Theorem 3.8: If U is a Banach space, then $\overline{\ell}\ (X, U, \|\cdot\|, \xi)$ is a GC - function space.
Proof:

Let $R_x : U \to \overline{\ell}\ (X, U, \|\cdot\|, \xi)$ defined by $R_x(u) = \overset{u}{\delta_x}$, where,

$$\overset{u}{\delta_x}(y) = \begin{cases} u, & \text{if } x = y \text{ and} \\ 0, & \text{otherwise.} \end{cases}$$

Since, $\overset{u}{\delta_x} \in \overline{\ell}\ (X, U, \|\cdot\|, \xi)$. Then for each $\varepsilon > 0$, we have

$$\sum_{x\in X} \xi\left(\frac{\|\overset{u}{\delta_x}(y)\|}{\varepsilon}\right) \le 1$$

$$\text{i.e,} \sum_{x\in X} \xi\left(\frac{\|u\|}{\varepsilon}\right) \le 1.$$

$$\text{Now, } \|\overset{u}{\delta_x}\|_1 = \inf\{\rho > 0 : \sum_{x\in X} \xi\left(\frac{\|\overset{u}{\delta_x}(y)\|}{\rho}\right) \le 1\}$$

$$= \inf\{\varepsilon > 0: \sum_{x\in X} \xi\left(\frac{\|u\|}{\varepsilon}\right) \le 1\} < \varepsilon$$

i.e., $\|\overset{u}{\delta_x}\|_1 < \varepsilon$.

Hence, $\|R_x(u)\|_1 = \|\overset{u}{\delta_x}\|_1 < \varepsilon$

which shows that R_x is continuous for each $x \in X$. Hence, $\overline{\ell}\ (X, U, \|\cdot\|, \xi)$ is a GC – function space.

4. CONTAINMENT RELATIONS AND LINEARITY OF $\ell\ (X, U, \|\cdot\|, \xi, \lambda, p)$

In this section, we explore some of the conditions pertaining to the containment relation between the classes $\ell\ (X, U, \|\cdot\|, \xi, \lambda, p)$ in terms of different values of λ and p. Throughout, we shall also frequently use the notation

$$t(x) = \left|\frac{\lambda(x)}{\mu(x)}\right|^{p(x)}.$$

Theorem 4.1: Let $p \in \ell_\infty(X, R^+)$. Then for any $\lambda, \mu \in s\ (X, C \setminus \{0\})$,
$\ell\ (X, U, \|\cdot\|, \xi, \lambda, p) \subset \ell\ (X, U, \|\cdot\|, \xi, \mu, p)$ if and only if $\liminf_x t(x) > 0$.

Proof:
For the sufficiency, suppose that $\liminf_x t(x) > 0$. Then there exists $m > 0$ such that

$$m\ |\mu(x)|^{p(x)} < |\lambda(x)|^{p(x)}$$

for all but finitely many $x \in X$.

Let $\phi \in \ell(X, U, \|.\|, \xi, \lambda, p)$, $\rho_1 > 0$ be associated with ϕ. Then we have

$$\sum_{x \in X} \xi\left(\frac{\|\lambda(x)\ \phi(x)\|^{p(x)}}{\rho_1}\right) \text{ is summable.}$$

Let us choose $\rho > 0$ such that $\rho_1 < m\rho$. Since ξ is non decreasing, so that

$$\begin{aligned}\xi\left(\frac{\|\mu(x)\ \phi(x)\|^{p(x)}}{\rho}\right) &= \xi\left(\frac{[\,|\mu(x)|\ \|\phi(x)\|\,]^{p(x)}}{\rho}\right)\\ &\le \xi\left(\frac{[\,|\lambda(x)|\ \|\phi(x)\|\,]^{p(x)}}{m\rho}\right)\\ &\le \xi\left(\frac{\|\lambda(x)\ \phi(x)\|^{p(x)}}{\rho_1}\right),\end{aligned}$$

for all but finitely many $x \in X$ and therefore $\sum_{x \in X} \xi\left(\frac{\|\mu(x)\ \phi(x)\|^{p(x)}}{\rho}\right)$ is summable. It clearly shows that $\phi \in \ell(X, U, \|.\|, \xi, \mu, p)$ and hence $\ell(X, U, \|.\|, \xi, \lambda, p) \subset \ell(X, U, \|.\|, \xi, \mu, p)$.

Conversely, assume that $\ell(X, U, \|.\|, \xi, \lambda, p) \subset \ell(X, U, \|.\|, \xi, \mu, p)$ but $\liminf_x t(x) = 0$. Then there exists a sequence (x_k) of distinct points in X such that for each $k \ge 1$,

$$k^3\,|\lambda(x_k)|^{p(x_k)} < |\mu(x_k)|^{p(x_k)}. \qquad \text{...(4.1)}$$

We now choose $u \in U$ such that $\|u\| = 1$ and define $\phi : X \to U$ by

$$\phi(x) = \begin{cases}(\lambda(x_k))^{-1}\, k^{-2/p(x_k)}\, u, & \text{if } x = x_k,\ k \ge 1, \text{and}\\ \theta, & \text{otherwise.}\end{cases} \qquad \text{...(4.2)}$$

Let $\rho > 0$. Then we have

$$\begin{aligned}M\left(\frac{\|\lambda(x)\ \phi(x)\|^{p(x)}}{\rho}\right) &= \xi\left(\frac{\|\lambda(x_k)\ \phi(x_k)\|^{p(x_k)}}{\rho}\right)\\ &= \xi\left(\frac{\|k^{-2/p(x_k)}\ u\|^{p(x_k)}}{\rho}\right)\\ &\le \frac{1}{k^2}\,\xi\left(\frac{\|u\|^{p(x_k)}}{\rho}\right)\\ &= \frac{1}{k^2}\,\xi\left(\frac{1}{\rho}\right).\end{aligned}$$

This implies that

$$\sum_{x\in X} \xi\left(\frac{\|\lambda(x)\ \phi(x)\|^{p(x)}}{\rho}\right) \le \xi\left(\frac{1}{\rho}\right)\sum_{k=1}^{\infty}\frac{1}{k^2} < \infty,$$

and shows that $\phi \in \ell(X, U, \|.\|, \xi, \lambda, p)$. But in view of (4.1) and (4.2),

$$\begin{aligned}\sum_{x\in X} \xi\left(\frac{\|\mu(x)\ \phi(x)\|^{p(x)}}{\rho}\right) &= \sum_{k=1}^{\infty} \xi\left(\frac{\|\mu(x_k)\ \phi(x_k)\|^{p(x_k)}}{\rho}\right)\\ &= \sum_{k=1}^{\infty} \xi\left(\frac{\|(\mu(x_k)\,(\lambda(x_k))^{-1}\,k^{-2/p(x_k)}\,u)\|^{p(x_k)}}{\rho}\right)\\ &= \sum_{k=1}^{\infty} \xi\left(\frac{\left|\frac{\mu(x_k)}{\lambda(x_k)}\right|^{p(x_k)} \frac{1}{k^2}\,\|u\|^{p(x_k)}}{\rho}\right)\\ &\ge \sum_{k=1}^{\infty} \xi\left(\frac{k}{\rho}\right),\end{aligned}$$

implies that $\phi \notin \ell(X, U, \|.\|, \xi, \mu, p)$, a contradiction. This completes the proof.

Theorem 4.2: Let $p \in \ell_\infty(X, R^+)$, then for any $\lambda, \mu \in s(X, C \setminus \{0\})$,

$\ell(X, U, \|.\|, \xi, \mu, p) \subset \ell(X, U, \|.\|, \xi, \lambda, p)$ if and only if $\limsup_x t(x) < \infty$.

Proof:

For the sufficiency of the condition, assume that $\limsup_x t(x) < \infty$. Then there exists $d > 0$ such that

$$|\lambda(x)|^{p(x)} < d\,|\mu(x)|^{p(x)}$$

for all but finitely many $x \in X$. Let $\phi \in \ell(X, U, \|.\|, \xi, \mu, p)$, $\rho_1 > 0$ be associated with ϕ. So that

$$\sum_{x\in X} \xi\left(\frac{\|\mu(x)\,\phi(x)\|}{\rho_1}^{p(x)}\right) \text{ is summable.}$$

Let us choose $\rho > 0$ such that $d\rho_1 < \rho$. Since ξ is non decreasing, we have

$$\xi\left(\frac{\|\lambda(x)\,\phi(x)\|}{\rho}^{p(x)}\right) \le \xi\left(\frac{d\,[\,|\mu(x)|\,\|\phi(x)\|\,]}{\rho}^{p(x)}\right)$$

$$\le \xi\left(\frac{\|\mu(x)\,\phi(x)\|}{\rho_1}^{p(x)}\right)$$

for all but finitely many $x \in X$ and therefore $\sum_{x\in X} \xi\left(\frac{\|\lambda(x)\,\phi(x)\|}{\rho}^{p(x)}\right)$ is summable. This shows that $\phi \in \ell(X, U, \|\,.\,\|, \xi, \lambda, p)$ and hence

$$\ell(X, U, \|\,.\,\|, \xi, \mu, p) \subset \ell(X, U, \|\,.\,\|, \xi, \lambda, p).$$

Conversely, suppose that $\ell(X, U, \|\,.\,\|, \xi, \mu, p) \subset \ell(X, U, \|\,.\,\|, \xi, \lambda, p)$ but $\limsup_x t(x) = \infty$. Then there exists a sequence (x_k) of distinct points in X such that for all $k \ge 1$,

$$|\lambda(x_k)|^{p(x_k)} > k|\mu(x_k)|^{p(x_k)}. \qquad \ldots(4.3)$$

Let us choose $u \in U$ such that $\|u\| = 1$ and define $\phi : X \to U$ by

$$\phi(x) = \begin{cases} (\mu(x_k))^{-1}\, k^{-2/p(x_k)}\, u \text{ , if } x = x_k,\ k \ge 1, \text{and} \\ \theta, \text{ otherwise.} \end{cases} \qquad \ldots(4.4)$$

Let $\rho > 0$. Then we have

$$\xi\left(\frac{\|\mu(x)\,\phi(x)\|}{\rho}^{p(x)}\right) = \xi\left(\frac{\|\mu(x_k)\,\phi(x_k)\|}{\rho}^{p(x_k)}\right)$$

$$= \xi\left(\frac{\|k^{-2/p(x_k)}\, u\|}{\rho}^{p(x_k)}\right)$$

$$\le \frac{1}{k^2}\,\xi\left(\frac{\|u\|}{\rho}^{p(x_k)}\right)$$

$$= \frac{1}{k^2}\,\xi\left(\frac{1}{\rho}\right).$$

This implies that

$$\sum_{x\in X} \xi\left(\frac{\|\mu(x)\,\phi(x)\|}{\rho}^{p(x)}\right) \le \xi\left(\frac{1}{\rho}\right) \sum_{k=1}^{\infty} \frac{1}{k^2} < \infty,$$

and shows that $\phi \in \ell(X, U, \|\,.\,\|, \xi, \mu, p)$. But in view of (4.3) and (4.4),

$$\sum_{x\in X} \xi\left(\frac{\|\lambda(x)\,\phi(x)\|}{\rho}^{p(x)}\right) = \sum_{k=1}^{\infty} \xi\left(\frac{\|\lambda(x_k)\,\phi(x_k)\|}{\rho}^{p(x_k)}\right)$$

$$= \sum_{k=1}^{\infty} \xi\left(\frac{\|(\lambda(x_k)\,(\mu(x_k))^{-1}\, k^{-2/p(x_k)}\, u)\|}{\rho}^{p(x_k)}\right)$$

$$= \sum_{k=1}^{\infty} \xi\left(\frac{\left|\frac{\lambda(x_k)}{\mu(x_k)}\right|^{p(x_k)} \frac{1}{k^2}\,\|u\|^{p(x_k)}}{\rho}\right)$$

$$\ge \sum_{k=1}^{\infty} \xi\left(\frac{k}{\rho}\right),$$

implies that $\phi \notin \ell(X, U, \|\,.\,\|, \xi, \lambda, p)$, a contradiction. This completes the proof.

If the Theorems 4.1 and 4.2 are combined, we get:

Theorem 4.3: If $p \in \ell_\infty(X, R^+)$. Then for any $\lambda, \mu \in s(X, C \setminus \{0\})$,

$\ell(X, U, \|\,.\,\|, \xi, \lambda, p) = \ell(X, U, \|\,.\,\|, \xi, \mu, p)$ if and only if

$0 < \liminf_x t(x) \le \limsup_x t(x) < \infty$.

Corollary 4.4: For $p \in \ell_\infty(X, R^+)$ and $\lambda \in s(X, C \setminus \{0\})$. Then

(i) $\ell(X, U, \|\,.\,\|, \xi, \lambda, p) \subset \ell(X, U, \|\,.\,\|, \xi, p)$ **iff** $\liminf_x |\lambda(x)|^{p(x)} > 0$;

(ii) $\ell(X, U, \|\,.\,\|, \xi, p) \subset \ell(X, U, \|\,.\,\|, \xi, \lambda, p)$ **iff** $\limsup_x |\lambda(x)|^{p(x)} < \infty$; **and**

(iii) $\ell(X, U, \|\,.\,\|, \xi, \lambda, p) = \ell(X, U, \|\,.\,\|, \xi, p)$ **iff**

$0 < \liminf_x |\lambda(x)|^{p(x)} \le \limsup_x |\lambda(x)|^{p(x)} < \infty$.

Proof:

If we take $\mu(x) = 1$ for each x in Theorems 4.1 , 4.2 and 4.3, we easily get the assertions (i), (ii) and (iii) respectively.

Theorem 4.5: For $p, q \in \ell_\infty (X, R^+)$ and $\lambda \in s (X, C \setminus \{0\})$, if $p(x) \le q(x)$ for all but finitely many $x \in X$, then $\ell(X, U, \| . \|, \xi , \lambda, p) \subset \ell(X, U, \| . \|, \xi, \lambda, q)$.

Proof:

Assume that $p(x) \le q(x)$ for all but finitely many $x \in X$.
Let $\phi \in \ell(X, U, \| . \|, \xi, \lambda, p)$, $\rho > 0$ be associated with ϕ , so that

$$\sum_{x\in X} \xi\left(\frac{\| \lambda(x)\, \phi(x) \|^{p(x)}}{\rho}\right) \text{ is summable.}$$

This implies that there exists a $J \in \mathcal{F}(X)$ such that

$$\xi\left(\frac{\| \lambda(x)\, \phi(x) \|^{p(x)}}{\rho}\right) \le \xi\left(\frac{1}{\rho}\right) \text{ for all } x \in X\setminus J$$

i.e., $\| \lambda(x)\, \phi(x) \| \le 1$ for all $x \in X\setminus J$. This clearly implies that

$$\left(\frac{\| \lambda(x)\, \phi (x) \|^{q(x)}}{\rho}\right) \le \left(\frac{\| \lambda(x)\, \phi(x) \|^{p(x)}}{\rho}\right).$$

Since ξ is non decreasing, therefore

$$\sum_{x \in X/J} \xi\left(\frac{\| \lambda(x)\, \phi (x) \|^{q(x)}}{\rho}\right) \le \sum_{x \in X/J} \xi\left(\frac{\| \lambda(x)\, \phi(x) \|^{p(x)}}{\rho}\right).$$

This proves that $\phi \in \ell(X, U, \| . \|, \xi, \lambda, q)$ and hence

$$\ell(X, U, \| . \|, \xi, \lambda, p) \subset \ell(X, U, \| . \|, \xi, \lambda, q).$$

After combining the Theorems 4.1 and 4.5, we get

Theorem 4.6: For $p, q \in \ell_\infty (X, R^+)$ and $\lambda, \mu \in s (X, C \setminus \{0\})$.
If (i) $\liminf_x t(x) > 0$; and
(ii) $p(x) \le q(x)$, for all but finitely many $x \in X$, then
$\ell (X, U, \| . \|, \xi, \lambda, p) \subset \ell (X, U, \| . \|, \xi, \mu, q)$.

In the following example, inspite of satisfaction of conditions (i) and (ii) of Theorem 4.6,we shall illustrate that the containment of $\ell(X, U, \| . \|, \xi, \lambda, p)$ in $\ell(X, U, \| . \|, \xi, \mu, q)$ may be strict.

Example 4.7:

Let (x_k) be a sequence of distinct points of X. Take $u \in U$ such that $\|u\| = 1$ and define $\phi : X \to U$ by the equation

$$\phi(x) = \begin{cases} k^{-3k}\, u, & \text{if } x = x_k,\ k = 1, 2, 3, \dots \text{,and} \\ \theta, & \text{otherwise.} \end{cases}$$

Further if $x = x_k$ and consider $p(x_k) = k^{-1}$, if k is odd ; $p(x_k) = k^{-2}$, if k is even.
$q(x_k) = k^{-1}$, $\lambda (x_k) = 3^k$, $\mu (x_k) = 2^k$ for all values of k, and $p(x) = \frac{1}{2}$, $q(x) = 1$, $\lambda (x) = 3$, $\mu(x) = 2$ otherwise. Then for $x = x_k$, $k \ge 1$, we have

$$t(x_k) = \left| \frac{\lambda(x_k)}{\mu(x_k)} \right|^{p(x_k)} = \frac{3}{2}\text{, if } k \text{ is odd; } t(x_k) = \left(\frac{3}{2}\right)^{1/k} \text{ if } k \text{ is even and}$$

$$t(x) = \left(\frac{3}{2}\right)^{1/2} \text{ otherwise.}$$

Thus, we see that $\liminf_x t(x) > 0$ and $p(x) \le q(x)$ for all but finitely many $x \in X$ and hence the conditions (i) and (ii) of Theorem 4.6 are satisfied. Then we have

$$\sum_{x\in X} \xi\left(\frac{\| \mu(x)\, \phi (x) \|^{q(x)}}{\rho}\right) = \sum_{k=1}^{\infty} \xi\left(\frac{\| \mu(x_k)\, \phi(x_k) \|^{q(x_k)}}{\rho}\right)$$

$$= \sum_{k=1}^{\infty} \xi\left(\frac{\| 2^k k^{-3k} u \|^{1/k}}{\rho}\right)$$

$$\le \xi\left(\frac{2}{\rho}\right) \sum_{k=1}^{\infty} \frac{1}{k^3} < \infty,$$

and hence $\phi \in \ell(X, U, \| . \|, \xi, \mu, q)$. But if k is even integer, then

$$\xi\left(\frac{\| \lambda(x)\, \phi (x) \|^{p(x)}}{\rho}\right) = \xi\left(\frac{\| \lambda(x_k)\, \phi (x_k) \|^{p(x_k)}}{\rho}\right)$$

$$= \xi\left(\frac{\| 3^k k^{-3k} u \|^{1/k^2}}{\rho}\right)$$
$$= \xi\left(\frac{3^{1/k} k^{-3/k} \| u \|^{1/k^2}}{\rho}\right)$$
$$\geq \xi\left(\frac{k^{-3/k}}{\rho}\right) \geq \xi\left(\frac{1}{3\rho}\right)$$

and shows that $\sum_{x\in X} \xi\left(\frac{\| \lambda(x)\, \phi(x) \|^{p(x)}}{\rho}\right)$ is not summable and hence $\phi \notin \ell(X, U, \| . \|, \xi, \lambda, p)$.

Thus the containment of $\ell(X, U, \| . \|, \xi, \lambda, p)$ in $\ell(X, U, \| . \|, \xi, \mu, q)$ is strict.

Theorem 4.8: $\ell(X, U, \| . \|, \xi, \lambda, p)$ **is normal .**

Proof:

Let $\phi \in \ell(X, U, \| . \|, \xi, \lambda, p)$, $\rho > 0$ be associated with ϕ and $\varepsilon > 0$ .Then there exists a $J \in \mathcal{F}(X)$ such that

$$\xi\left(\frac{\| \lambda(x)\, \phi(x) \|^{p(x)/L}}{\rho}\right) < \varepsilon, \text{ for every } x \in X \setminus J.$$

This shows that

$$\sum_{x\in X} \xi\left(\frac{\| \lambda(x)\ \phi(x) \|^{p(x)}}{\rho_1}\right) \text{ is summable.}$$

Now, if we take scalars $\alpha(x)$, $x \in X$ such that $|\alpha(x)| \leq 1$, then

$$\xi\left(\frac{\| \alpha(x)\, \lambda(x)\, \phi(x) \|^{p(x)/L}}{\rho}\right) \leq \xi\left(\frac{|\alpha(x)|^{p(x)/L} \| \lambda(x)\, \phi(x) \|^{p(x)/L}}{\rho}\right)$$
$$\leq \xi\left(\frac{\| \lambda(x)\, \phi(x) \|^{P(x)/L}}{\rho}\right) < \varepsilon,$$

and hence $\sum_{x\in X} \xi\left(\frac{\| \alpha(x)\, \lambda(x)\, \phi(x) \|^{p(x)/L}}{\rho}\right)$ is summable.

This shows that $\alpha\phi \in \ell(X, U, \| . \|, \xi, \lambda, p)$ and hence $\ell(X, U, \| . \|, \xi, \lambda, p)$ is normal.

Theorem 4.9: $\ell(X, U, \| . \|, \xi, \lambda, p)$ **forms a linear spaces over** $\boldsymbol{C}$**.**

Proof:

Let $\phi, \psi \in \ell(X, U, \| . \|, \xi, \lambda, p)$, $\rho_1 > 0$ and $\rho_2 > 0$ are associated with ϕ and ψ respectively and $\alpha, \beta \in \boldsymbol{C}$. Then we have

$$\sum_{x\in X} \xi\left(\frac{\| \lambda(x)\, \phi(x) \|^{p(x)/L}}{\rho_1}\right) < \infty;$$
$$\text{and} \sum_{x\in X} \xi\left(\frac{\| \lambda(x)\, \psi(x) \|^{p(x)/L}}{\rho_2}\right) < \infty.$$

We now set ρ such that $\rho \geq 2\rho_1 A[\alpha]$ and $\rho \geq 2\rho_2 A[\beta]$, where $A[t] = \max(1, |t|)$. Then for such ρ and using non decreasing and convex properties of ξ we have

$$\sum_{x\in X} \xi\left(\frac{\| \lambda(x)\, (\alpha\phi(x) + \beta\psi(x)) \|^{p(x)/L}}{\rho}\right)$$
$$\leq \sum_{x\in X} \xi\left(\frac{\| \lambda(x)\, \alpha\phi(x) + \| \lambda(x)\, \beta\psi(x) \|^{p(x)/L}}{\rho}\right)$$
$$\leq \sum_{x\in X} \xi\left(\frac{\| \lambda(x)\, \alpha\phi(x) \|^{p(x)/L}}{\rho} + \frac{\| \lambda(x)\, \beta\psi(x) \|^{p(x)/L}}{\rho}\right)$$
$$\leq \sum_{x\in X} \xi\left(\frac{|\alpha|^{p(x)/L}}{\rho} \| \lambda(x)\, \phi(x) \|^{p(x)/L} + \frac{|\beta|^{p(x)/L}}{\rho} \| \lambda(x)\, \psi(x) \|^{p(x)/L}\right)$$
$$\leq \sum_{x\in X} \xi\left(\frac{A[\alpha]}{\rho} \| \lambda(x)\, \phi(x) \|^{p(x)/L} + \frac{A[\beta]}{\rho} \| \lambda(x)\, \psi(x) \|^{p(x)/L}\right)$$
$$\leq \sum_{x\in X} \xi\left(\frac{1}{2\rho_1} \| \lambda(x)\, \phi(x) \|^{p(x)/L} + \frac{1}{2\rho_2} \| \lambda(x)\, \psi(x) \|^{p(x)/L}\right)$$
$$\leq \frac{1}{2} \sum_{x\in X} \xi\left(\frac{\| \lambda(x)\, \phi(x) \|^{p(x)/L}}{\rho_1}\right) + \frac{1}{2} \sum_{x\in X} \xi\left(\frac{\| \lambda(x)\, \psi(x) \|^{p(x)/L}}{\rho_2}\right)$$
$$< \infty.$$

This implies that $\alpha\phi + \beta\psi \in \ell(X, U, \| . \|, \xi, \lambda, p)$ and hence $\ell(X, U, \| . \|, \xi, \lambda, p)$ forms a linear space over $\boldsymbol{C}$. This completes the proof.

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