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ON EPIMORPHICALLY CLOSED HOMOTYPICAL PERMUTATIVE VARIETIES

Noor Mohammad Khan

Department of Mathematics
Aligarh Muslim University, Aligarh- 202002(INDIA).

Corresponding Author E-mail: khan_ noormohammad@redifmail.com

ABSTRACT
We find some sufficient conditions that a homotypical identity be preserved under epis of semigroups in conjunction with any nontrivial permutation identity.

INTRODUCTION
The general question of which identities are preserved under epis has been studied in semigroup theory, ring theory and elsewhere (Burgess, 1975). For example, in (Gardner, 1979) Gardner has shown that certain identities weaker than commutativity are not preserved under epis of rings although commutativity is preserved under epis of rings (Bulazewska, 1965). It has been shown by Higgins (Higgins, 1984), that identities for which both sides contain repeated variables are not preserved under epis of semigroups. In (Howie, 1967), Howie and Isbell have shown that commutativity is preserved under epis of semigroups. The author (Khan, 1982, Khan, 1985) has extended this result and shown that all identities in conjunction either with commutativity or semicommutative permutation identity are preserved under epis of semigroups. In (Clfford, 1967, Proof of Theorem 8.3), Higgins has shown that, in general, all homotypical identities for which both sides contain repeated variables are not preserved under epis in conjunction with any non trivial permutation identity. In (Khan, 1985, Theorem 4.7), the author has found some sufficient conditions that a homotypical identity containing repeated variables on both sides be preserved under epis of semigroups in conjunction with any nontrivial permutation identity. In this paper we extend further [Khan, 1985, Theorem 4.7(iii)] by finding some sufficient conditions that a homotypical identity containing repeated variables on both sides be preserved under epis of semigroups in conjunction with any nontrivial permutation identity. However, finding a complete determination of all identities containing repeated variables on both sides which are preserved under epis of semigroups still remains an open problem.[1]

Preliminaries:

Amorphisms α: A→B in the category **C** of semigroups is called an epimorphisms (epi for short) if $\forall$ C $\in$ ***C*** and for all morphisms β, γ: B → C, $\alpha\beta = \alpha\gamma$ implies $\beta = \gamma$. It can be easily verified that a morphism α: **S**→**T** is epi if and only if the inclusion map *i*: **S**α →**T** is epi, and the inclusion map *i*: **U** →**S** from any subsemigroup **U** of **S** is epi if and only if **Dom (U, S) = S**.

The following, Isbell's zigzag theorem, provides a most useful characterization of semigroup dominions.

Result 2.1([9,Theorem 2.3] or [7,Theorem VII.2.13]). Let **U** be a subsemigroup of any semigroup **S** and let d be any element of **S**. Then $d \in$ **Dom (U, S)** if and only if either $d\ \varepsilon$ **U** or there are elements $a_0, a_1, a_2, \ldots, a_{2m} \in$ **U**; $t_1, t_2, t_m, y_1, y_2, \ldots, y_m \in$ **S** such that

$$d = a_0 t_1, \qquad a_0 = y_1 a_1$$

$$a_{2i-1}t_i = a_{2i}t_{i+1}, \quad y_i a_{2i} = y_{i+1}a_{2i+1} \qquad (i = 1, 2,3, \ldots, m-1) \qquad (1)$$

$$a_{2m-1}t_m = a_{2m}, \qquad y_m a_{2m} = d.$$

These equations are called a zigzag of length m over U with value d and spine $a_0, a_1, a_2, \ldots, a_{2m}$..

An identity of the form

$$x_1x_2x_3\ldots x_n = x_{i1}x_{i2}\ldots x_{in} \qquad (n\geq 3),$$

is called a permutation identity, where i is any permutation of the set {1,2,...,n}. Again a permutation identity of the form

$$x_1x_2x_3\ldots x_n = x_{i1}x_{i2}\ldots x_{in} \qquad (2)$$

is called nontrivial if i is any nontrivial permutation of the set {1,2,...,n}. Further a permutation identity is said to be semicommutative if $i_1 \neq 1$ and $i_n \neq n$. A semigroup **S** is said to be permutative if it satisfies a nontrivial permutation identity (2) and a permutative semigroup **S** is said to be semicommutative if $i_1 \neq 1$ and $i_n \neq n$.

An identity u = v is said to be preserved under epis if for all semigroups

U and **S** with **U** a subsemigroup of **S** and such that **Dom(U, S) = S**, **U** satisfying u = v implies **S** satisfies u = v.

Result 2.2 ([12],Theorem 3.1) All permutation identities are preserved under epis.

Result 2.3 ([11], Result 3). Let **U** be any subsemigroup of a semigroup **S**. Then for any $d\ \varepsilon$ **Dom(U, S)\U**, if (1) be a zigzag of shortest possible length m over **U** with value d, then $y_j, t_j \in$ **S\U** $\forall$ j = 1,2,...,m.

In the following results, let **U** and **S** be any semigroups with **U** a subsemigroup of **S** and such that **Dom(U, S)** = **S**.

Result 2.4 ([11], Result 4). For any $d \in$ **S\U**, if (1) be a zigzag of shortest possible length m over **U** with value d and k be any positive integer, then there exist $a_1, a_2, \dots, a_k \in$ **U** and $d_k \in$ **S\U** such that $d = a_1 a_2 \dots a_k d_k$.

Result 2.5 ([11],Corollary 4.2). If **U** be permutative, then

$$s x_1 x_2 \dots x_k t = s x_{j1} x_{j2} \dots x_{jk} t$$

$\forall\ x_1, x_2, \dots x_k \in$ ***S***, s, t $\in$ **S\U**, and any permutation j of the set {1,2,...,k}.

Result 2.6 ([12], Proposition 4.6). Let **U** be a permutative semigroup. If $d \in$ **S\U** and (1) be a zigzag of length m over **U** with value d and with $y_1 \in$ **S\U**(for example if the zigzag (1) is of the shortest possible length), then $d_k = a_0^k t_1^k$ for any positive integer k.

The notations and conventions of Cllifford and Preston or Howie will be used throughout without explicit mention.

3. Main Result: An identity $u = v$ is said to be homotypical if C(u) = C(v)*;* where C(u), for any word u, is the set of all variables appearing in u; otherwise heterotypical.

Theorem 3.1. Let (2) be any nontrivial permutation identity. Then any nontrivial homotypical identity I (one which is not satisfied by the class of all semigroups) of the following form is preserved under epis in conjunction with (2):

$$x_1^{p_1} x_2^{p_2} \dots x_r^{p_r} = x_1^{q_1} x_2^{q_2} \dots x_r^{q_r}, \qquad (*)$$

where $0 < p_r \le p_{r-1} \le \dots \le p_2 \le p_1$ and $0 < q_1 \le q_2 \le \dots \le q_{r-1} \le q_r$ and $r \ge 0$.

Proof: Take any semigroups **U** and **S** with **U** epimorphically embedded in **S**, and such that **U**(and hence, **S**, by Result 2.2) satisfies the identity (2). We show that the identity($*$) satisfied by **U** is also satisfied by **S**. Assume that **U** satisfies the given identity($*$).

For k = 1,2,...,r; consider the word $x_1^{p_1} x_2^{p_2} \dots x_k^{p_k}$ of length $p_1 + p_2 + \dots + p_k$. We shall prove that **S** satisfies ($*$) by induction on k , assumming that the remaining elements $x_{k+1}, x_{k+2}, \dots, x_r \in$ **U**. First for k = 0, the identity is satisfied by **S** vacuously. So assume next that the identity($*$) is satisfied $\forall\ x_1, x_2, \dots, x_{k-1} \in$ **S** and $\forall\ x_k, x_{k+1}, \dots, x_r \in$ **U**. Without loss we can assume that $x_k \in$ **S\U**. As $x_k \in$ **S\U** and **Dom(U, S) = S**, by Result 2.1, we may let (1) be a zigzag of shortest possible length m over **U** with value x_k. We assume first that 1 < k < r.

Now

$$x_1^{p_1} x_2^{p_2} \dots x_r^{p_r} = x_1^{p_1} x_2^{p_2} \dots x_{k-1}^{p_{k-1}} a_0^{p_k} t_1^{p_k} x_{k+1}^{p_{k+1}} \dots x_r^{p_r} \qquad \text{(by eq. (1) and Result 2.6)}$$

= $x_1^{p_1}x_2^{p_2} \ldots x_{k-1}^{p_{k-1}}a_0^{p_k}b_{k+1}^{(1)p_k}b_{k+2}^{(1)p_k}\ldots b_r^{(1)p_k}t_1^{(1)p_k}x_{k+1}^{p_{k+1}}\ldots x_r^{p_r}$

(by eq. (1) and Results 2.4 and 2.5, for some $b_{k+1}^{(1)}, b_{k+2}^{(1)}, b_r^{(1)} \in$ **U** and $t_1^{(1)} \in$ **S\U**)

= $x_1^{p_1}x_2^{p_2} \ldots x_{k-1}^{p_{k-1}}a_0^{p_k}b_{k+1}^{(1)p_{k+1}}b_{k+2}^{(1)p_{k+2}}\ldots br^{(1)p_r}w^{(1)}t_1^{(1)p_k}z$

(where $w^{(1)} = b_{k+1}^{(1)\{p_k-p_{k+1}\}}b_{k+2}^{(1)\{p_k-p_{k+2}\}}\ldots b_r^{(1)\{p_k-p_r\}}$ and $z = x_{k+1}^{p_{k+1}}\ldots x_r^{p_r}$, by Result 2.5)

= $x_1^{q_1}x_2^{q_2}\ldots x_{k-1}^{q_{k-1}}a_0^{q_k}b_{k+1}^{(1)q_{k+1}}b_{k+2}^{(1)q_{k+2}}\ldots b_r^{(1)q_r}w^{(1)}t_1^{(1)p_k}z$

(by inductive hypothesis as $a_0 \in$ **U**)

= $vy_1^{(1)q_k}\ c_1^{(1)q_k}c_2^{(1)q_k}\ \ldots c_{k-1}^{(1)q_k}a_1^{q_k}b_{k+1}^{(1)q_{k+1}}b_{k+2}^{(1)q_{k+2}}\ldots b_r^{(1)q_r}w^{(1)}t_1^{(1)p_k}z$ (by eq.(1) and Result 2.5 and dual of Result 2.4, for some $c_1^{(1)},c_2^{(1)},\ldots,c_{k-1}^{(1)} \in$ **U**; where $v = x_1^{q_1}x_2^{q_2}\ldots x_{k-1}^{q_{k-1}}$ and $y_1^{(1)}, t_1^{(1)} \in$ **S\U**)

= $vy_1^{(1)q_k}v^{(1)}c_1^{(1)q_1}c_2^{(1)q_2}\ldots c_{k-1}^{(1)q_{k-1}}a_1^{q_k}b_{k+1}^{(1)q_{k+1}}b_{k+2}^{(1)q_{k+2}}\ldots b_r^{(1)q_r}w^{(1)}t_1^{(1)p_k}z$ (by Result 2.5, where $v^{(1)} = c_1^{(1)\{q_k-q_1\}}\ c_2^{(1)\{q_k-q_2\}}\ \ldots c_{k-1}^{(1)\{q_k-q_{k-1}\}}$ as $y_1^{(1)}, t_1^{(1)} \in$ **S\U**

=$y_1^{(1)q_k}v^{(1)}c_1^{(1)p_1}c_2^{(1)p_2}\ldots c_{k-1}^{(1)p_{k-1}}a_1^{p_k}b_{k+1}^{(1)p_{k+1}}b_{k+2}^{(1)p_{k+2}}\ldots b_r^{(1)p_r}w^{(1)}t_1^{(1)p_k}z$

(as **U** satisfies ($*$))

= $vy_1^{(1)q_k}v^{(1)}c_1^{(1)p_1}c_2^{(1)p_2}\ldots c_{k-1}^{(1)p_{k-1}}a_1^{p_k}b_{k+1}^{(1)p_k}b_{k+2}^{(1)p_k}\ldots b_r^{(1)p_k}w^{(1)}t_1^{(1)p_k}z$

(by Result 2.5, as $y_1^{(1)}, t_1^{(1)} \in$ **S\U** and $w^{(1)} = b_{k+1}^{(1)\{p_k-p_{k+1}\}}\ b_{k+2}^{(1)\{p_k-p_{k+2}\}}\ \ldots b_r^{(1)\{p_k-p_r\}}$)

= $vy_1^{(1)q_k}v^{(1)}c_1^{(1)p_1}c_2^{(1)p_2}\ldots c_{k-1}^{(1)p_{k-1}}a_1^{p_k}\ t_1^{(1)p_k}z$

(by Result 2.5, as $t_1^{(1)p_k} = b_{k+1}^{(1)p_k}b_{k+2}^{(1)p_k}\ \ldots b_r^{(1)p_k}t_1^{(1)p_k}$)

= $vy_1^{(1)q_k}v^{(1)}c_1^{(1)p_1}c_2^{(1)p_2}\ldots c_{k-1}^{(1)p_{k-1}}a_2^{p_k}\ t_2^{p_k}z$

(by Result 2.5 and equations (1))

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= $vy_{m-1}^{(1)q_k}v^{(m-1)}c_1^{(m-1)p_1}c_2^{(m-1)p_2}\ldots c_{k-1}^{(m-1)p_{k-1}}a_{2m-2}^{p_k}\ t_m^{p_k}z$

= $vy_{m-1}^{(1)q_k}v^{(m-1)}c_1^{(m-1)p_1}c_2^{(m-1)p_2}\ldots c_{k-1}^{(m-1)p_{k-1}}a_{2m-2}^{p_k}b_{k+1}^{(m)p_k}b_{k+2}^{(m)p_k}\ldots b_r^{(m)p_k}t_m^{(1)p_k}z$

(by Results 2.4 and 2.5 as $y_{m-1}^{(1)}, t_m^{(1)} \in$ **S\U** for some $b_{k+1}^{(m)}, b_{k+2}^{(m)},\ldots,b_r^{(m)} \in$ **U**)

= $vy_{m-1}^{(1)q_k}v^{(m-1)}c_1^{(m-1)p_1}c_2^{(m-1)p_2}\ldots c_{k-1}^{(m-1)p_{k-1}}a_{2m-2}^{p_k}b_{k+1}^{(m)p_{k+1}}b_{k+2}^{(m)p_{k+2}}\ldots b_r^{(m)p_r}\,w^{(m)}t_m^{(1)p_k}z$

(by Result 2.5 as $y_{m-1}^{(1)}, t_m^{(1)} \in$ **S**\U, where $w^{(m)} = b_{k+1}^{(m)\{p_k-p_{k+1}\}}\ b_{k+2}^{(m)\{p_k-p_{k+2}\}}\ \ldots b_r^{(m)\{p_k-p_r\}}$)

= $vy_{m-1}^{(1)q_k}v^{(m-1)}c_1^{(m-1)q_1}c_2^{(m-1)q_2}\ldots c_{k-1}^{(m-1)q_{k-1}}a_{2m-2}^{q_k}b_{k+1}^{(m)q_{k+1}}b_{k+2}^{(m)q_{k+2}}\ldots b_r^{(m)q_r}\,w^{(m)}t_m^{(1)p_k}z$

(since **U** satisfies ($*$))

= $vy_{m-1}^{(1)q_k}v^{(m-1)}c_1^{(m-1)q_k}c_2^{(m-1)q_k}\ldots c_{k-1}^{(m-1)q_k}a_{2m-2}^{q_k}b_{k+1}^{(m)q_{k+1}}b_{k+2}^{(m)q_{k+2}}\ldots b_r^{(m)q_r}\,w^{(m)}t_m^{(1)p_k}z$

(by Result 2.5, as $v^{(m-1)} = c_1^{(m-1)\{q_k-q_1\}} c_2^{(m-1)\{q_k-q_2\}} \dots c_{k-1}^{(m-1)\{q_k-q_{k-1}\}}$
and $y_{m-1}^{(1)}, t_m^{(1)} \in$ **S\U**)

$= v y_{m-1}^{q_k} a_{2m-2}^{q_k} b_{k+1}^{(m)q_{k+1}} b_{k+2}^{(m)q_{k+2}} \dots b_r^{(m)q_r} w^{(m)} t_m^{(1)p_k} z$

(by Result 2.5, as $y_{m-1}^{q_k} = y_{m-1}^{(1)q_k} c_1^{(m-1)q_k} c_2^{(m-1)q_k} \dots c_{k-1}^{(m-1)q_k}$ and $y_{m-1}^{(1)}, t_m^{(1)} \in$ **S\U**)

$= v y_m^{q_k} a_{2m-1}^{q_k} b_{k+1}^{(m)q_{k+1}} b_{k+2}^{(m)q_{k+2}} \dots b_r^{(m)q_r} w^{(m)} t_m^{(1)p_k} z$

(by Result 2.5 and equations (1) as $y_{m-1}^{(1)}, t_m^{(1)} \in$ **S\U**)

$= v y_m^{(1)q_k} c_1^{(m)q_k} c_2^{(m)q_k} \dots c_{k-1}^{(m)q_k} a_{2m-1}^{q_k} b_{k+1}^{(m)q_{k+1}} b_{k+2}^{(m)q_{k+2}} \dots b_r^{(m)q_r} w^{(m)} t_m^{(1)p_k} z$

(by Result 2.5 and dual of Result 2.4 for some $c_1^{(m)}, c_2^{(m)}, \dots, c_{k-1}^{(m)}$ ε **U** and $y_m^{(1)} \in$ **S\U**)

$= v y_m^{(1)q_k} v^{(m)} c_1^{(m)q_1} c_2^{(m)q_2} \dots c_{k-1}^{(m)q_{k-1}} a_{2m-1}^{q_k} b_{k+1}^{(m)q_{k+1}} b_{k+2}^{(m)q_{k+2}} \dots b_r^{(m)q_r} w^{(m)} t_m^{(1)p_k} z$

(by Result 2.5 as $y_{m-1}^{(1)}, t_m^{(1)} \in$ **S\U**, where $v^{(m)} = c_1^{(m)\{q_k-q_1\}} c_2^{(m)\{q_k-q_2\}} \dots c_{k-1}^{(m)\{q_k-q_{k-1}\}}$)

$= v y_m^{(1)q_k} v^{(m)} c_1^{(m)p_1} c_2^{(m)p_2} \dots c_{k-1}^{(m)p_{k-1}} a_{2m-1}^{p_k} b_{k+1}^{(m)p_{k+1}} b_{k+2}^{(m)p_{k+2}} \dots b_r^{(m)p_r} w^{(m)} t_m^{(1)p_k} z$

(as **U** satisfies (*))

$= v y_m^{(1)q_k} v^{(m)} c_1^{(m)p_1} c_2^{(m)p_2} \dots c_{k-1}^{(m)p_{k-1}} a_{2m-1}^{p_k} b_{k+1}^{(m)p_k} b_{k+2}^{(m)p_k} \dots b_r^{(m)p_k} w^{(m)} t_m^{(1)p_k} z$

(by Result 2.5, as $y_m^{(1)}, t_m^{(1)} \in$ **S\U**
and $w^{(m)} = b_{k+1}^{(m)\{p_k-p_{k+1}\}} b_{k+2}^{(m)\{p_k-p_{k+2}\}} \dots b_r^{(m)\{p_k-p_r\}}$)

$= v y_m^{(1)q_k} v^{(m)} c_1^{(m)p_1} c_2^{(m)p_2} \dots c_{k-1}^{(m)p_{k-1}} a_{2m-1}^{p_k} t_m^{p_k} z$

(by Result 2.5, as $y_m^{(1)}, t_m \in$ **S\U** and $t_m^{p_k} = b_{k+1}^{(m)p_k} b_{k+2}^{(m)p_k} \dots b_r^{(m)p_k} w^{(m)} t_m^{(1)p_k}$)

$= v y_m^{(1)q_k} v^{(m)} c_1^{(m)p_1} c_2^{(m)p_2} \dots c_{k-1}^{(m)p_{k-1}} a_{2m}^{p_k} z$

(by Result 2.5 and equations (1), as $y_m^{(1)}, t_m \in$ **S\U**)

$= v y_m^{(1)q_k} v^{(m)} c_1^{(m)p_1} c_2^{(m)p_2} \dots c_{k-1}^{(m)p_{k-1}} a_{2m}^{p_k} x_{k+1}^{p_{k+1}} x_{k+2}^{p_{k+2}} \dots x_r^{p_r}$

(as $z = x_{k+1}^{p_{k+1}} x_{k+2}^{p_{k+2}} \dots x_r^{p_r}$)

$= v y_m^{(1)q_k} v^{(m)} c_1^{(m)q_1} c_2^{(m)q_2} \dots c_{k-1}^{(m)q_{k-1}} a_{2m}^{q_k} x_{k+1}^{q_{k+1}} x_{k+2}^{q_{k+2}} \dots x_r^{q_r}$

(as U satisfies (*))

$= v y_m^{(1)q_k} v^{(m)} c_1^{(m)q_k} c_2^{(m)q_k} \dots c_{k-1}^{(m)q_k} a_{2m}^{q_k} x_{k+1}^{q_{k+1}} x_{k+2}^{q_{k+2}} \dots x_r^{q_r}$

(by Result 2.5, as $y_m^{(1)}, t_m \in$ **S\U** and $v^{(m)} = c_1^{(m)\{q_k-q_1\}} c_2^{(m)\{q_k-q_2\}} \dots c_{k-1}^{(m)\{q_k-q_{k-1}\}}$)

$= v y_m^{q_k} a_{2m}^{q_k} x_{k+1}^{q_{k+1}} x_{k+2}^{q_{k+2}} \dots x_r^{q_r}$

(by Result 2.5, as $y_m^{q_k} = y_m^{(1)q_k} c_1^{(m)q_k} c_2^{(m)q_k} \dots c_{k-1}^{(m)q_k}$)

$= x_1^{q_1} x_2^{q_2} \dots x_r^{q_r}$ (by Result 2.6 as v = v = $x_1^{q_1} x_2^{q_2} \dots x_{k-1}^{q_{k-1}}$),

as required.

Finally, a proof in the remaining cases, namely when k = 1 or k = r,
can be obtained from the proof above by making the following conventions:
First when k = 1,

(i) the word v = 1,

(ii) the word

$c_1{}^{(i)q}{}_k c_2{}^{(i)q}{}_k \ldots c_{k-1}{}^{(i)q}{}_k = c_1{}^{(i)q}{}_1 c_2{}^{(i)q}{}_2 \ldots c_{k-1}{}^{(i)q}{}_{k-1} = 1$ for $i = 1, 2, \ldots, m$.

and

$c_1{}^{(i)p}{}_1 c_2{}^{(i)p}{}_2 \ldots c_{k-1}{}^{(i)p}{}_{k-1} = v^{(i)} = 1$ and $y_i{}^{(1)} = y_i$ for $i = 1, 2, \ldots, m$.

Dually when $k = r$,

(i) word $z = 1$,

(ii) the word

$b_{k+1}{}^{(i)p}{}_k b_{k+2}{}^{(i)p}{}_k \ldots b_r{}^{(i)p}{}_k = b_{k+1}{}^{(i)p}{}_{k+1} b_{k+2}{}^{(i)p}{}_{k+2} \ldots b_r{}^{(i)p}{}_r = 1$ for $i = 1, 2, \ldots, m$.

and

$b_{k+1}{}^{(i)q}{}_{k+1} b_{k+2}{}^{(i)q}{}_{k+2} \ldots b_r{}^{(i)q}{}_r = w^{(i)} = 1$ for $i = 1, 2, \ldots, m$.

Remark. The Theorem 3.1 extends [12, Theorem (iii)].

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