



## NUMERICAL EXPERIMENTS TO OBTAIN THE SCALING LAWS FOR NEUTRON YIELD ON MATHER-TYPE PLASMA FOCUS MACHINES BELOW 500 JOULES

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### ABSTRACT

In this paper, we explain how we have fitted Lee's 5-phase and 6-phase model code to analyze the current waveforms of 6 small plasma focus machines (below 500Joules) working in deuterium gas medium. Using the information from the code fitted to these current waveforms, the scaling laws for these small- energy machines were obtained as follows

$$Y_n \approx E_0^{2.1}; \quad Y_n \approx I_{peak}^{3.9}; \quad Y_n \approx I_{pinch}^{5.0}$$

**Key words:** Lee's model code, neutron yield, scaling law, plasma focus neutron scaling laws

### INTRODUCTION

According to S Lee and SH Saw [1] the current trace of the plasma focus is one of the best indicators of gross performance of the plasma focus machine. The axial and radial phase dynamics and the crucial energy transfer into the focus pinch are among the most important information that is quickly apparent from the current trace. The exact time profile of the total current trace is governed by the bank parameters, by the focus tube geometry and the operational parameters. The current trace is also dependent on the fraction of mass swept-up and the fraction of sheath current and the variation of these fractions through the axial and radial phases. These parameters determine the axial and radial dynamics, specifically the axial and radial speeds which in turn affect the profile and magnitudes of the discharge current. The discharge current waveform contains information on all the dynamic, electrodynamic, thermodynamic and radiation processes that occur in the various phases of the plasma focus. This explains the importance attached to matching the computed total current trace to the measured total current trace in the procedure adopted by the Lee model code [2-16]. Once matched, the fitted model parameters assure that the computation proceeds with all physical mechanisms accounted for, at least in the gross energy and mass balance sense. One of the most important procedures therefore is to connect the numerical experiment to the reality of the actual machine by fitting the computed current trace to a measured current trace. Using this information, we look into the existing scaling laws such as those assembled by H. Kromholz et al in their paper "A Scaling



Law for Plasma Focus Devices” [17] and by S Lee and S H Saw in their papers “The Plasma Focus- Scaling Properties to Scaling Laws” [1] and “Neutron Scaling Laws from Numerical Experiments” [5] in which are proposed general scaling laws. The papers by Lee and Saw are based on large (up to MJ) and small machines and cover the whole range of energy from 400 J to MJ, using data derived from both measured results and numerical experiments. We now examine machines below 400 J.

## MATERIALS AND METHODS

Using the Lee model code, the computed total current waveform was fitted to the measured waveform (obtained from the published articles [18, 19, 20, 21, 22, 23] current or current derivative waveforms which were digitalized with Engauge [24]) by changing model factors  $f_m$ ,  $f_c$ ,  $f_{mr}$  and  $f_{cr}$  one by one, till the computed waveform agrees with the measured waveform. First,  $f_m$  and  $f_c$  are tuned sequentially until the features of the computed rising slope of the total current trace and the rounding off of the peak current as well as the peak current itself are in reasonable fit with the measured total current trace. We then continue to fit the radial  $f_{mr}$  and  $f_{cr}$  until features of the computed slope and the depth of the dip agree with the measured current waveform. Using the Lee Model 5-phase code the fitting ends here. If there is an extended part of the measured current dip which cannot be fitted by the computed current dip no matter how the model parameters are varied, then Lee Model 6-phase code is used. The 6-phase code has an additional phase between the end of pinch and the expanded column phase. This additional phase is fitted by adding anomalous resistance terms into the circuit equation (typically 3 sequential anomalous resistance terms) [16].

## RESULTS AND DISCUSSION

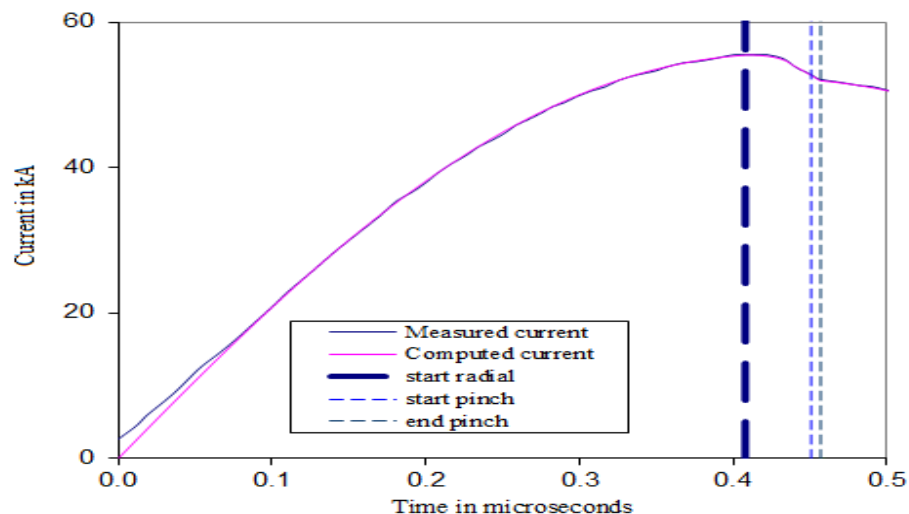


Figure 1: The fitting of the computed current trace to the measured current trace obtained from Argentina Nanofocus operating at 16kV, 1.5 Torr deuterium gas. The measured current waveform was extracted from shot number 572, Figure 7 in the paper entitled “D-D neutron yield in the 125 J dense plasma focus Nanofocus” [19]. Figure 1 shows a typical fitting of



computed current waveform to measured current waveform for the case of the Argentinian Nanofocus ANF. Once the current waveform is fitted, the model parameters are obtained. This process was repeated for all 6 machines. Four machines were fitted with Lee’s 5-phase code whereas the other 2 machines (AASC and \*India PF) have extended dips and required the extension of the 6-phase code. Using the fitted model parameters, the Lee code was configured for each machine at different pressures to find the optimum neutron yield for each machine. The machine configurations are recorded in Table 1. The information obtained at optimum yield for each machine is recorded in Table 4. Additionally Tables 2 and 3 record the anomalous resistance data required to fit the two machines with extended dips ED that could not be fitted completely with the 5-phase code.

Table 1: Machine and operating parameters and fitted model parameters of each of the machines.

	PF50	ANF	AASC	FMPF-1	*India PF	PF400
Capacitance $C_0$ ( $\mu\text{F}$ )	0.2	1.1	10.88(ES)	2.4	4	0.95
Static inductance $L_0$ (nH)	38	74	15.8(ES)	32.9	46	40
Circuit resistance $r_0$ (m $\Omega$ )	20	25	3.9	60	10	10
Outer radii, ‘b’(cm)	1.35	2.1	2	1.5	1.6	1.55
Inner anode ‘a’(cm)	0.3	0.75	0.75	0.6	0.5	0.6
Anode length ‘ $z_0$ ’(cm)	0.48	1.8	3	1.7	2	1.7
Charging voltage $V_0$ (kV)	25,29	16	4.5(ES)	12	10	28
Fill pressure $P_0$ (Torr)	6.8	1.5	2.9	2.25	6	6.6
Fill gas(molecular weight)	4	4	4	4	4	4
Fill gas(atomic number)	1	1	1	1	1	1
Fill gas(molecule(2))	2	2	2	2	2	2
Axial phase mass factor, $f_m$	0.13	0.055	0.065	0.155	0.135	0.08
Axial phase current factor, $f_c$	0.7	0.7	0.7	0.7	0.7	0.7
Radial phase mass factor, $f_{mr}$	0.1	0.1	0.1	0.165	0.1	0.11
Radial phase current factor, $f_{cr}$	0.75	0.95	0.75	0.75	0.8	0.71
<b>Taper type machine</b>						
Taper starts at (cm)				1.0		
Final tapered radius (cm)				0.3		

Note: (i) PF 50 was charged at 25kV and 29kV [18];so we have two set of results for PF50 in Table 4 (ii) ES is equivalent secondary; (iii)\*India PF is India smallest sealed type machine; (iv)ANF is Argentina Nanofocus

Table 2: The fitted anomalous resistance terms for AASC Plasma Focus machine.

	$R_0(\Omega)$	Characteristic of fall time, $\tau_2$ (ns)	Characteristic of rise time, $\tau_1$ (ns)	End fraction time
Dip 1	0.8	24	10	1
Dip 2	0.03	100	10	1
Dip 3	0.01	100	10	1



Table 3: The fitted anomalous resistance terms for India smallest sealed type machine.

	$R_0(\Omega)$	Characteristic of fall time, $\tau_2$ (ns)	Characteristic of rise time, $\tau_1$ (ns)	End fraction time
Dip 1	0.5	22	10	1
Dip 2	0.02	80	10	1
Dip 3	0.03	100	10	1

Table 4: Some of the information obtained from the Lee Model code configured when optimum yield was obtained.

	PF 50	PF 50	ANF	AASC	FMPF-1	India PF	PF 400
Energy(J)	62.5	84.1	140.8	110.2	172.8	200	372.4
Peak current (kA)	54	63	54	87	69	82	125
Pinch start current (kA)	39	45	50	57	48	62	84
Pinch minimum temperature( $10^6$ K)	16.2	15.5	9.5	5.8	18.3	7.4	8.5
Pinch maximum temperature( $10^6$ K)	16.5	15.9	9.7	6.2	18.9	7.6	8.7
Peak axial speed (cm/ $\mu$ s)	6.5	6.4	8.6	9.1	7.3	5.3	10
Peak radial shock speed (cm/ $\mu$ s)	54.7	53.8	42.1	35.4	36.9	37.4	40.3
Peak radial piston speed (cm/ $\mu$ s)	36.7	36.0	28.2	24.9	25	25.7	26.9
Final pinch radius $r_{min}$ (cm)	0.04	0.04	0.10	0.11	0.06	0.07	0.08
Pinch length $z_{max}$ (cm)	0.4	0.4	1.1	1.1	0.6	0.7	0.8
Pinch duration (ns)	1.6	1.6	5.2	6.7	3.5	3.9	4.4
Peak induced voltage (kV)	15.1	17.2	13.2	10.2	17.3	14.6	20.6
Neutron yield ( $\times 10^4$ n)	2.2	5.2	6.2	7	8.8	25.3	125

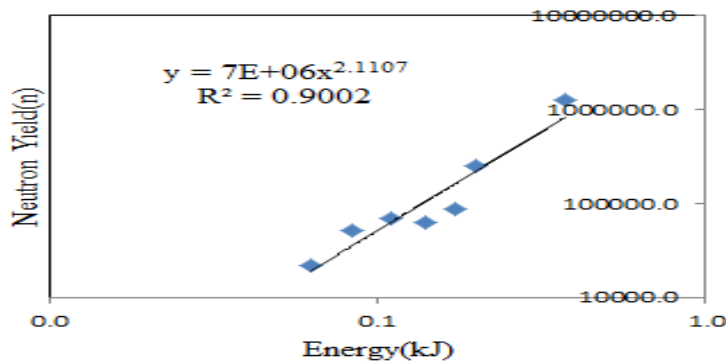


Figure 2: Neutron yield versus stored energy for the 6 machines studied.



Next, we use some of the information obtained from Table 4 and plot the graphs as shown in Figure 2, 3 and 4. Figure 2 shows the plot of neutron yield versus the energy input into the plasma focus machine in log-log scale. From Figure 2, we have:  $Y_n = 7 \times 10^6 E^{2.1}$  where  $E$  is in kJ, which is close to the generally accepted neutron yield versus energy scaling law of  $Y_n \approx E^2$  [1, 5, 7, 17].

Figure 3 shows the plot of neutron yield versus the peak current in the plasma focus machine in log-log scale. From Figure 3, we obtain:  $Y_n = 0.0058 I_{peak}^{3.9}$  where  $I_{peak}$  is in kA, which is close to  $Y_n \approx I_{peak}^{3.9}$  which was obtained by S Lee and SH Saw in their paper “Neutron Scaling Laws from Numerical Experiments” [5].

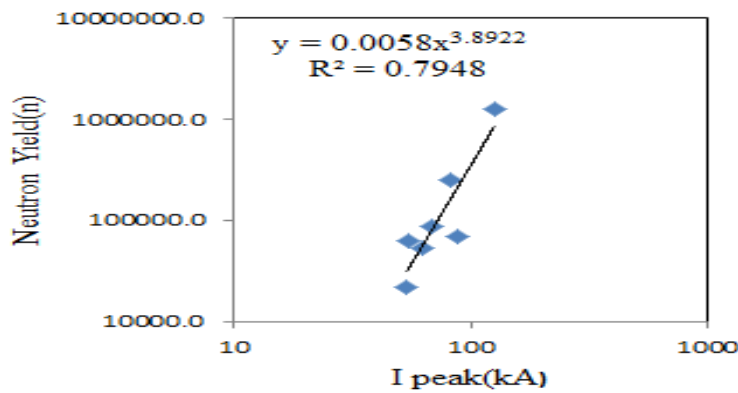


Figure 3: Neutron yield versus peak current for the 6 machines studied.

Figure 4 shows the plot of neutron yield versus the pinch current in the plasma focus machine. Figure 4 gives us the relationship of  $Y_n = 0.0002 I_{pinch}^{5.0}$  where  $I_{pinch}$  is in kA, which is close to  $Y_n \approx I_{pinch}^{4.7}$  which was obtained by S Lee and SH Saw in their paper “Neutron Scaling Laws from Numerical Experiments”[5].

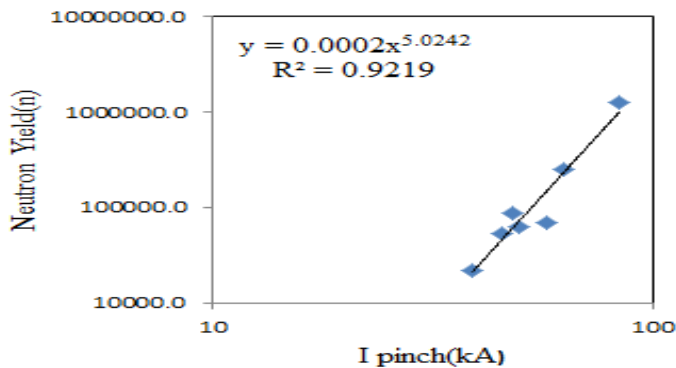


Figure 4: Neutron yield versus pinch current for the 6 machines studied.

## CONCLUSION

Combining the computed data of the 6 plasma focus machines studied, we obtain the following scaling laws. Neutron yield  $Y_n$  in deuterium as functions of stored energy  $E_0$ , peak circuit current  $I_{peak}$  and pinch current  $I_{pinch}$ :

$$Y_n \approx E_0^{2.1}; \quad Y_n \approx I_{peak}^{3.9}; \quad Y_n \approx I_{pinch}^{5.0}$$

Thus we can conclude that the scaling laws from the two papers [1, 17] are still valid for plasma focus machines below 500 joules.

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