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Does Mccutcheon's mortality polynomial matrix actually account for mortality decline at ten years?

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Abstract

This paper intends to employ a non-parametric technique as an alternative technique of modelling and estimating the instantaneous mortality rate intensities which serves as the underlying basis in modeling the distribution of future lifetime. It relies heavily on the analytic properties of life table survival functions l_x. The specific objectives of the study are (i) to derive models for the force of mortality using polynomial function (ii) to derive the survival function (iii) to detect the age at which mortality actually declines and (iv) estimate the curve of death. Computational evidence from our results confirms that in the models 1-3, the mortality intensity μ_x and the curve of death $\mu_x l_x$ are not both defined within the age band $0 \leq x \leq 2$. The implication is that the infant mortality cannot be captured and the model is not admissible within this interval. Furthermore, it is also observed that $\mu_x = \mu$ is constant within the interval $2 \leq x \leq 9$ and mortality declines at age x=10. Consequently, there is a visible improvement in the care of infants which accounts for the decline in infant mortality.

In model 4 since $l_x < l_{(x-1)} < l_{(x-2)} < l_{(x-3)} < l_{(x-4)} < l_{(x-5)} < l_{(x-6)}$, it then becomes apparent that $\mu_x < 0$. The fact that the force of mortality becomes negative represents a phantom detected from the McCutcheon's mortality matrix.

Keywords: Survival function; Instantaneous mortality; Intensities; Life table; Mortality matrix; Phantom

1. Introduction

Interpolation defines approximation of a value in between two interval values over a given set of values (Das & Chakrabarty 2016a) [1]. In the case of survival data points, interpolation describes a process of estimating intermediate value of such survival function from a set of its given values. According to Das and Chakrabarty (2016b) [2], polynomial interpolation is a technique of computing values between known data survival values. We infer from Neil (1979) [3] and Das and Chakrabarty (2016b) [2], that in fractional age at death problems where survival data has a gap, but data is given on either side of the gap or at a few definite points within the gap, interpolation permits for computation of the values within the gap. This partly accounts for the concept of the fractional distribution of deaths in mortality analysis. Where mortality tables is dependent on fractional age at death, the actuarial determination of the survival functionl_x at fractional age when required cannot be achieved unless by linear interpolation (Dickson, Hardy, & Waters, 2013) [4]. Consequently, the problem is to derive the approximate values instead of their real analytical values using tools of approximation. Functional parsimonious parametric mortality models have been developed allowing actuaries to determine risk of uncertainty connected with mortality intensities. As a result of the direct application of these models on mortality functions depending on age, actuaries have observed that mortality rates could likely yield consequential reaction to any change in demographic conditions. According to Putra, Fitriyati and Mahmudi (2019) [5], in actuarial statistics, computing the curve of death in the life tables one of which is mortality rate intensities has constituted hydra-headed problems particularly where relevant survival data is not available to model the intensity function analytically. In Neil (1997) [3], it is viewed that when survival curve $l_x$is measured at differing ages and the underlying mathematical expression is not available, then μ_x can only be obtained by approximation. Following Neil (1997) [3], Rabbi and Karmaker (2013) [6], the problem of estimating death rate μ_x at any given instant appears very often in mortality statistics. Neil (1997) [3], Kovacheva (2017) [7], Siswono, Azmi and Syaifudin (2021) [8] argue further that ifl_x denotes expected number of lives surviving to age x and μ_x is the death rate at an instant, then it is feasible to obtain the value of μ_x analytically either from the first order ordinary differential equation described by $\mu_x l_x = -(dl_x)/dx$ or if $l_{(x+t)}$ is functionally expressed as a convergent series polynomial function. In either case, an analytical framework for establishing the functional relationship between the number of lives l_x expected to survive to age x and instantaneous rate of mortality is derived Neil (1997) [3]. Furthermore, in order to justify the motives for invoking instantaneous rate of change, we can change time in steps. The issue is to estimate at an instant using Taylor's series expansion under an assumption that $l_x$is a convergent series polynomial function by interpolation at the commencement of the mortality table. Consequently, $\mu_x$can now be computed from the numerical point of view by invoking limiting processes so that inference can be drawn about the probability of death occurring in a defined interval of time Neil (1997) [3]. The first order differential equation above could also be employed to estimate the transition probabilities in a Markov process in a two state decrement model given the transition intensities.

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Theorem 1. *Let the minimum point of mortality μ_x be n and suppose the mortality in the age interval 0<x<1 can be approximated by*

$$\int_0^1 K_1 r_1 e^{(r_1 x)} + K_2 r_2 e^{(r_2 x)} dx \tag{1}$$

Then (i) $n = \frac{1}{(r_2 - r_1)} log_e \frac{r_1(2-e^{(r_2)})}{r_2(2-e^{(r_1)})}$

(ii)

$$\int_0^1 K_1 r_1 e^{(r_1 x)} + K_2 r_2 e^{(r_2 x)} dx$$
$$\rightarrow K_1 \left[e^{(r_1)} - 1 + \frac{(e^{(r_1)} - 2)}{(2 - e^{(r_2)})} e^{(r_2)} - \frac{(e^{(r_1)} - 2)}{(2 - e^{(r_2)})} \right]$$

Proof. The mortality interpretation for the rate of change of sickness Sis given as

$$S^{'}(x) = dS/dx = \alpha S - \beta S + \theta R \tag{2}$$

The mortality interpretation for the rate of change of recovery as age increases with probability β if the insured life was sick, recovers and decreases by probabilityθ if the insured life was healthy and becomes sick is given as:

$$R^{'}(x) = dR/dx = \beta S - \theta R \tag{3}$$

The matrix of co-efficient is:

$$M = \begin{pmatrix} \alpha - \beta & \theta \\ \beta & -\theta \end{pmatrix} \begin{pmatrix} S \\ R \end{pmatrix} \tag{4}$$

$$\frac{d}{dx}\mu(x) = K_1 r_1 e^{(r_1 x)} + K_2 r_2 e^{(r_2 x)} \tag{5}$$

where r_1 and r_2 are the eigenvalues corresponding to the matrix of co- efficient.

with boundary condition:

$$\mu^{'}(n) = 0$$
$$0 = \int_0^1 K_1 r_1 e^{(r_1 x)} + K_2 r_2 e^{(r_2 x)} dx \tag{6}$$

$$\mu^{'}(n) = K_1 r_1 e^{(r_1 n)} + K_2 r_2 e^{(r_2 n)} = 0 \tag{7}$$

$$K_1 r_1 e^{(r_1 n)} = -K_2 r_2 e^{(r_2 n)} \tag{8}$$

$$K_1 = \frac{-K_2 r_2 e^{(r_2 - r_1)n}}{r_1} \tag{9}$$

$$\int_0^1 K_1 r_1 e^{r_1 x} + K_2 r_2 e^{r_2 x} dx = K_1 + K_2 = \mu(0) \tag{10}$$

$$[K_1 e^{(r_1 x)} + K_2 e^{(r_2 x)}]_0^1 = K_1 + K_2 \tag{11}$$

$$K_1 e^{(r_1)} + K_2 e^{(r_2)} - K_1 - K_2 = K_1 + K_2 \tag{12}$$

$$K_1 e^{(r_1)} - 2K_1 = 2K_2 - K_2 e^{(r_2)} \tag{13}$$

$$\frac{-(e^{(r_1)} - 2) K_2 r_2 e^{(r_2 - r_1)n)}}{r_1} = (2 - e^{(r_2)}) K_2 \tag{14}$$

$$\frac{(2 - e^{(r_1)}) r_2 e^{(r_2 - r_1)n)}}{r_1} = (2 - e^{(r_2)}) \tag{15}$$

$$e^{(r_2 - r_1)n} = \frac{r_1(2 - e^{(r_2)})}{r_2(2 - e^{(r_1)})} \tag{16}$$

$$n = \frac{1}{(r_2 - r_1)} log_e \left[\frac{r_1(2 - e^{(r_2)})}{r_2(2 - e^{(r_1)})} \right] \tag{17}$$

But in (17),

$$n = \frac{1}{(r_2 - r_1)} log_e \left[\frac{-r_1 K_1}{r_2 K_2} \right] \tag{18}$$

$$log_e \left[\frac{r_1(2 - e^{r_2})}{r_2(2 - e^{r_1})} \right] = log_e \left[\frac{-r_1 K_1}{r_2 K_2} \right] \Rightarrow K_2 = \left[\frac{(e^{r1} - 2)K_1}{(2 - e^{r2})} \right] \tag{19}$$

$K_2 = \left[\frac{(e^{r1} - 2)K_1}{(2 - e^{r2})} \right]$ substituting K_2 in (1), the result in (ii) follows

$$K_1 \int_0^1 (r_1 e^{(r_1 x)} + \frac{e^{(r_1)} - 2}{2 - e^{(r_2)}} * r_2 e^{(r_2 x)}) dx$$
$$= K_1 \left(e^{(r_1)} - 1 + \frac{e^{(r_1)} - 2}{2 - e^{(r_2)}} e^{(r_2)} - \frac{e^{(r_1)} - 2}{2 - e^{(r_2)}} \right) \tag{20}$$

Q.E.D. □

2. Materials and methods

The McCutcheon(1983) [9] applied the polynomial

$$f(x) = (x - x_1)(x - x_2)...(x - x_(k-1))(x - x_(k+1))...(x - x_m)$$

$$= \prod_{\substack{R=1 \\ R \neq k}}^{m} (x - x_R)$$

to develop a general matrix of coefficients C_{ij} called mortality matrices. From our observation, the assumption is to show that mortality models can alternatively be constructed through different orders of polynomial. The contribution of this paper is anchored on the arguments presented below. The general mortality matrix is being decomposed into 4matrix models and tested to advance evidences of a decline in mortality at 10. This polynomial matrix technique is conveniently extended to an arbitrary number of age terms. The number of age terms in the mortality model is hence chosen with reference to the data instead of being specified in advance. A major importance over this polynomial technique is that the deformation of the age dependent functions are chosen to maximize the estimation and modelling to the data such that the polynomial term employs the maximum amount of information from the mortality data. The polynomial technique seems more computationally flexible because it can be employed over diverse range of mortality data. Since parametric age dependent functions are only adequate for a restricted age bound, the polynomial technique have been applied over the full age range. The polynomial method avoids the subjective judgement in formulating the mortality models as terms are constructed to maximize the fit to the mortality data. Parametric models parsimoniously specifies the governing distribution assumptions of mortality model and the techniques applied in estimating mortalities using data. Parsimonious age dependent functions are only applicable over limited age ranges. Although this represents an edge in permitting higher level of interpretation with respect to demographic relevance, it explains that the models with parametric age functions are usually not adequate over the full age range. The nature and the mathematical behaviour of parsimonious parametric mortality models have developed in complexity. However, the parsimonious parametric models especially the generalized Makeham's class have not provided actuarial evidence of any decline in mortality at10. The polynomial mortality has been introduced to evince evidence that the mortality computation functionally depends on the number of coefficients which varies with the order of the polynomials and this is meant to prove the hypothesis of mortality decline and then generate better mortality rates when compared with the parsimoniously parametric models. The essence of polynomial modelling is to provide a trade-off between the cardinality of parameters needed to generate good estimates of mortality rates such that as the order of the polynomial grows, the parameters required correspondingly increases and consequently generates the targeted estimates approaching the true mortality rates. This mortality modelling will contribute to the Nigerian population dynamics as crucial to the field of Nigerian demographic statistics and health care planning. Furthermore, this will be of good use as Nigeria has not developed any mortality tables for life insurance underwriting but continues to depend on European mortality tables for underwriting purposes.

2.1. Model 1

Let α be arbitrary age
McCutcheon (1983) [9] defines

$$\mu_{\alpha+i} = \frac{-\sum_{j=1}^{n} c_{ij} l_{\alpha+j}}{l_{\alpha+i}} \tag{21}$$

i=1,2,3,4,... where $c_{ij} = \begin{pmatrix} -147 & 360 & -450 & 400 & -225 & 72 & -10 \\ -10 & -77 & 150 & -100 & 50 & -15 & -2 \\ 2 & -24 & -35 & 80 & -30 & 8 & -1 \\ -1 & 9 & -45 & 0 & 45 & -9 & 1 \end{pmatrix}$ defines the mortality matrix.

If $n = 7$,$i = 4$, then $\alpha = x-4$ in Eq. (21)

$$\mu_{x-4+4} = \frac{-\sum_{j=1}^{7} c_{4j} l_{x-4+j}}{l_{x-4+j}} \tag{22}$$

$$\mu_x = \frac{-\sum_{j=1}^{7} c_{4j} l_{x-4+j}}{l_x} \tag{23}$$

$$\mu_x = \frac{-1}{l_x}[c_{4,1}l_{(x-3)} + c_{4,2}l_{(x-2)} + c_{4,3}l_{(x-1)} + c_{4,4}l_x + c_{4,5}l_{(x+1)} + c_{4,6}l_{(x+2)} + c_{4,7}l_{(x+3)}] \tag{24}$$

$$\mu_x = \frac{-1}{60l_x}[-l_{(x-3)} + 9l_{(x-2)} - 45l_{(x-1)} + 0 \times l_x + 45l_{(x+1)} - 9l_{(x+2)} + 1 \times l_{(x+3)}] \tag{25}$$

$$\mu_x \frac{1}{60l_x}[l_{(x-3)} - 9l_{(x-2)} + 45l_{(x-1)} - 45l_{(x+1)} + 9l_{(x+2)} - l_{(x+3)}] \tag{26}$$

2.2. Model 2

$$n = 7, i = 3, \alpha = x - 3$$

$$\mu_{\alpha+i} = \frac{-\sum_{j=1}^{n} c_{ij} l_{\alpha+j}}{l_{\alpha+i}} \tag{27}$$

$$\mu_{x-3+3} = \frac{-\sum_{j=1}^{7} c_{3j} l_{x-3+j}}{l_{x-3+3}} \tag{28}$$

$$\mu_x = \frac{-\sum_{j=1}^{7} c_{3j} l_{x-3+j}}{l_x} \tag{29}$$

$$\mu_x = \frac{-1}{l_x}[c_{3,1}l_{(x-2)} + c_{3,2}l_{(x-1)} + c_{3,3}l_{(x)} + c_{3,4}l_{x+1} + c_{3,5}l_{(x+2)} + c_{3,6}l_{(x+3)} + c_{3,7}l_{(x+4)}] \tag{30}$$

$$\mu_x = \frac{-1}{60l_x}[2l_{(x-2)} - 24l_{(x-1)} - 35 \times l_x + 80l_{(x+1)} - 30l_{(x+2)} + 8 \times l_{(x+3)} - l_{x+4}] \tag{31}$$

$$\mu_x = \frac{1}{60l_x}[-2l_{(x-2)} + 24l_{(x-1)} + 35 \times l_x - 80l_{(x+1)} + 30l_{(x+2)} - 8 \times l_{(x+3)} + l_{x+4}] \tag{32}$$

2.3. Model 3

$$n = 7, i = 2, \alpha = x - 2$$

$$\mu_{\alpha+i} = \frac{-\sum_{j=1}^{n} c_{ij} l_{\alpha+j}}{l_{\alpha+i}} \tag{33}$$

$$\mu_{x-2+2} = \frac{-\sum_{j=1}^{7} c_{2j} l_{x-2+j}}{l_{x-2+2}} \tag{34}$$

$$\mu_{x} = \frac{-\sum_{j=1}^{7} c_{2j} l_{x-2+j}}{l_{x}} \tag{35}$$

$$\mu_x = \frac{-1}{l_x}[c_{2,1}l_{(x-1)} + c_{2,2}l_x + c_{2,3}l_{(x+1)} + c_{2,4}l_{(x+2)} + c_{2,5}l_{(x+3)} + c_{2,6}xl_{(x+4)} + c_{2,7}l_{(x+5)}] \tag{36}$$

$$\mu_x = \frac{-1}{60l_x}[-10l_{(x-1)} - 77l_x + 150l_{(x+1)} - 100l_{(x+2)} + 50l_{(x+3)} - 15l_{(x+4)} + 2l_{(x+5)}] \tag{37}$$

$$\mu_x = \frac{1}{60l_x}[10l_{(x-1)} + 77l_x - 150l_{(x+1)} + 100l_{(x+2)} - 50l_{(x+3)} + 15l_{(x+4)} - 2l_{(x+5)}] \tag{38}$$

2.4. Model 4

Theorem 2. $\mu_x = \frac{1}{60l_x}[147l_x - 360l_{(x-1)} + 450l_{(x-2)} - 400l_{(x-3)} + 225l_{(x-4)} - 72l_{(x-5)} + 10l_{(x-6)}]$
Given that
then, $\mu_x < 0$
Proof.

$$n = 7, i = 1, \alpha = x - 1$$

$$\mu_{\alpha+i} = \frac{-\sum_{j=1}^{n} c_{ij} l_{\alpha+j}}{l_{\alpha+i}} \tag{39}$$

$$\mu_{x-1+1} = \frac{-\sum_{j=1}^{7} c_{1j} l_{x-1+j}}{l_{x-1+1}} \tag{40}$$

$$\mu_{x} = \frac{-\sum_{j=1}^{7} c_{1j} l_{x-1+j}}{l_{x}} \tag{41}$$

$$\mu_x = \frac{-1}{l_x}[c_{1,1}l_x + c_{1,2}l_{(x-1)} + c_{1,3}l_{(x-2)} + c_{1,4}l_{(x-3)} + c_{1,5}l_{(x-4)} + c_{1,6}l_{(x-5)} + c_{1,7}l_{(x-6)}] \tag{42}$$

$$\mu_x = \frac{1}{60l_x}[147l_x - 360l_{(x-1)} + 450l_{(x-2)} - 400l_{(x-3)} + 225l_{(x-4)} - 72l_{(x-5)} + 10l_{(x-6)}] \tag{43}$$

Observe that,

$$l_x < l_{(x-1)} < l_{(x-2)} < l_{(x-3)} < l_{(x-4)} < l_{(x-5)} < l_{(x-6)} \tag{44}$$

Consequently, $\mu_x < 0$ Where, $l_x = \int_0^\infty l_{x+\xi}\mu_{x+\xi}d\xi$
Q.E.D

3. Data presentation and analysis

The tables in the page 5-6-7 show the data for model 1, 2 and 3 respectively.

4. Discussion of results

In table 1, the mortality intensity $\mu_x$and the curve of death $\mu_x l_x$ are not both defined within the restricted age band $0 \leq x \leq 2$ and consequently, the intensitiesμ_2, μ_1 and$\mu_0$cannot be captured. At adult ages, the risk of ageing will escalate and the cause of death will either increase at higher degree of intensity or even cause severe ageing as the force of mortality increases. Furthermore, $\mu_x = \mu$ is constant within the interval $3 \leq x \leq 9$. In this interval, this constant force of mortality can be observed from the survival probability $\theta p_x = e^{-\int_0^\theta \mu dy} = e^{-\mu\theta}$and corresponds to the exponential failure distribution. Under this constant force assumption, the probability that a life survives to age $\theta + x$ within the age band $3 \leq x \leq 9$ is independent of x. This assumption of the constant $\mu_x = \mu$ may result in a step function for the force of mortality over successive years of age. The intensity first declines at age$x = 10$ and the reason is that there is a local minimum of mortality in the neighborhood of $x = 10$. At some other pointsy, the intensities oscillates. From the theorem 1 above, n=10years. This condition improves in table 2 where the mortality intensity μ_x and the curve of death $\mu_x l_x$ are not both defined within the restricted age band $0 \leq x \leq 1$. Furthermore, this implies that the death intensitiesμ_1 and μ_0 cannot be admissible. However, $\mu_x = \mu$ is constant within the interval $2 \leq x \leq 8$but slightly increases at 9 and decreases at age x=10. Consequently, this may be as a result of a higher level of health care or a healthy lifestyle and also a better environment. In table 3, there is no age where the mortality intensities are constant. Although, the mortality intensityμ_x and the curve of death$\mu_x l_x$ are not both defined at age $x = 0$, it is constant in the age interval $1 \leq x \leq 6$. However, it declines at both ages $x = 8$and $x = 10$. In the three cases, it is clear that the estimation of μ_x is by far a difficult problem where part of the difficulty is the computation ofμ_0 at integral ages when the only information given is l_xSince $\mu_x$usually varies rapidly in the interval$0 \leq x \leq 1$, there may not be a universally acceptable measure ofμ_0. Although, a major advantage of models 1-3 over other numerical methods involving $l_{(-1)}$ is the flexibility to obtain a rough estimate value forμ_0, the point of singularity is reason why $\mu_{74} < 0$ in table 3.

Table 1: Model 1

x	l_x	μ_x	$l_x\mu_x$
0	1000000	-	-
1	999917	-	-
2	999834	-	-
3	999751	0.000083	83
4	999668	0.000083	83
5	999585	0.000083	83
6	999502	0.000083	83
7	999419	0.000083	83
8	999336	0.000083	83
9	999253	0.000083	83.25
10	999170	0.000081	81.33
11	999087	0.00009	90.1
12	998989	0.000102	101.4
13	998886	0.000107	106.67
14	998772	0.000124	123.52
15	998632	0.000161	160.65
16	998440	0.000233	232.58
17	998165	0.000302	301.53
18	997801	0.000489	487.63
19	997207	0.000623	620.95
20	996610	0.000595	593.15
21	996014	0.000598	595.2
22	995419	0.000598	595.5
23	994823	0.000599	595.75
24	994228	0.000598	594.37
25	993634	0.000598	593.9
26	993040	0.000598	594
27	992446	0.000599	594.12
28	991852	0.000598	593.48
29	991259	0.000598	593.12
30	990666	0.000598	592.03
31	990074	0.0006	593.67
32	989475	0.000613	606.33
33	988856	0.000642	634.65
34	9888200	0.000689	6811.94
35	987495	0.000734	724.85
36	986751	0.000777	766.27
37	985957	0.000837	824.97
38	985098	0.000906	892.6
39	984172	0.000975	959.42
40	983180	0.001041	1023.8
41	982125	0.001106	1086.6
42	981008	0.001168	1145.72
43	979834	0.001228	1203.52
44	978600	0.001292	1264.33
45	977304	0.00136	1329.07
46	975940	0.001434	1399.3
47	974503	0.001516	1476.88
48	972985	0.001601	1557.98
49	971386	0.001691	1642.33
50	969700	0.001782	1727.62
51	967930	0.001875	1815
52	966068	0.001976	1908.72
53	964113	0.002075	2000.87
54	962068	0.00217	2087.95
55	959940	0.002257	2166.45
56	957738	0.002335	2236.17
57	955468	0.002413	2305.1
58	953126	0.002496	2379.43
59	950705	0.002594	2466.28
60	948187	0.002713	2572.25
61	945550	0.002866	2709.52
62	942754	0.003064	2888.73
63	939760	0.003304	3105.17
64	936535	0.003575	3348.15
65	933063	0.003851	3593.62
66	929349	0.004125	3833.4
67	925381	0.004453	4120.47
68	921096	0.004825	4444.48
69	916495	0.005191	4757.63
70	911580	0.005566	5074.3
71	906343	0.005959	5401.15
72	900769	0.006388	5754.12
73	894820	0.006875	6152.15
74	888447	0.007431	6602.3
75	885186	0.008101	7170.5
76	874128	0.008911	7789.7
77	865966	0.009884	8559.13
78	856958	0.011068	9484.63
79	946937	0.012502	11838.71
80	935708	0.014253	13336.95
81	823033	0.016374	13476.65
82	808671	0.01891	15292.2
83	792366	0.021903	17355.58
84	773885	0.025381	19642.35
85	753013	0.029394	22134.3
86	729563	0.033974	24785.88
87	703405	0.03916	27545.27
88	674454	0.045011	30357.85
89	642694	0.05159	33156.42
90	608203	0.058814	35770.68
91	571297	0.066428	37950.37
92	532501	0.074244	39534.85
93	492452	0.082135	40447.52
94	451844	0.08996	40647.75
95	411397	0.09754	40127.67
96	371817	0.104691	38926.02
97	333751	0.111176	37105.27
98	297747	0.117066	34856.07
99	264088	0.122994	32481.33
100	232897	0.127721	29745.92
101	204588	0.132142	27034.63
102	178720	0.138013	24665.72
103	155253	0.143548	22286.33
104	134119	0.149114	19999.08
105	115218	0.154683	17822.28
106	98432	0.160237	15772.48
107	83626	0.165782	13863.72
108	70655	0.171304	12103.47
109	59368	0.1768	10496.27
110	49611	0.182287	9043.42
111	41231	0.187747	7740.98
112	34081	0.193143	6582.5
113	28020	0.198503	5562.05
114	22914	0.203815	4670.22
115	18640	0.209073	3897.12
116	15084	0.214217	3231.25
117	12145	0.227699	2765.4
118	9729	0.137791	1340.57
119	7755	0.738414	5726.4

Table 2: Model 2

x	l_x	μ_x	$l_x\mu_x$
0	1000000	-	-
1	999917	-	-
2	999834	0.000083	83.01
3	999751	0.000083	83.01
4	999668	0.000083	83.01
5	999585	0.000083	83.01
6	999502	0.000083	83.01
7	999419	0.000083	83.01
8	999336	0.000083	82.76
9	999253	0.000084	84.42
10	999170	0.000079	79.07
11	999087	0.000092	92.26
12	998989	0.000101	100.46
13	998886	0.000107	106.96
14	998772	0.000124	123.8
15	998632	0.000157	157.11
16	998440	0.000247	246.87
17	998165	0.000273	272.35
18	997801	0.000521	519.68
19	997207	0.000606	604.2
20	996610	0.000599	597.19
21	996014	0.000598	595.29
22	995419	0.000599	596.17
23	994823	0.000599	596.01
24	994228	0.000598	594.66
25	993634	0.000598	594.34
26	993040	0.000598	594.26
27	992446	0.000599	594.66
28	991852	0.000598	593.47
29	991259	0.000599	593.97
30	990666	0.000598	592.09
31	990074	0.0006	594.21
32	989475	0.000613	606.71
33	988856	0.000641	633.94
34	988200	0.000691	683.25
35	987495	0.000734	724.4
36	986751	0.000777	766.43
37	985957	0.000838	826.29
38	985098	0.000907	893.24
39	984172	0.000976	960.6
40	983180	0.001042	1024.43

x	l_x	μ_x	$l_x\mu_x$
41	982125	0.001108	1088.49
42	981008	0.001169	1146.51
43	979834	0.00123	1205.43
44	978600	0.001293	1265.69
45	977304	0.001362	1331.32
46	975940	0.001435	1400.7
47	974503	0.001519	1480.09
48	972985	0.001603	1559.48
49	971386	0.001695	1646.41
50	969700	0.001784	1729.79
51	967930	0.001879	1818.9
52	966068	0.00198	1912.5
53	964113	0.00208	2005.1
54	962068	0.002175	2092.6
55	959940	0.002262	2171.7
56	957738	0.00234	2240.99
57	955468	0.002419	2311.27
58	953126	0.002502	2384.89
59	950705	0.002602	2473.74
60	948187	0.00272	2578.64
61	945550	0.002874	2717.89
62	942754	0.003074	2897.72
63	939760	0.003315	3115.27
64	936535	0.003589	3361.38
65	933063	0.003869	3610.4
66	929349	0.004137	3844.36
67	925381	0.004478	4143.49
68	921096	0.004849	4466.16
69	916495	0.005219	4782.73
70	911580	0.0056	5104.41
71	906343	0.006062	5494.21
72	900769	0.005894	5309.01
73	894820	0.008953	8010.92
74	888447	0.002033	1806.25
75	885186	0.010575	9360.45
76	874128	0.010657	9315.68
77	865966	0.009848	8527.86
78	856958	0.0112	9597.99
79	846937	0.012668	10728.97
80	835708	0.014475	12096.79

x	l_x	μ_x	$l_x\mu_x$
81	823033	0.016663	13714.42
82	808671	0.019301	15607.86
83	792366	0.022427	17770.12
84	773885	0.026084	20185.77
85	753013	0.030341	22847.08
86	729563	0.035236	25707.01
87	703405	0.040844	28729.7
88	674454	0.047228	31852.94
89	642694	0.054518	35038.56
90	608203	0.062616	38083.43
91	571297	0.071268	40714.96
92	532501	0.080281	42749.45
93	492452	0.089517	44082.68
94	451844	0.098805	44644.27
95	411397	0.107917	44396.85
96	371817	0.116652	43373.1
97	333751	0.124624	41593.49
98	297747	0.131864	39262.09
99	264088	0.139753	36907.01
100	232897	0.145061	33784.34
101	204588	0.151465	30987.89
102	178720	0.158831	28386.21
103	155253	0.166165	25797.54
104	134119	0.173579	23280.21
105	115218	0.18106	20861.41
106	98432	0.188608	18565.04
107	83626	0.196218	16408.89
108	70655	0.203872	14404.57
109	59368	0.211569	12560.45
110	49611	0.219338	10881.57
111	41231	0.227136	9365.04
112	34081	0.234916	8006.18
113	28020	0.242745	6801.7
114	22914	0.250534	5740.73
115	18640	0.258385	4816.3
116	15084	0.257585	3885.41
117	12145	0.349712	4247.25
118	9729	-0.039916	-388.34
119	7755	-	-

Table 3: Model 3

x	l_x	μ_x	$l_x\mu_x$
0	1000000	-	-
1	999917	0.000083	83
2	999834	0.000083	83
3	999751	0.000083	83
4	999668	0.000083	83
5	999585	0.000083	83
6	999502	0.000083	83
7	999419	0.000084	83.5
8	999336	0.00008	80.42
9	999253	0.000089	88.95
10	999170	0.000075	74.77
11	999087	0.000094	94.15
12	998989	0.0001	99.88
13	998886	0.000107	106.42
14	998772	0.000131	130.92
15	998632	0.000129	128.65
16	998440	0.000306	305.37
17	998165	0.000209	208.78
18	997801	0.000555	553.6
19	997207	0.000598	596.47
20	996610	0.000599	597.37
21	996014	0.000597	594.3
22	995419	0.000599	596.02
23	994823	0.000599	595.78
24	994228	0.000598	594.13
25	993634	0.000598	594.18
26	993040	0.000598	593.53
27	992446	0.0006	595.03
28	991852	0.000597	592.12
29	991259	0.000599	594.22
30	990666	0.000597	591.37
31	990074	0.0006	593.85
32	989475	0.000615	608.6
33	988856	0.000637	629.52
34	988200	0.000693	684.77
35	987495	0.000734	724.75
36	986751	0.000775	764.62
37	985957	0.000838	825.97
38	985098	0.000905	891.97
39	984172	0.000976	960.57
40	983180	0.00104	1022.03
41	982125	0.001108	1088.42
42	981008	0.001166	1144.33
43	979834	0.001229	1204.55
44	978600	0.001291	1263.22
45	977304	0.001362	1330.8
46	975940	0.001431	1396.83
47	974503	0.001519	1479.92
48	972985	0.001598	1554.48
49	971386	0.001694	1645.52
50	969700	0.00178	1725.83
51	967930	0.001876	1815.57
52	966068	0.001976	1908.67
53	964113	0.002075	2000.82
54	962068	0.00217	2087.43
55	959940	0.002258	2167.68
56	957738	0.002333	2234.68
57	955468	0.002414	2306.8
58	953126	0.002494	2377.03
59	950705	0.002597	2468.75
60	948187	0.002711	2570.8
61	945550	0.002866	2710.28
62	942754	0.003065	2889.68
63	939760	0.003302	3103.05
64	936535	0.003571	3344.08
65	933063	0.003866	3606.93
66	929349	0.004111	3820.28
67	925381	0.004458	4125.57
68	921096	0.004826	4444.95
69	916495	0.005189	4755.52
70	911580	0.005437	4956.55
71	906343	0.006952	6300.75
72	900769	0.003054	2750.52
73	894820	0.013588	12159.23
74	888447	-0.002707	-2404.75
75	885186	0.013292	11765.52
76	874128	0.009597	8388.67
77	865966	0.009881	8556.92
78	856958	0.011073	9489.32
79	846937	0.012496	10583.32
80	835708	0.014259	11916.42
81	823033	0.01637	13473.17
82	808671	0.018911	15293.03
83	792366	0.021906	17357.65
84	773885	0.025377	19638.78
85	753013	0.029398	22136.75
86	729563	0.033967	24781.37
87	703405	0.039177	27556.97
88	674454	0.044999	30349.48
89	642694	0.051587	33154.5
90	608203	0.058818	35773.12
91	571297	0.06643	37951.17
92	532501	0.074242	39534.17
93	492452	0.082134	40447.25
94	451844	0.08997	40652.28
95	411397	0.097503	40112.25
96	371817	0.104702	38930.13
97	333751	0.111374	37171.28
98	297747	0.11651	34690.38
99	264088	0.123763	32684.22
100	232897	0.127127	29607.47
101	204588	0.13238	27083.37
102	178720	0.137981	24659.97
103	155253	0.143541	22285.18
104	134119	0.149119	19999.7
105	115218	0.154681	17822.07
106	98432	0.160237	15772.45
107	83626	0.165783	13863.75
108	70655	0.171307	12103.67
109	59368	0.176795	10495.97
110	49611	0.182287	9043.45
111	41231	0.187755	7741.32
112	34081	0.193127	6581.95
113	28020	0.198529	5562.78
114	22914	0.203771	4669.22
115	18640	0.220131	4103.25
116	15084	0.122943	1854.47
117	12145	0.55188	6702.58
118	9729	-0.501365	-4877.78
119	7755	1.492424	11573.75

In model 4the force of mortality is derived as follows

$\mu_x = \frac{1}{60 l_x}[147 l_x - 360 l_{(x-1)} + 450 l_{(x-2)} - 400 l_{(x-3)} + 225 l_{(x-4)} - 72 l_{(x-5)} + 10 l_{(x-6)}]$.

It is apparent that

$l_x < l_{(x-1)} < l_{(x-2)} < l_{(x-3)} < l_{(x-4)} < l_{(x-5)} < l_{(x-6)}$.

Consequently, it becomes apparent that$\mu_x < 0$. The fact that the force of mortality becomes negative represents a phantom detected from the McCutcheon's mortality matrix.

5. Conclusion

In this paper, we have given a detailed analysis of mortality modelling based on McCutcheon's mortality matrices to explain decline in mortality at 10 years. Four models were developed to that effect. The first three models yield positive mortality intensities while the forth model was shown to produce negative mortality intensity. However in generalized Makeham's mortality class $GM(m,n) = \sum_{k=1}^{m} \beta_k C_{k-1}(\xi(x)) + exp \sum_{k=m+1}^{m+n} \beta_k C_{k-m-1}(\xi(x))$, this concept of mortality decline is not usually explained. This is because of the general level of exponential increase of ageing.

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