Journal of Nepal Geological Society, 2000, Vol. 22, pp. 25–30

Evaluation on slope stability of unsaturated soils

L. J. Wang and M. Zhang
Geotechnical Engineering Institute, Department of Civil Engineering,
Northern Jiaotong University, Beijing, China

ABSTRACT

In drought-prone and semiarid areas, the groundwater table is deep and the soils are at an unsaturated state because of evaporation or transpiration. The negative pore water pressure or matric suction (u_a-u_w) is an important property of unsaturated soils that are situated above the groundwater table. In the conditions of rainfall, ground seepage, or drainpipe leakage, the matric suction will decrease with the increase of the degree of saturation, and the soils will lose their part of shear strength, which is the main reason why many unsaturated soil slopes become unstable. This paper discusses the engineering properties of unsaturated soils. Following the limit equilibrium principle, the unsaturated soil slopes are evaluated by applying the slice method.

INTRODUCTION

In the drought-prone and semiarid areas, the groundwater table is deep and the soils are at an unsaturated state owing to evaporation or transpiration. Because of heavy exploitation of the groundwater in the urban areas, the groundwater table rapidly goes down. On the other hand, various civil structures are founded on this unsaturated soil stratum.

The presence of negative pore water pressure or matric suction (u_a–u_w) in the soils above the groundwater table is an important engineering property of unsaturated soils, which makes them differ from saturated ones. In the conditions of rainfall, ground seepage, or drainpipe leakage, the matric suction in soils will decrease with the increase of the degree of saturation, consequently they will lose their part of shear strength leading to slope instability. The engineering properties of the unsaturated soils are not known clearly and the evaluation method for this kind of soil slope is not properly developed.

Bishop et al. (1960) studied the effect of suction generated by the reduction in the confining pressure on clays in undrained conditions. The Second International Conference on Unsaturated Soils (1998) summed up the research of last forty years. The two stress variable theories put forward by Fredlund and Morgenstern (1976) have been accepted and applied widely. Willy and Fernando (1998) and Lu (1989) have discussed the engineering uses of unsaturated soil mechanics. The effect of seasonal variation of soil suction on slope stability has been described by many authors (Oberg 1995; Lim et al. 1996; Anderson 1983). Due to their contribution to the apparent cohesion, soil suction has a particular importance on near-surface stability. Oberg (1995) emphasised the importance of matric suction, whereas Krahn et al. (1989) presented a case where the instability of a road slope was associated with the loss of suction due to rainfall. Johnson and Sitar (1990) presented results that improved understanding of hydrological conditions leading to debris flow initiation.

ENGINEERING PROPERTIES OF UNSATURATED SOILS

The unsaturated soil is a kind of multi-phase admixture. It is stated that the unsaturated soil is composed of soil grain, water, and air. But Fredlund (1993) considers that the air-water interphase, the contractile skin, is the fourth phase in the unsaturated soil. Under the outer stress gradient, the action between the soil grain and the contractile skin reaches to an equilibrium state, and the air and water come into flow.

Classification of unsaturated soils

According to the distribution characteristics of the volumetric water content (θ) of in situ soils (Xu 1991), the unsaturated soils in the vadose region can be divided into the following five zones:

Bond water zone

It is located at the top of unsaturated soil, and is a zone in which q is influenced considerably by the outside environment. When the soil is exposed to an outer water source, the water content will markedly increase. Under evaporation, the moisture in soil is consumed and the surface layer becomes dry. The value of q is 0.03–0.1 for sandy soils and 0.1–0.15 for clayey soils. We can see that the water belongs to the film water when q is less than 0.1 for sandy soils and 0.15 for clayey soils. Because of the presence of film water, the soil is highly susceptible to moisture absorption. The matric suction is so high that the vertical fissures occur on the surface.

Corner capillary zone

It is located below the bond water zone. The value of q increases to 0.1–0.2 for sandy soils and 0.2 or so for clayey soils. There is a significant humidity difference between the suspended capillary zone and the two zones above, which makes the soil in the zone possess very strong ability for heading off and saving up the infiltrating water.

Suspended capillary zone

It is the thickest zone, which is distributed evenly in the unsaturated soils. In most of cases, q exceeds 0.2 tending to or crossing the limit of maximum water content of the corner capillary zone, and comes into the range of suspended capillary zone. Through the transfer of capillary pressure, the infiltrating water can successfully supply the groundwater. This zone plays an important role of transmitter by keeping the water content unchanged. The shallower the upper edge of this zone is, the shorter is the time required for the moisture infiltration into the groundwater. The matric suction in the zone usually is between -10 and -60 kPa, and its magnitude plays a very important role in the slope stability of unsaturated soils, since a majority of slip surfaces will pass through this zone.

Sustaining capillary zone

Its thickness is less. The value of θ is close to saturation (i.e. about 0.3 for sand and 0.4 for clay). The water transmission ability of the zone becomes stronger. The matric suction in the zone is close to zero.

Saturation zone

It is located below the groundwater table where pore water pressure becomes positive.

The above five zones are developed universally in the vadose zone. Especially, the third zone is thick and uniform with a shallow upper edge. It plays a role of transmitter of surface water into the groundwater.

Jiang (1989) analysed the classifications of unsaturated soils. Barden (1965) classified compacted unsaturated soils into the following five types based on the degree of saturation (S_r): (1) Type A, S_r<50%, very dry or a bit wet, the pore water is absorbed by the framework of soil grains, the air in pores is interconnected. During consolidation, only the air flows out of the soil, and the effective stress $\sigma' = \sigma - u_a$, where u_a is the pore air pressure. (2) Type B, 50%<S_r<90%, wet or very wet, the pore water cannot flow, but the air is still interconnected, the air can flow only in the pores. The matric suction ($u_a - u_w$) in soil is very high, pore water pressure is negative, the effective stress equation is $\sigma' = (\sigma - u_a) + \chi(u_a - u_w)$. (3) Type C, S_r=90%, the water content is optimum. As S_r increases, the coefficient of permeability of air (K_a) decreases, and the coefficient of permeability of water (K_w) increases. The matric suction is lower, $u_w > 0$, the air and water flow at the same time, both of them being the same in quantity. (4) Type D, S_r>90%, very wet or close to saturation, the air is closed and cannot flow. The effective stress equation is $\sigma' = \sigma - u_w$. (5) Type E, S_r>95%, very wet or saturated, only the uncompressible water in the pores of soil can flow.

Matric suction in unsaturated soils

The geotechnical problems are complicated and of multiform. They are generally related to the following three dominant properties of soil: (1) coefficient of permeability, (2) parameters of shear strength, and (3) parameters of volumetric deformation. The unsaturated soil is described as the soil with negative pore water pressure or the matric suction. The presence of the matric suction is the main reason, which makes the engineering properties of unsaturated soils differ considerably from those of the saturated ones. The three dominant properties of unsaturated soils are related to the matric suction. The changes in pore water pressure with depth are also related to such environmental changes as climate, evaporation, rainfall, immersion, and water table fluctuations.

The pore water pressure curve can provide the relationship between θ and matric suction (Fig. 1). The pressure plate test was used for measuring the water-soil characteristic curve of the samples taken from a practical foundation pit in Northern Jiaotong University in Beijing; the results are shown in Fig. 2. We can find that the range of intense matric suction in this soil is from 0.15 to 0.4 of θ. And this range is characteristic for the foundation pit soils up to a depth of 2–7 m.

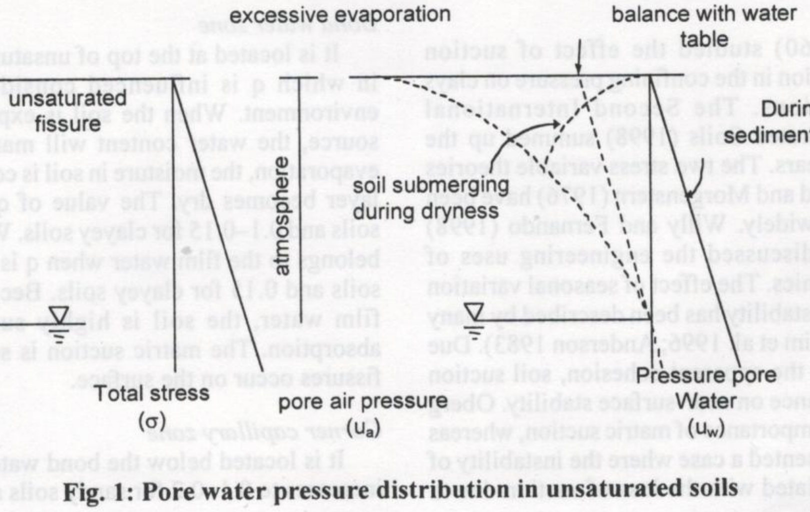

Fig. 1: Pore water pressure distribution in unsaturated soils

Shear strength of unsaturated soils

There are two famous failure criteria for unsaturated soils. One of them is the Coulomb failure criterion based on the effective stress put forward by Bishop:

$$\tau_f = c' + [(\sigma - u_a) + \chi(u_a - u_w)]\ tg\phi' \qquad (1)$$

where, τ_f is the shear strength of unsaturated soil; σ is the total stress; u_a is the pore air pressure; u_w is the pore water pressure; χ is a parameter whose value is from 0 to 1, which is based on the saturation conditions, soil type, dry and wet circulation, and the load and stress path of matric suction.

The other is the extended Mohr-Coulomb failure criterion based on two stress components (σ-u_a) and (u_a-u_w), put forward by Fredlund:

$$\tau_f = c' + (\sigma - u_a)tg\phi' + (u_a - u_w)tg\phi^b \qquad (2)$$

where, c′ expresses the real cohesion of soils, or structural cohesion; (σ-u_a) tgϕ' expresses the frictional force caused by the outside effective stress, or frictional strength; (u_a-u_w)tgϕ^b expresses the appended frictional strength caused by the matric suction or negative pore water pressure, also called adsorptive strength or apparent cohesion. Equation (2) can be considered as a three-dimensional relationship figure in which there is the third axis called the matric suction axis (u_a-u_w), as shown in Fig. 3. When the saturation S_r reaches to 100%, u_a is close to u_w so that the matric suction is zero, the remaining two terms of the Equation (2) are the parameters of saturated soil, c′ and ϕ'.

CALCULATION MODEL FOR SLOPE STABILITY OF UNSATURATED SOILS

A notable feature of the model is that the magnitude and direction of the thrust on the each slice of the sliding body is not assumed but the location of action point in the thrust is assumed. Because the thrust line location has less influence in the factor of safety, the lower third point of the soil slice side is taken as the action location of the thrust in the model. According to this, a thrust line can be drawn in Fig. 4.

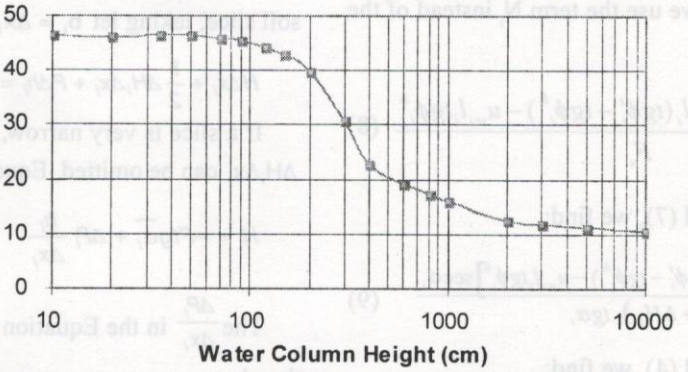

Fig. 2: Soil-water characteristic curve for silty clay in a foundation pit

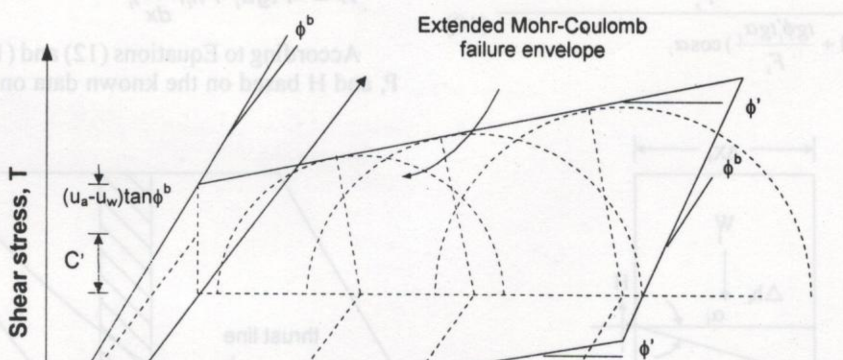

Fig. 3: Extended Mohr-Coulomb failure envelope of unsaturated soils

After assuming the location of thrust point P, the action force system on the soil slice No. i is analysed according to the static equilibrium conditions. The summation of total forces acted on the slice No. i should be zero in the vertical and horizontal directions (Fig. 4), namely:

$$\sum F_X = 0, \quad \sum F_Y = 0 \tag{3}$$

$$\sum F_X = -T_i \cos\alpha_i + N_i \sin\alpha_i - P - \Delta P_i + P = 0 \tag{4}$$

$$\sum F_Y = -N_i \cos\alpha_i - T_i \sin\alpha_i + W_i + H + \Delta H_i - H = 0 \tag{5}$$

Substituting Equation (5) in (4), we find:

$$\Delta P_i = -T_i(\cos\alpha_i + \frac{\sin^2\alpha_i}{\cos\alpha_i}) + (W_i + \Delta H_i)tg\alpha_i \tag{6}$$

$$\sum \Delta P_i = -\sum T_i(\cos\alpha_i + \frac{\sin^2\alpha_i}{\cos\alpha_i}) + \sum(W_i + \Delta H_i)tg\alpha_i = 0 \tag{7}$$

In allusion to the unsaturated soils, we adopt the failure criterion of shear strength, Equation (2), put forward by Fredlund. When the slope is on the state of stability, the portion of the shear strength on the soil slice No. i is equipoise with the shear stress, and we use the term N_i instead of the term $\sigma_{ni} l_i$, namely :

$$T_i = \frac{c_i' l_i + N_i tg\phi_i' - u_{ai} l_i (tg\phi_i' - tg\phi_i^b) - u_{wi} l_i tg\phi_i^b}{F_s} \tag{8}$$

From Equations (8) and (7), we find:

$$F_s = \frac{\sum\left[c_i' l_i + N_i tg\phi_i' - u_{ai} l_i (tg\phi_i' - tg\phi_i^b) - u_{wi} l_i tg\phi_i^b\right]\sec\alpha_i}{\sum(W_i + \Delta H_i)\ tg\alpha_i} \tag{9}$$

From Equations (7) and (4), we find:

$$N_i = \frac{W_i + \Delta H_i - \dfrac{\left[(c_i' l_i + u_{ai} l_i\ (tg\phi_i - tg\phi_i^b) + u_{wi} l_i tg\phi b\right] \sin\alpha_i}{F_s}}{(1 + \dfrac{tg\phi_i' tg\alpha_i}{F_s})\cos\alpha_i} \tag{10}$$

Substituting Equation (10) into Equation (9), under commonly conditions, the pore air pressure is the atmospheric pressure (u_a=0), and by simplifying we get:

$$F_s = \frac{\Sigma\left\{\left[c_i' l_i - u_{wi} l_i tg\phi_i^b\right]\cos\alpha_i + (W_i + \Delta H_i)tg\phi_i'\right\}\dfrac{\sec^2\alpha_i}{1 + tg\phi_i' tg\alpha_i / F_s}}{\Sigma(W_i + \Delta H_i)\ tg\alpha_i} \tag{11}$$

Equation (11) is the formula of general limit equilibrium slice method for slope stability of unsaturated soils.

Calculation method

We take the soil slice No.i as an example, substituting Equations (8) and (10) into Equation (6), We obtain:

$$\Delta P_i = (W_i - \Delta H_i)tg\alpha_i - \frac{(c_i' l_i - u_{wi} l_i tg\phi_i^b)\cos\alpha_i + (W_i + \Delta H_i)tg\phi_i'}{F_s} \times \frac{\sec^2\alpha_i}{1 + tg\phi_i' tg\alpha_i / F_s} \tag{12}$$

In the same way, by taking the action point O of the reaction N_i on the slip surface of the soil slice as the centre of moment, we write out the equilibrium conditions on the soil slice, taking let $b_i = \Delta x_i$, $\sum M_0 = 0$:

$$H\Delta x_i + \frac{1}{2}\Delta H_i \Delta x_i + P\Delta h_i = \Delta P_i h_i \tag{13}$$

If a slice is very narrow, the higher orders of minimum $\Delta H_i \Delta x_i$ can be omitted, Equation (13) may take the form of

$$H = -Ptg\overline{\alpha_i} + \Delta P_i \frac{h_i}{\Delta x_i} \tag{14}$$

The $\frac{\Delta P_i}{\Delta x_i}$ in the Equation (14) can be written as $(\frac{dP}{dx})_i$, thereby

$$H = -Ptg\overline{\alpha_i} + h_i(\frac{dP}{dx})_i \tag{15}$$

According to Equations (12) and (15), determine the ΔP_i, P, and H based on the known data on the soil slice, we can

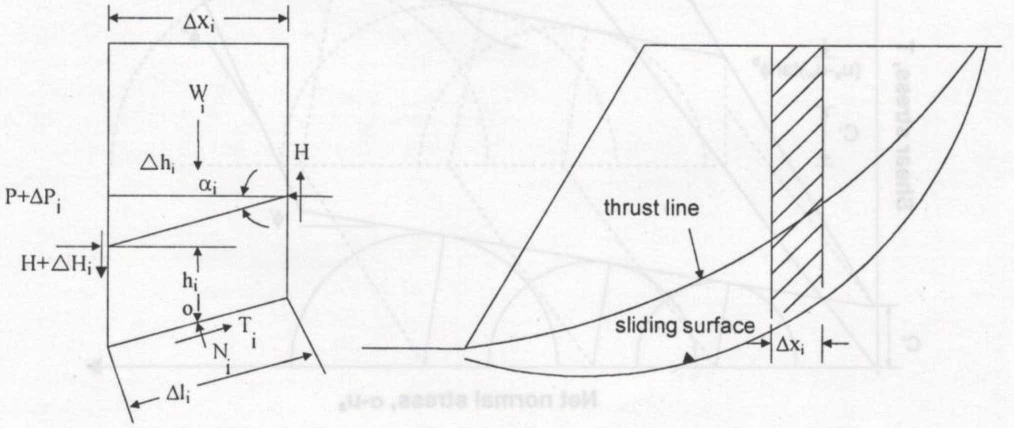

Fig. 4: Calculation model of non-arcuate method for unsaturated soils

calculate the factor of slope stability. In order to simplify the calculation, induct symbols as follows:

$$A_j' = (c_i' - u_{wi} tg\phi_i^{b})\Delta x_i + (W_i + \Delta H_i) tg\phi_i' \tag{16}$$

$$A_j = \frac{(c_i' - u_{wi} tg\phi_i^{b})\Delta x_i + (W_i + \Delta H_i) tg\phi_i'}{n_{\alpha i}} \tag{17}$$

$$B_i = (W_i + \Delta H_i) tg\alpha_i \tag{18}$$

$$n_{\alpha i} = \frac{1 + \frac{tg\phi_i'}{F_s} tg\alpha_i}{sec^2\alpha_i} = \frac{1 + tg\phi_i' \frac{tg\alpha_i}{F_s}}{1 + tg^2\alpha_i} \tag{19}$$

The Equation (11) can be simplified as:

$$F_s = \frac{\sum A_i' / n_{\alpha i}}{\sum B_i}, \quad \Delta P_i = B_i - \frac{A_i}{F_s} \tag{20}$$

Since both sides of Equations (11) have the factor of safety Fs, the hit-and-trial method can be used in calculation. First, we can let $\Delta H_i = 0$, obtain a factor of safety, until determine the minimum factor of slope stability in multi-sliding surfaces. For most of practical problems with non-arcuate sliding surface, let $\Delta H_i = 0$, we can get more accurate results. If we want to get high precision result, iterative method can be used. We can take ΔH_i obtained from the last calculation into Equation (11) and calculate Fs again. When the absolute error between the former and latter results is less than 0.01, we consider the result applicable. Generally, it can be achieved after 5 or 6 iterations.

Under the positive pore water pressure, we can let $\phi^b = \phi'$. The above calculation model not only is used to analyse the stability of unsaturated soil slope, but also applied to evaluate the stability of saturated soil slope.

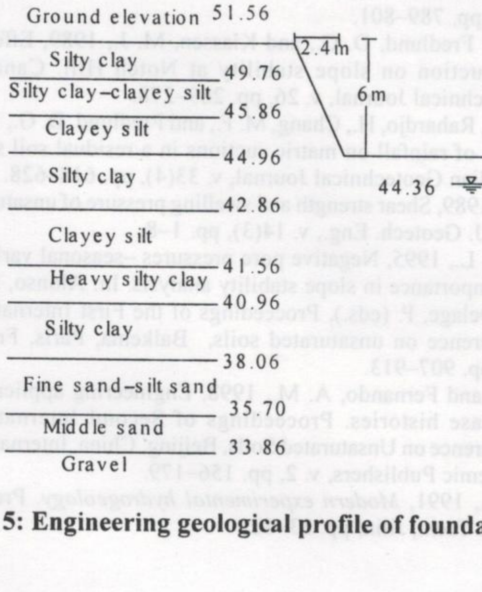

Fig. 5: Engineering geological profile of foundation pit

A CASE STUDY

The East Teaching Building in Northern Jiaotong University in Beijing is a multi-function teaching building, which is composed of eight storeys above the ground and one storey as a cellar. The pithead size of the foundation is 67×23 m²; the excavated depth is 6 m, which was dug on 31 October 1998. The designed gradient of the foundation pit slope is 2.5:1. There is no any support and measures for the pit slope.

Engineering properties of unsaturated silty clay

The engineering geological profile of the foundation pit is shown in Fig. 5. The soil of the whole pit is composed of yellow silty clay. The entire slope is above the groundwater table, which is 7.2 m deep. The soil has the following engineering geological parameters: specific gravity γ=19.0 kN/m³, water content w=18%, saturation S_r=65%, cohesion c=20 kPa, internal frictional angle ϕ=28°.

In order to study the matric suction characteristics in the slope of the foundation pit, the type WM-1 negative pressure meter was used. We installed the instruments on the southern slope and measure the matric suction. The matric suction probes were inserted into the soil up to 0.5 m horizontally and with the spacing of 0.5 m in vertical direction. During two days of observation, we got the matric suction distribution characteristics in the slope profile, shown in Fig. 6.

From the matric suction characteristic curve (Fig. 6), we can know that the matric suction in the slope soil is changeable with the depth. It is higher near the ground surface, and then decreases along the depth. It can be seen that 0–1.5 m is a higher matric suction zone, 1.5–2.5 m is the decreased zone of matric suction, and below 3 m is the basically unchangeable zone of the suction.

Assessment of unsaturated soil slope

The matric suction, which exists in the unsaturated soils, is one of important properties. Many foundation pit slopes lose their stability often because the engineering properties of the unsaturated soil are not known distinctly and the

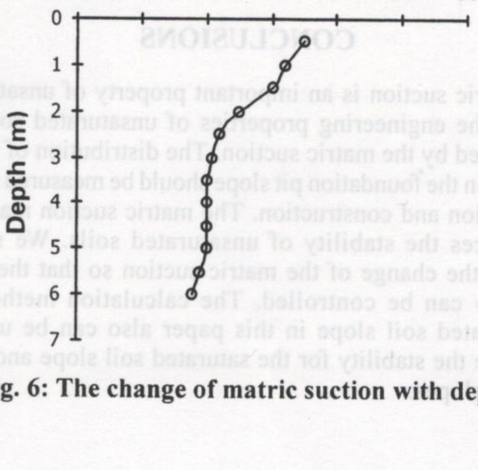

Fig. 6: The change of matric suction with depth

Table 1: Calculation results for foundation pit slope stability of unsaturated soils

Case	Calculation parameters	Matric suction u_a-u_w(kPa)	Factors of stability (FOS) 1st sliding surface	Factors of stability (FOS) 2nd sliding surface
1	φ=29.7° c =10kPa φ^b=15°	-50	1.540	1.740
2		-40	1.453	1.651
3		-30	1.367	1.562
4		-20	1.281	1.474
5		-10	1.195	1.387
6		0	1.110	1.300

evaluation method for this kind of slope is not proper. Here the general slice analysis model for unsaturated soil slope stability is used to evaluate the foundation pit slope in Northern Jiaotong University.

In order to make the results accurate and reliable, a computed program for unsaturated soil slope stability was developed. In calculation, we took different matric suction values to evaluate the factors of safety of the slope. At the same time, many possible slip surfaces were chosen so that the most dangerous slip surface can be determined. The results for two sliding surfaces are shown in the Table 1. A variety of slope stability factors with the matric suction are shown in Fig. 7.

Because of the matric suction increase or water content decrease, we can find that the slope stability increases accordingly. The matric suction intensively controls the foundation pit slope stability in unsaturated soils. After rainstorm or long time raining, the humidity or saturation of the slope soils will increase, and the matric suction, which is related to the water content of soils will decrease rapidly. It is the decrease of matric suction in soils during or after rainfall that results in many foundations pit slope failures. So when the foundation pit slope is designed, the slope stability is unreliable if the variety of matric suction is not considered, especially when the construction of the foundation pit is in the rainy season.

CONCLUSIONS

Matric suction is an important property of unsaturated soils. The engineering properties of unsaturated soils are controlled by the matric suction. The distribution of matric suction in the foundation pit slope should be measured during excavation and construction. The matric suction markedly influences the stability of unsaturated soils. We should predict the change of the matric suction so that the slope stability can be controlled. The calculation method for unsaturated soil slope in this paper also can be used to evaluate the stability for the saturated soil slope and other natural slopes.

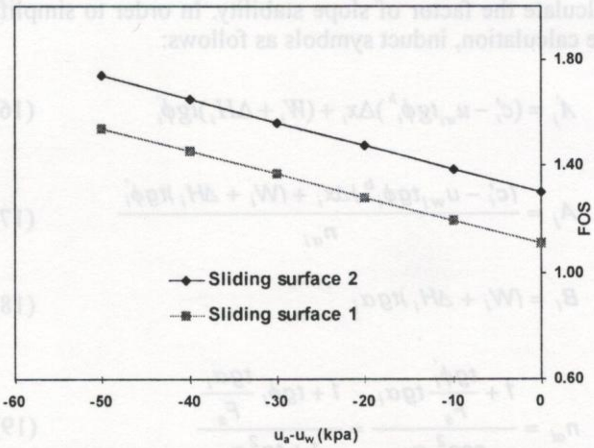

Fig. 7: Slope stability influenced by the matric suction

ACKNOWLEDGEMENT

This work was supported by the key research project (No. 59738160) of the National Natural Science Fund of China.

REFERENCES

Anderson, M. G., 1983, *Prediction of soil suction for slope in Hong Kong*. Geotechnical Control Office, Hong Kong. 243 p.

Barden, L., 1965, Consolidation of compacted and unsaturated clays. Geotechnique, v. 13(3), pp. 267–286.

Bishop, A. W., Alpan, I., Blight, G. E., and Donald, I. B., 1960, Factors controlling the shear strength of partly saturated cohesive soils. ASCE. Res. Conf. on the Shear Strength of Cohesive Soils, Univ. of Colorado, Boulder. pp. 503–532.

Fredlund, D. G. and Morgenstern, N., 1976, Constitutive relations for volume change in unsaturated soils. Canadian Geotechnical Journal, v. 13, pp. 261–276.

Fredlund, D. G., 1993, *Soil mechanics for unsaturated soils*. John Wiley & Sons. New York. pp. 377–387.

Jiang, P. N., 1989, On the engineering properties of unsaturated soil. J. Geotech. Eng., v. 11(6), pp. 39–59.

Jonson, K. A. and Sitar, N., 1990, Hydrologic conditions leading to debris-flow initiation. Canadian Geotechnical Journal, v. 27, pp. 789–801.

Krahn, J., Fredlund, D. G., and Klassen, M. J., 1989, Effect of soil suction on slope stability at Notch Hill. Canadian Geotechnical Journal, v. 26, pp. 269–278.

Lim, T. T., Rahardjo, H., Chang, M. F., and Fredlund, D. G., 1996, Effect of rainfall on matric suctions in a residual soil slope. Canadian Geotechnical Journal, v. 33(4), pp. 618–628.

Lu, Z. J., 1989, Shear strength and swelling pressure of unsaturated soils. J. Geotech. Eng., v. 14(3), pp. 1–8.

Oberg, A. L., 1995, Negative pore pressures –seasonal variation and importance in slope stability analysis. In: Alonso, E. E., and Delage, P. (eds.), Proceedings of the First International Conference on unsaturated soils. Balkema, Paris, France, v. 2, pp. 907–913.

Willy, L. and Fernando, A. M., 1998, Engineering applications and case histories. Proceedings of Second International Conference on Unsaturated Soils, Beijing, China, International Academic Publishers, v. 2, pp. 156–179.

Xu, Q. S., 1991, *Modern experimental hydrogeology*. Press of Sci. & Tech., Jilin, pp. 33–40.