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# Estimation of S-wave velocity structure and its application to ground motion simulation

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## ABSTRACTS

Microtremor array is the most inexpensive and easy to perform technique for the estimation of S-wave velocity structure. Microtremor array measurements have been carried out in the Shizuoka Prefecture, Japan to estimate S-wave velocity structure up to the basement. Phase velocities at wide period range were determined by frequency-wavenumber spectral analysis of vertical microtremor array records. The determined phase velocity is inverted to obtain one-dimensional S-wave velocity profile by genetic algorithm inversion method. A four layer S-wave velocity model with a basement velocity of 3.5 km/s was constructed. Simulation of ground motion has been carried out with two-dimensional finite difference method. Simulation of subsurface structural model was derived from the microtremor array measurement and previous seismic refraction survey. Two profiles were taken for simulation one from Hamaoka to Ryuhyoh and another from Hamaoka to Shimada. 2-D effect of subsurface structure is observed in the propagation of ground motion in the basin. The importance of determination of 2-D subsurface structure for the estimation of ground motion is shown.

## INTRODUCTION

S-wave velocity in a subsurface layer is the most crucial factor for characterizing strong ground motion at a site. The lack of parameters of subsurface structure data like shear wave velocity, density, Q–value (Quality factor Q, is dimensionless quantity describing attenuation in wave amplitude because of internal friction) make it difficult to estimate strong ground motion using numerical simulation techniques, so the primary step for the simulation of ground motion is to estimate the subsurface structure at a site. There are many other geophysical and geotechnical methods, to obtain velocity structure, such as seismic refraction, but these methods are time consuming, not economic, required a lot of manpower and difficult to conduct in the densely populated cities. So microtremor array technique is comparatively better technique.

Microtremors are ambient ground noises from different sources like sea wave, cultural noise from traffic, factories etc. Microtremors are classified into two types; short-period microtremors with period less than 1 second related to shallow subsurface structure of tens of meters thick and long-period microtremors with more than 1 sec period and related to deeper soil structure up to the basement. In an array measurement vertical microtremors are measured which are mainly composed of Rayleigh wave (Horike 1985).

In this study microtremor array measurement has been carried out to estimate S–wave velocity structure. On the basis of estimated S-wave velocity subsurface 2-D ground motion simulation has been done by finite difference method.

## ANALYSIS METHOD

Microtremors are surface waves from various sources and their vertical motions are generally composed of Rayleigh waves. As there is systematic variation of phase velocity with increasing period (Tokosoz 1964), this is interpreted as phase velocity of microtremors.

Phase velocities of Rayleigh waves from microtremors are determined from frequency –wavenumber (f-k) spectral analysis. There are two methods for the estimation of f-k spectra beam forming method (BFM) (LaCoss 1969) and maximum likelihood method (MLM) (Capon 1969). As MLM method gives high resolution it is applied here. In MLM method, f-k spectra at frequency $f$ and wavenumber k is given by

$$P(f,\mathbf{k})=[\mathbf{E}(\mathbf{k})^*\mathbf{S}^{-1}(f)\,\mathbf{E}(\mathbf{k})]^{-1} \tag{1}$$

where, $\mathbf{E}(\mathbf{k})=[(e^{ikr_1},e^{ikr_2},......e^{ikr_n})]^T$

where, $\mathbf{S}(f)$ is the matrix of coherent power spectral density matrix between seismometers, **E(k)** is the matrix of relative locations of stations in the array, and $\mathbf{E}(k)^*$ is the complex transpose of **E(k)**. For the two sensors at $i^{th}$ and $j^{th}$ positions the f-k spectra is given by

$$P(f,\mathbf{k})=A_i(f,\mathbf{k})\,A_j(f,\mathbf{k})S_{ij}(f)e^{ik(r_i-r_j)} \tag{2}$$

After the estimation of f-k spectrum at a frequency, the spectrum is drawn in two dimensional $\mathbf{k}_x$-$k_y$ space. The phase velocity and propagation direction are determined by locating the peak wavenumber ($k_{mx}$, $k_{my}$), using the formula

$$V = \frac{2\pi f}{\sqrt{k_{mx}^2 + k_{my}^2}} \tag{3}$$

$$\theta = \tan^{-1}\left(\frac{k_{my}}{k_{mx}}\right) \tag{4}$$

By repeating this computation for several frequencies phase velocity dispersion curve for wide range of period is determined. This phase velocity is used for the inversion analysis.

The phase velocity estimated is inverted to a one-dimensional S-wave velocity profile by genetic inversion. This is a global optimization method. In this method density is kept fix and P-wave velocity of each layer and the shear wave velocity and thickness of each layer is determined. Search limits for both S-wave velocity and thickness are defined. Initially 20 models are randomly generated. Then three genetic operations, selection, crossover and mutation are performed in initial models. The models with smaller misfit can survive more in the next generation and the models with larger misfits are replaced by newly generated models. For the j-th individual, misfit function $\phi_j$ is defined as the average of root mean squares of the difference between calculated $C_c$ and $C_0$ observed phase velocities.

$$\phi_j = \frac{1}{N}\sum_{i=1}^{N} \frac{C_0(T_i) - C_c(T_i)}{\sigma(T_i)} \tag{5}$$

where N is number of observed data and $\sigma$(Ti) is the standard deviation of the observed group velocity at a period of Ti. The fitness of an individual is defined as

$$F_j = \frac{1}{\phi_j} \tag{6}$$

By iteration of these three genetic operations the initial model approached a global optimal solution. The number of iterations is set to 100.

## DATA ACQUISITION AND PROCESSING

The investigated area is in Shizuoka prefecture, Japan, near the coastal city of Hamaoka (Fig. 1). Surface geology of the area is very shallow quaternary deposits and below that there are two layers of Tertiary deposits and the basement lies at a depth of about 5 km (Kobayashi and Seo 1980). Microtremor array measurements were carried out in 2 different array sizes with the installation of 7 vertical seismometers, with a spacing of 0.1 to 1.5 km (Fig. 2). Thirty minutes recording and 1 hour recording were carried out in small and large arrays, respectively.

The observed data are divided into data with time window of 327.68 sec. All the data sets from each window are used for the f-k spectral analysis. The f-k spectra are estimated by the MLM of Capon (1969). Phase velocity and propagation direction are estimated from a wavenumber whose spectral amplitude is the largest. This procedure is repeated for all the dataset for each period of interest. Phase velocity dispersion curve is estimated in wide period range.

Fig. 3 shows the f-k spectra at various periods. It is clearly seen that at long period (Fig. 3a) the f-k spectra have single peak and there are many peaks even in short period ( Fig. 3b and 3c) range which indicate the complexity of the source of

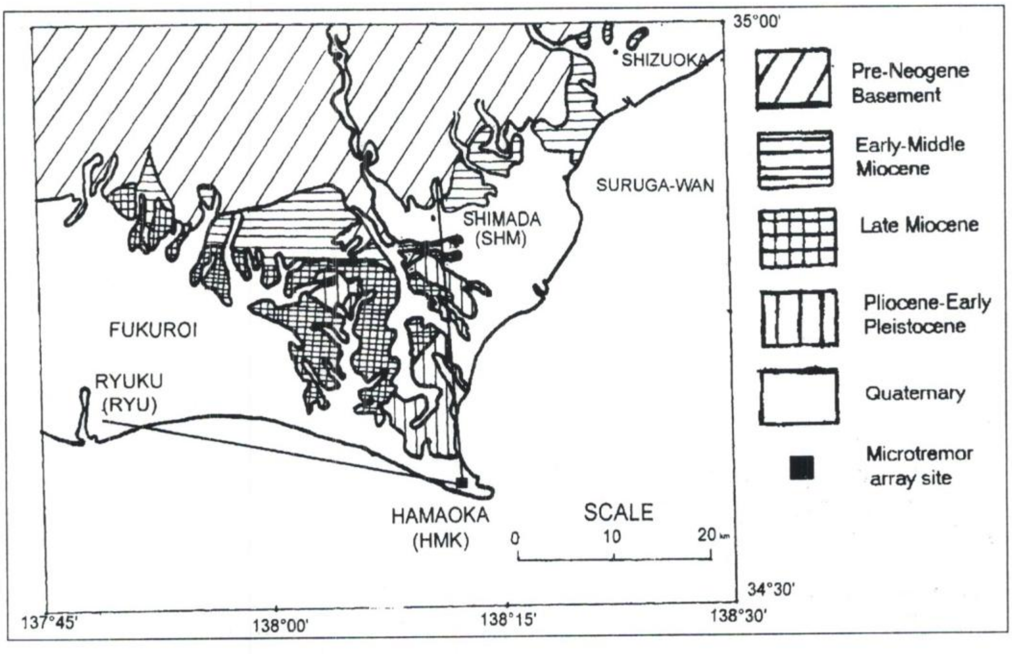



**Fig. 1: Location of microtremor array site and geology.**

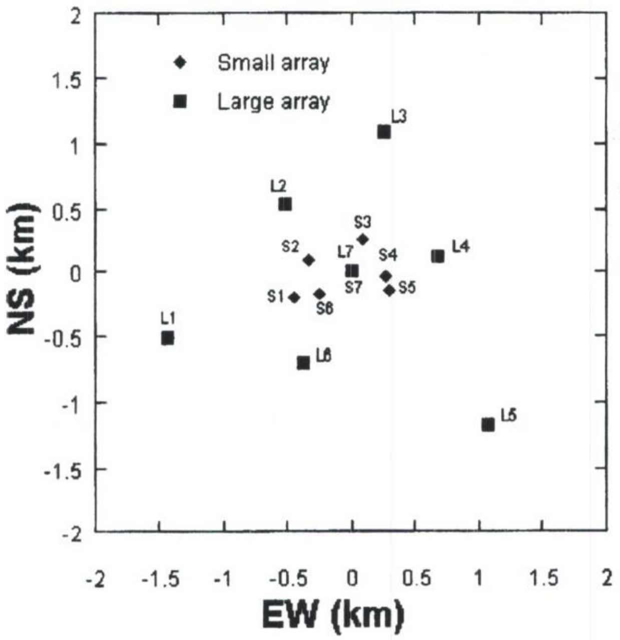



**Fig. 2: Microtremor array configuration.**

microtremors at short period range. Fig. 4 shows the azimuthal distribution of microtremor source. In the long period (Fig. 4a) the source has fixed azimuthal distribution along SE direction but for the short period (Fig. 4b and 4c) it is scattered in every direction. The observed phase velocity is shown in Fig. 5.

The obtained phase velocity is inverted to get S-wave velocity structure by genetic algorithm (GA) inversion method (Yamanaka and Ishida 1996). In the GA method, density is fixed, P-wave velocity is determined by empirical relation Vp=1.11*Vs+1.29 (km/s).

A four-layer model is taken for the inversion. S-wave velocity and thickness are searched within search limits. Narrow search area is set for S-wave velocity and large area for the thickness. The search limits used for GA is given in the Table 1.

Fig. 6 shows the variation of average misfit with increasing generation. In the first 10 generations, it is very high, but after that it almost converges. There are local minimum solutions in the phase velocity inversion because

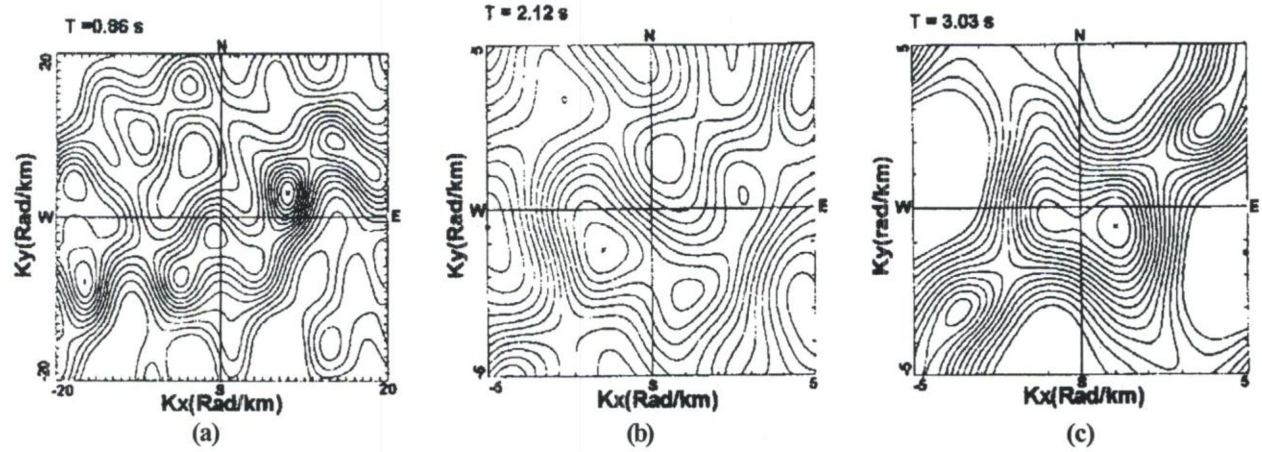



**Fig. 3: Representative examples of frequency wavenumber spectra between period range of 0.6 sec to 3.0 sec.**

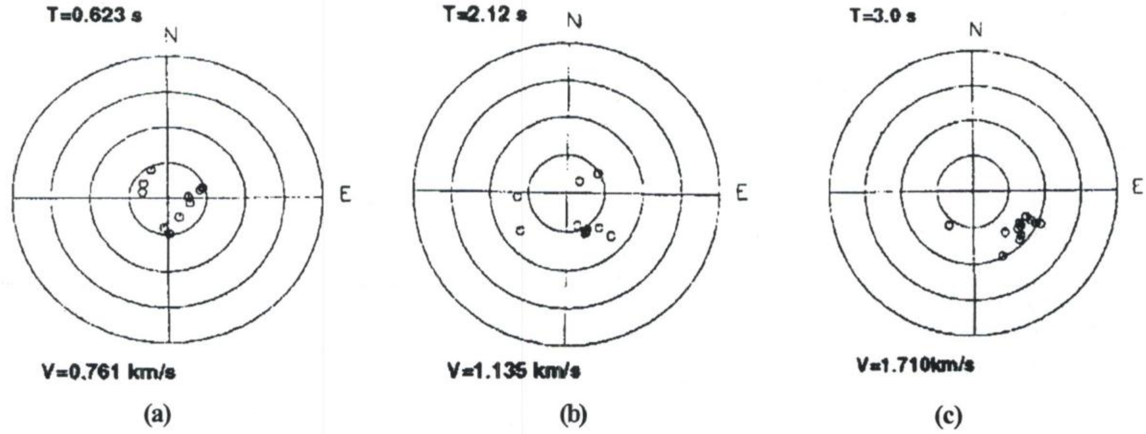



**Fig. 4: Propagation direction of microtremors at different periods.**

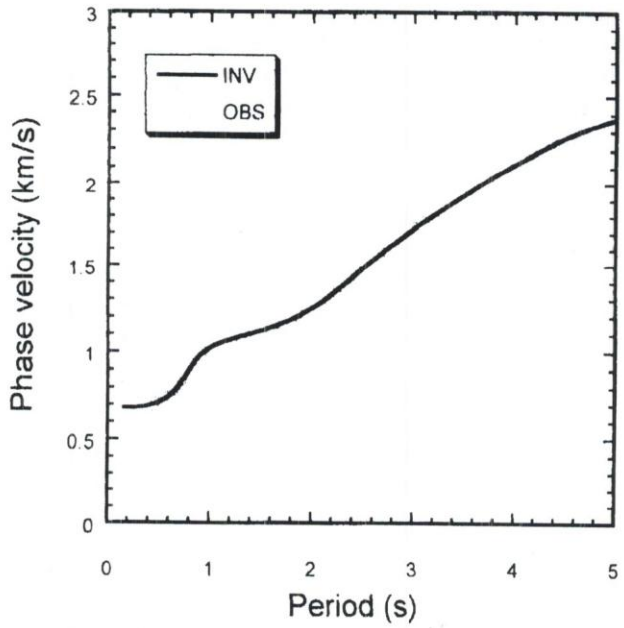



Fig. 5: Observed and inverted phase velocities.

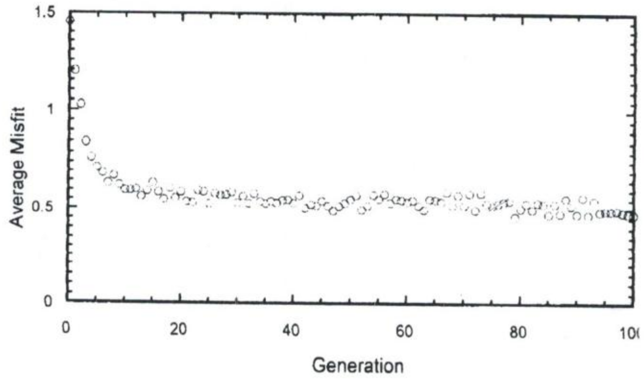


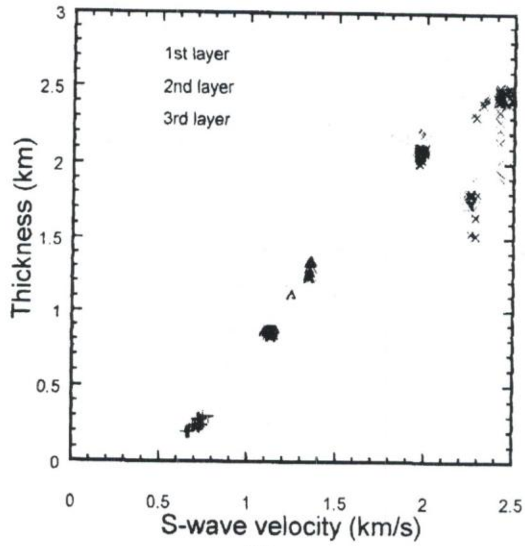

Fig. 6: Average misfit with increasing generation in generic inversion of phase velocity.

Table 1: Search limits for genetic algorithm (GA).

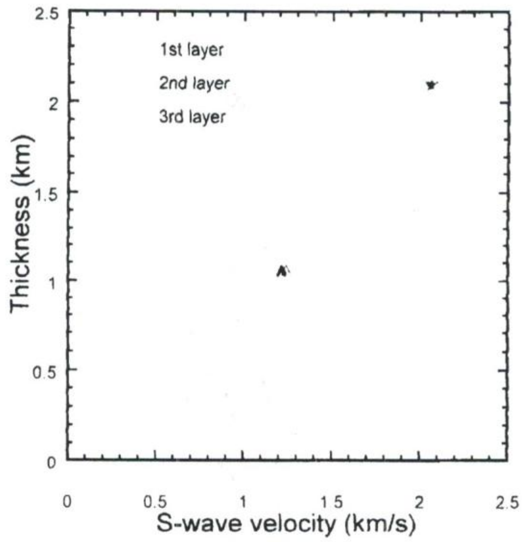
| Layer no. | Vs(km/s) | Thickness (km) | $\rho$(ton/m$^3$) |
|---|---|---|---|
| 1 | 0.5-1 | 0.1-1.1 | 1.8 |
| 2 | 0.9-1.6 | 0.2-2.0 | 2.0 |
| 3 | 1.7-2.5 | 0.4-2.5 | 2.3 |
| 4 | 2.9-4.1 | $\infty$ | 2.5 |

of nonlinearity of the misfit function. All the models generated were examined following the idea of acceptable solution, defined as the models having misfit less than certain value. First acceptable solution level used was 10% of the minimum misfits for all models. Fig. 7a shows the distribution of the parameters for all acceptable models. The parameters for the first layer converge to a point but for the second and third layers they show 2 to 3 local minimum. Then the acceptable limit was decreased to 3% of the minimum misfit. Now almost all the parameters cluster to a single point (Fig. 7b). The final S-wave model is acquired averaging from these parameters.

The inverted phase velocity is compared with the observed one (Fig. 5) and it shows a good fit. The final S-wave velocity structure is displayed in Fig. 8 and tabulated in Table 2. The dashed line indicates the S-wave velocity structure obtained from empirical relation with P-wave velocity, determined by seismic refraction (Kobayashi and Seo 1980). The two profiles don't show a good fit because the S-wave velocity from refraction is calculated with empirical formula from P-wave velocity. The velocity determined for multi layer media by refraction method has no good resolution. This method gives the good estimation




(a)




(b)

Fig. 7: Parameters distribution in acceptable solutions in genetic inversion.

**Table 2: S-wave velocity determined by microtremor array.**

| Layer number | Vs(km/s) | Thickness(km) |
|---|---|---|
| 1 | 0.72 | 0.25 |
| 2 | 1.21 | 1.02 |
| 3 | 2.11 | 2.08 |
| 4 | 3.52 | ∝ |

of velocity of the basement only. Because of this reason the S-wave velocity structure determined by array measurement and refraction does not fit well.

## GROUND MOTION SIMULATION BY FINITE DIFFERENCE METHOD

The second part of this study consists of simulation of ground motion in the area of the microtremor array measurement by finite difference method.

The analysis of FD scheme of Virieux (1984) is used that includes velocity and stress in the equation of motion. The two-dimensional SH-wave propagation equation in heterogeneous medium is given by

$$\rho(x,z)\frac{\delta^2 v}{\delta t^2} = \frac{\delta}{\delta x}\left[\mu(x,z)\frac{\delta v}{\delta x}\right] + \frac{\delta}{\delta z}\left[\mu(x,z)\frac{\delta v}{\delta z}\right] \tag{7}$$

where, ρ(x,z) is density and μ(x,z) is shear modulus at point M(x, z).

$$\frac{\delta v}{\delta t} = l(x,z)\left[\frac{\delta}{\delta x}(\sigma_{xy}) + \frac{\delta}{\delta z}(\sigma_{zy})\right] \tag{8}$$

where, $l(x,z)$ is inverse of density, $\sigma_{xy}$ and $\sigma_{zy}$ are shear stresses.

$$\sigma_{xy} = \mu(x,z)\frac{\delta v}{\delta x} \text{ and } \sigma_{zy} = \mu(x,z)\frac{\delta v}{\delta z}$$

In the finite difference method derivatives are discretized by centered finite differences. The finite difference equation for 2-D SH-wave propagation is given by

$$V_{i,j}^{k+1/2} = V_{i,j}^{k-1/2} + \frac{dt}{dx}L_{i,j}\left[\Sigma^{k}_{i+1/2,j} - \Sigma^{k}_{i-1/2,j}\right] + \frac{dt}{dz}L_{i,j}\left[T^{k}_{i,j+1/2} - T^{k}_{i,j-1/2}\right]$$
$$\Sigma^{k+1}_{i+1/2,j} = \Sigma^{k}_{i+1/2,j} + \frac{dt}{dx}M_{i+1/2,j}\left[V^{k+1/2}_{i+1,j} - V^{k+1/2}_{i,j}\right] \tag{9}$$

and

$$T^{k+1}_{i,j+1/2} = T^{k}_{i,j+1/2} + \frac{dt}{dz}M_{i,j+1/2}\left[V^{k+1/2}_{i,j+1} - V^{k+1/2}_{i,j}\right]$$

where k is the index for time discretization, i for x-axis discretization and j for z-axis discretization, dt grid step in time, dx and dz are grid steps along x and z axis. L is lightness (inverse of density) inside the medium and M is shear modulus. Velocity at time (k+1/2)dt, numerical stress $(\sum, T) = (\sigma_{xy}, \sigma_{zy})$ at time kdt is computed explicitly from velocity at (k-1/2)dt and stress at kdt.

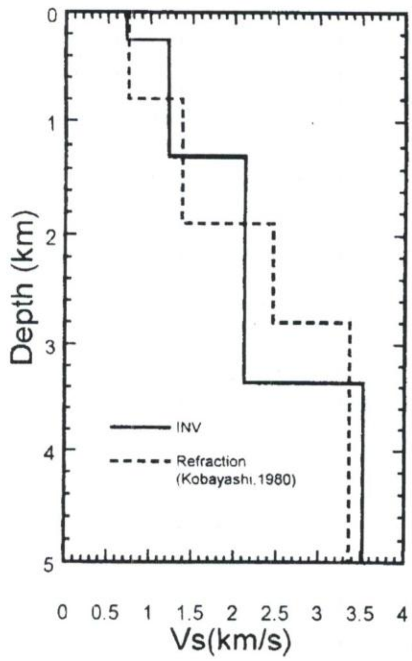



**Fig. 8: S-wave velocity profile inverted phase velocity of surface wave and dashed line indicates S-wave profile from empirical relation with P-wave velocity determined from seismic refraction.**

Because of limitation in computer memory, boundary condition should be applied at the most outer side of the FD models. But these boundaries will cause some non-physical reflection, to suppress this reflection absorbing boundary condition is applied on the bottom of the model. The boundary conditions depend upon the problem and the input source. In this study symmetric boundary condition and vertical plane S-wave as input is used.

## GROUND MOTION SIMULATION

Subsurface structural model is estimated from the microtremor array measurement. On the basis of this model the propagation of wave was observed using plane wave as the source. The incident velocity is given by Ricker wavelet

$$f(t) = \frac{\pi}{2}\left(a - \frac{1}{2}\right)e^{-a} \tag{10}$$

where, $a = \pi(t - t_s)^2 / t_p^2$

In this study $t_p$ is 1.25 sec. and $t_s$ is 1.36 sec.

In order to model correctly the plane wave, symmetric conditions were applied along the two vertical boundaries. The model is extended horizontally for about 50 km on both sides for numerical stability. The incident wave is applied beneath the basin.

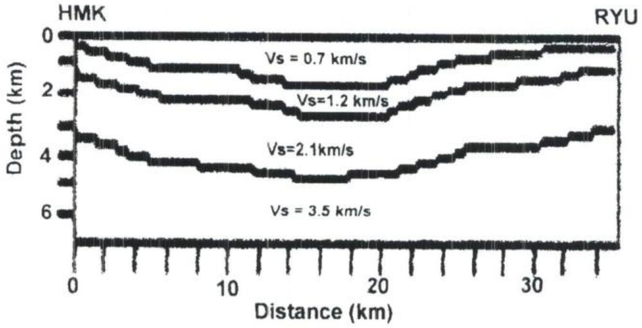



Fig. 9: Subsurface structural model along the profile between Hamaoka and Ryuhyoh (HMK-RYH).

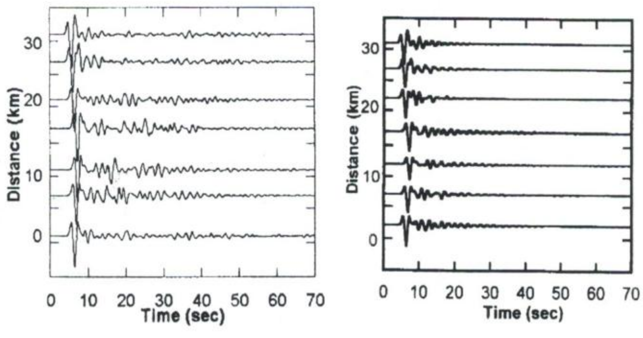



Fig. 10: Computed velocity along the profile HMK-RYU (a) 2-dimensional subsurface model (b) 1-dimensional subsurface model.

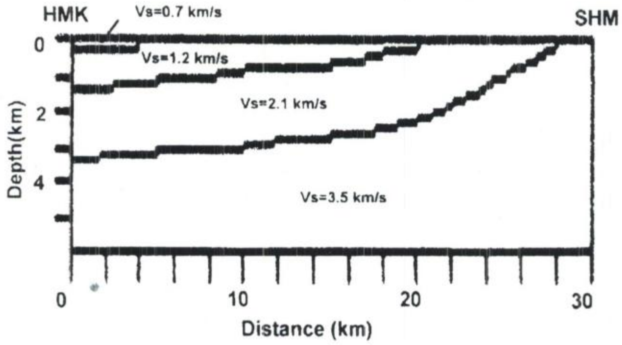



Fig. 11: Subsurface structural model along the profile between Hamaoka and Shimada (HMK-SHM).

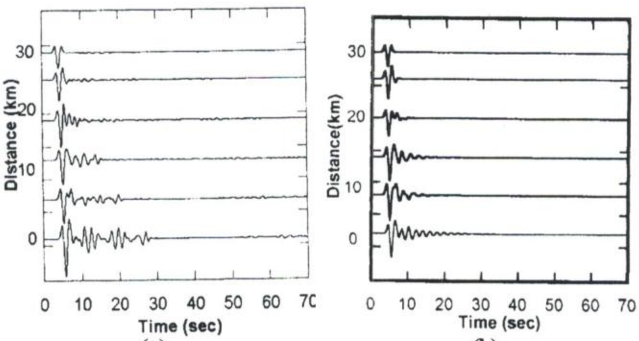



Fig. 12: Computed velocity along the profile HMK-SHM (a) 2-dimensional subsurface model (b) 1-dimensional subsurface model.

The simulation is carried out along two profile lines, one along the coastal line between Hamaoka and Ryuhyoh (hereafter HMK-RYU) and another towards north from Hamaoka to Shimada (hereafter HMK-SHM). The first profile line falls all along the sediments filled basin whereas the second profile line meet the basement about 28 km to the north of Hamaoka. The models along these two lines are shown in Fig. 9 and Fig. 11. The subsurface structural model along HMK-RYU is derived from keeping the shape of the profile same as the one from refraction (Kobayashi 1980) and using thickness and velocity determined from microtremor array survey. Similarly the model along HMK-SHM profile is also derived by the same procedure as in HMK-RYU profile. The Q-value in the computation is determined by the empirical formula Q = Vs/10.

The computed velocities on the surface of the 2-dimensional model along HMK-RYU is shown in the Fig. 10a. Each trace is normalized with the maximum of all the traces. The ground motion at a distance more than 6 km from Hamaoka show dispersive wave characteristics and with higher amplitude and longer duration. This implies that the thickness of upper layer with S-wave velocity of 0.72 km/s has significant influence in the characteristics of long-period ground motion as the thickness of this layer increases high amplitude and long duration ground motion is observed around the center of the basin. Ground velocities are computed in 1-dimensional subsurface structures at various points (Fig. 10b), the effect of 2–dimensional structure is clearly observed with more complicated wave pattern and long duration.

The computed velocities on the surface of the model along HMK-SHM is shown in Fig 12. Each trace is normalized with the maximum of all the traces. In this model the surface layer vanishes after 5km from Hamaoka, up to 5 km dispersive wave characterized with large amplitude is clearly visible. In Fig 12a the ground motions at various points on the surface have complicated pattern with large amplitude and long duration compared with 1-D model in Fig 12b. It is observed that duration of ground motion differs accordingly with the shape of the basin, which shows the strong 2-D effects of sedimentary layer on ground motion.

## CONCLUSIONS

Microtremor array measurements technique has been applied to estimate the deep S-wave velocity profile. Subsurface S-wave velocity structure is estimated from the inversion of phase velocity of microtremors in the wide period range with the global optimization method of genetic inversion. From the simulation of ground motion it is clear that the effect of 2-D subsurface structure is very important in the estimation of ground motion at a site. Microtremor array technique is the most promising and economical method for the determination of deep S-wave velocity structure in a sedimentary basin, which in turn helps in the estimation of ground motion. Better understanding of 2-D or 3-D basin geometry would allow a better simulation of the ground motion.

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