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Analysis of elastic behaviour of granite using homogenisation theory

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ABSTRACT

We carried out photographic analysis of randomly distributed discrete elements in the Inada granite. The modal analysis of granite was also conducted simultaneously with the above photographic analysis. The results show that quartz and feldspar including mica occupy 99.4% of the total volume of the fine-grained granite. Based on the results of previous studies, an elastic homogenisation method is applied to analyse the macro-level stress distribution in the Inada granite, which is a composite material of rock-forming minerals with micro discontinuities.

For proper rock sampling and specimen preparation, the representative elementary volume (REV) should be determined in rock mechanical tests and numerical analyses. We determined the REV of the Inada granite using a stereoscopic microscope and applying a homogenisation numerical analysis.

INTRODUCTION

Sanchez (1980) developed a homogenisation method from partial differentiation and applied it in the estimation of effective physical properties of composite materials. Bensoussan et al. (1978) demonstrated that if the size of a micro structure is small enough as compared to its overall structure, the solution by the homogenisation method tends to a limiting value. Guedes and Kikuchi (1990) estimated the effective properties of an elastic material with complicated micro structures using the homogenisation method, and Fish et al. (1994) analysed the errors incurring in the homogenisation method. Hollister and Kikuchi (1992) established that homogenisation is a better method than a standard mechanics approach in estimating the effective properties of elastic materials when the size of a micro structure is small. In the field of rock engineering, Bear and Verruijt (1987) determined the representative elementary volume (REV) using the void ratio (U_v/U), which is the ratio of the void volume (U_v) to the rock volume (U). Seo et al. (2000a) conducted a study on the viscoelastic behaviour of granite.

We carried out a series of analysis on the Inada granite (a fine-grained granite from Ibaraki, Japan) to calculate its elastic modulus based on the homogenisation method. In order to select the REV, each result acquired from the interpretation models of different sizes was used after reviewing the content of rock-forming minerals, proportion of micro joints, and the change in elastic modulus. The effectiveness of the REV selection method was also reviewed by comparing the values of the homogenisation method with the real experimental results.

EXPERIMENT

The rock specimens used in this study were collected from the Inada granite formed during the Cretaceous–Palaeocene time (Takahasi 1982). Table 1 summarises its mineral composition and physical properties. The rock samples (40 mm × 20 mm × 5 mm) were collected from a quarry. Their flatness and perpendicularity indices were made less than 4/1000 for obtaining accurate test results. Then uniaxial compressive tests were conducted along their long axes with a velocity of 0.2 MPa/sec (Seo et al. 2000a).

Estimation of elastic modulus from experimental results

In order to estimate the elastic modulus, we used the stress-strain curve and the 50% tangential elastic modulus represented by the gradient of the tangent at 50% of the maximum stress during the uniaxial compression of each sample. According to the experimental results, the average uniaxial compressive strength and average elastic modulus of the granite are 178.4 MPa and 101.4 GPa respectively.

NUMERICAL ANALYSIS

Microcrack properties

It is known that 99% of the total volume of the fine-grained Inada granite is composed of quartz, feldspar, and mica. Its mechanical behaviour is known to be greatly influenced by the uniformly distributed microcracks in the rock mass (Seo et al. 2000a). Therefore, we studied the distribution of microcracks which form rift, grain, and hardway as well as three types of rock-forming minerals listed in Table 2. According to Seo et al. (2000a), the microcracks of the Inada granite can be classified into the following four types: between quartz and quartz (JQQ), between quartz and feldspar (JQF), between feldspar and feldspar (JFF), and mica-related (JMQF). The material coefficient of the microcracks is calculated by taking the time t as zero (0). It means that the viscoelastic properties of the granite are ignored in the elastic analysis.

(Joint element 1, JQQ):

$$K(t) = 38.2+0.41\exp(-t/54.7)+0.21\exp(-t/1.41) \quad (1)$$

$$G(t) = 32.2+0.31\exp(-t/0.5)+0.88\exp(-t/42.7) \quad (2)$$

(Joint element 2, JQF):

$$K(t) = 160.3-2.65\exp(-t/0.55)-3.15\exp(-t/9.21)-5.13\exp(-t/74.9) \quad (3)$$

$$G(t) = 34.1+0.39\exp(-t/0.5)+0.69\exp(-t/33.4) \quad (4)$$

(Joint element 3, JFF):

$$K(t) = 96.7 \quad (5)$$

$$G(t) = 56.8+0.17\exp(-t/0.3)+0.52\exp(-t/3.72)+0.74\exp(-t/60.0) \quad (6)$$

(Joint element 4, JMQF):

$$K(t) = 10.1+3.35\exp(-t/31.1) \quad (7)$$

$$G(t) = 11.3+151.7\exp(-t/0.3) \quad (8)$$

Table 1: Modal composition and physical properties of Inada fine-grained granite (Seo et al. 2000b)

Components	Properties
Quartz (Q)	25.5 %
Feldspar (F)	66.8%
Mica	7.1%
Others	0.6 %
Colour	Grayish white
Texture	Equigranular
Grain size of Q and F	1 ~ 4 mm

Calculation model

The calculation model was constructed on the basis of the following procedure. Considering that the grain size in the Inada granite ranges between 1 and 4 mm, the maximum side of the element in the calculation model was restricted to 0.6 mm so that an element could be completely included in the mineral. The area of each element was 0.22 mm^2. The element shape was decided to be rhombus so that it could well represent the granular boundary curve. In addition, nine gradual interpretation models were prepared whose element numbers were 36 (6×6, 7.9 mm^2), 100 (10×10, 22 mm^2), 196 (14×14, 43.1 mm^2), 324 (18×18, 71.3 mm^2), 484 (22×22, 106.5 mm^2), 676 (26×26, 148.7 mm^2), 900 (30×30, 198 mm^2), 1156 (34×34, 254.3 mm^2), and 1444 (38×38, 317.7 mm^2). They were made at 6 locations for 196-element (or lower) models, at 4 locations for 484-element models, and at 2 locations for 1444-element models. Fig. 1 shows one of the calculation models that has 1444 (38×38) elements.

Table 2: Young's modulus and Poisson's ratio of minerals (Seo et al. 2000b)

Minerals	E(GPa)	υ
Quartz	95.6	0.079
Feldspar	69.7	0.301
Mica	88.1	0.248

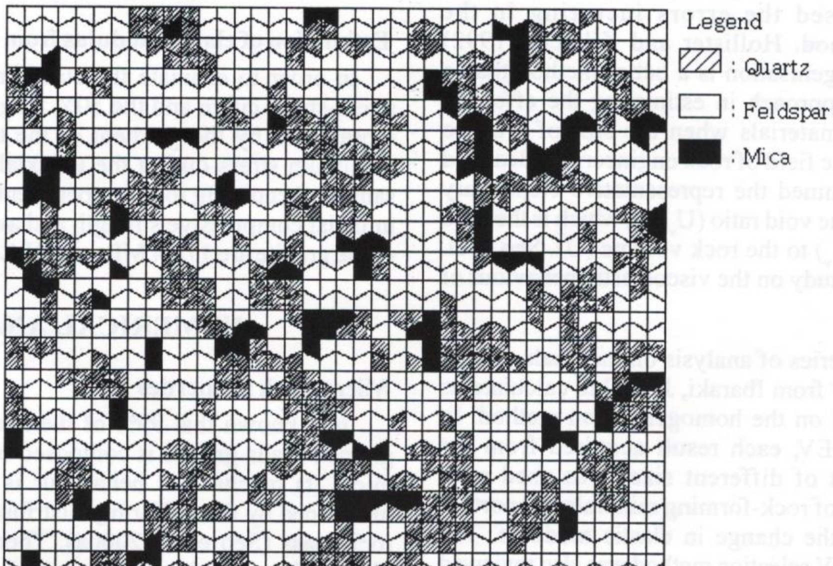

Fig. 1: A homogenisation analysis mesh for 1444-element model

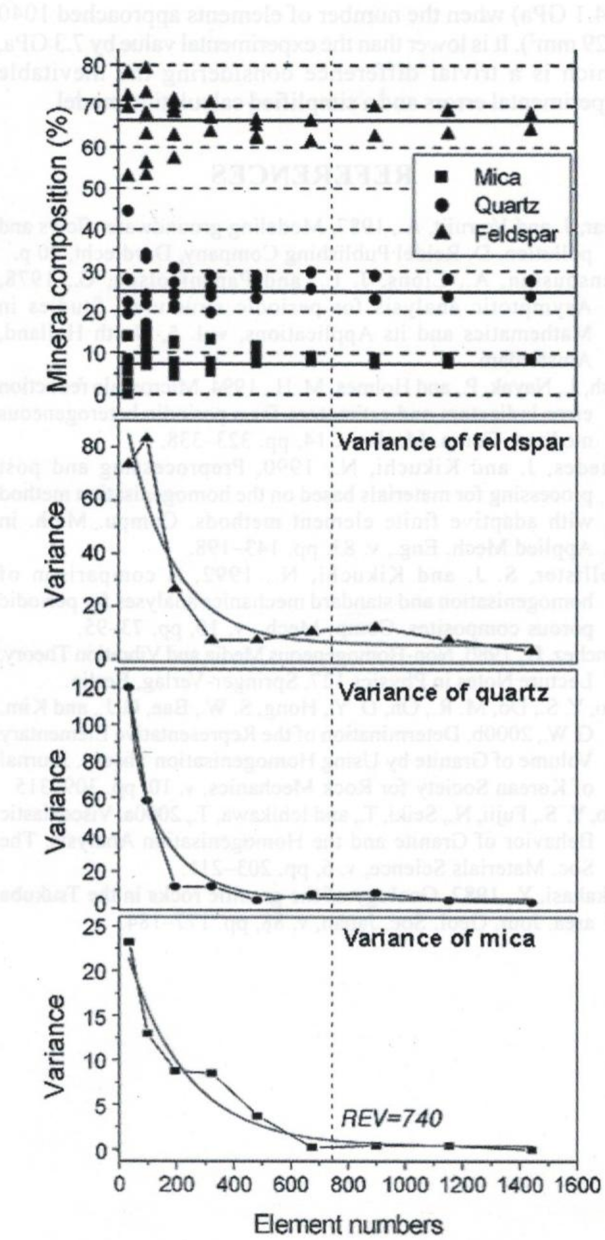

Fig. 2: Variation of mineral composition with element number

ANALYSIS RESULTS AND DISCUSSION

Ratio of rock-forming minerals

We calculated the content of each rock-forming mineral using the number of elements occupied by that mineral in the gradual calculation model. Fig. 2 shows the content of rock-forming minerals in the gradual interpretation model and the variance of each distribution from the average distribution of rock-forming minerals in the Inada granite.

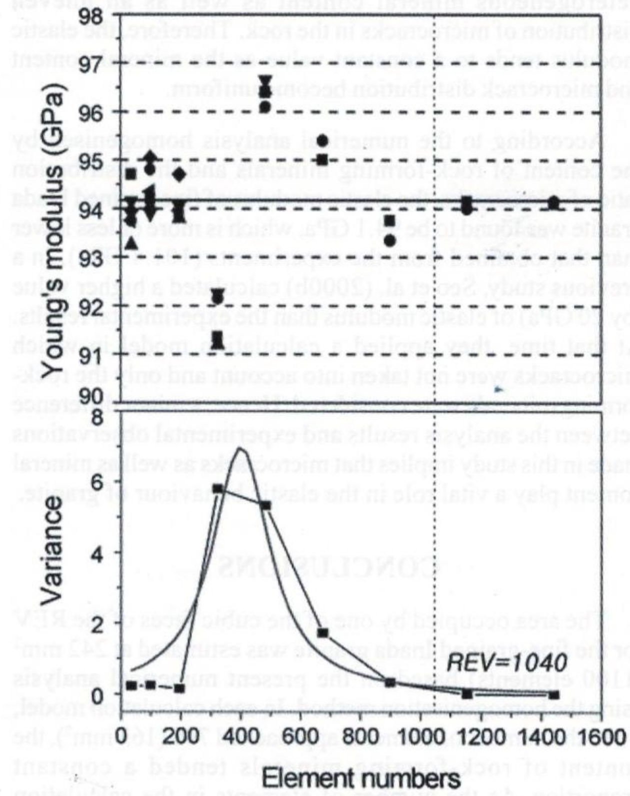

Fig. 3: Variation of Young's modulus with respect to element numbers

The solid line in the mineral content graph is the content of each rock-forming mineral observed under the polarised microscope. All of three rock-forming minerals tended to certain values as the number of elements increased and the proportions of mineral changes decreased. Regarding feldspar and quartz, the values were found similar to those observed under the microscope. Although mica showed somewhat a higher ratio, it also had a similar tend. The greater variance in the calculation model with a small number of elements means an irregular distribution of rock-forming minerals. The variance of mineral content did not change after the 740 (162.8 mm^2) element model, indicating that the proportion of mineral content in the fine-grained Inada granite becomes constant when its surface area is larger than 162.8 mm^2.

Elastic modulus

The results of stepwise numerical calculations and corresponding variances are shown in Fig. 3. The study revealed that the variance between the gradual elastic modulus and average elastic modulus is insignificant. The elastic modulus tends to a constant value in the 1040-element (228.8 mm^2) or higher interpretation model. The reason behind a greater variance of elastic modulus in the models with a smaller number of elements can be attributed to their heterogeneous mineral content as well as an uneven distribution of microcracks in the rock. Therefore, the elastic modulus tends to a constant value as the mineral content and microcrack distribution become uniform.

According to the numerical analysis homogenised by the content of rock-forming minerals and the distribution ratio of microcracks, the elastic modulus of fine-grained Inada granite was found to be 94.1 GPa, which is more or less lower than that obtained from the experiments (101.4 GPa). In a previous study, Seo et al. (2000b) calculated a higher value (by 20 GPa) of elastic modulus than the experimental results. At that time, they applied a calculation model in which microcracks were not taken into account and only the rock-forming minerals were considered. Hence, a minor difference between the analysis results and experimental observations made in this study implies that microcracks as well as mineral content play a vital role in the elastic behaviour of granite.

CONCLUSIONS

The area occupied by one of the cubic faces of the REV for the fine-grained Inada granite was estimated at 242 mm^2 (1100 elements) based on the present numerical analysis using the homogenisation method. In each calculation model, when the number of elements approached 740 (163 mm^2), the content of rock-forming minerals tended a constant proportion. As the number of elements in the calculation model increased, the variance between the ratio of rock-forming minerals at each step and their average ratio decreased.

The elastic modulus acquired from numerical analysis using the homogenisation method became constant (94.1 GPa) when the number of elements approached 1040 (229 mm^2). It is lower than the experimental value by 7.3 GPa, which is a trivial difference considering the inevitable experimental errors and a simplified calculation model.

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