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Earthquake-induced slope vibration and its relationship with material properties, slope height, and input wave frequency

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ABSTRACT

The relationship of the amplification of vibration on the surface with the slope geometry, earthquake input signal (mainly frequency of the wave) and the material property of the slope is studied using the finite difference program FLAC. A sinusoidal wave with acceleration of 1 m/s^2 is applied for the duration of 0.25 sec. The influence of varying input frequency is investigated applying the frequencies in the range of 3 Hz to 15 Hz. The variation of amplification due to the change in shear modulus of the material in the slope and that due to slope height is also investigated.

It is found that the crests of the higher slopes are amplified most by the lower input frequency and smaller slopes are amplified most by the higher input frequency. The overall magnitude of the amplification is maximum for smaller slopes when input signals of higher frequency are applied. The generation of standing waves at certain harmonic frequencies could be the reason behind such a high amplification. It is also seen that the amplification peaks of vibrations repeat at different slope heights, which is considered reflection of the harmonic effect. A clear resonance effect is seen when the amplification of vibration is plotted against the shear modulus. The peaks of such amplification also repeat periodically as a result of varying shear modulus. Present study provides some preliminary relationship of amplification with slope height and shear modulus of the material in the slope. The horizontal amplification of vibration as much as 17 (horizontal acceleration in the order of 1.7 g) is obtained at the crest of normal limestone slope with 20 m height when an input signal of 15 Hz frequency is applied. It is also revealed that for extremely lower values of shear modulus, there is mostly attenuation instead of amplification and for extremely high values of shear modulus, amplification is negligible as compared to the certain range of intermediate shear modulus. Maximum amplification in the order of 6.5 (horizontal acceleration of 0.66 g) is achieved for the shear modulus of 3000 MPa with slope height of 40 m when the input acceleration of 0.1 g is applied. This type of relationship is believed to have great importance for seismic microzonation study as the results might be used along with the digital elevation model for separating different zones of hazard levels. However, one should be aware that this result is for the slope with particular combination of material properties, geometry and earthquake input signals, and generalisation could lead to misleading results.

BACKGROUND

Ground response analysis plays a vital role in predicting the ground surface motion during dynamic loading as well as in evaluating the earthquake-induced forces. A number of techniques have been developed to analyse the ground response in terms of one, two, and three dimensions. Since it is not always possible to arrive at an analytical solution to such problems (Desai and Abel 1972), numerical modelling techniques are some of the convenient ways of dealing with them. The effect of topography on the ground response was investigated by a number of authors. They all agreed in general that topography and site conditions played a significant role in amplifying the ground shaking. Boore (1972) used finite difference techniques and found that the surface waves were amplified at the crest and were slightly amplified or attenuated at the flanks, and that the effects were frequency-dependent. Similarly, Davis and West (1973), measured the height of crests and bases of three mountains and found that the mountain crests were amplified significantly more than their bases. They also pointed out that the shear waves caused resonance of the mountain if their wavelengths were comparable to the dimensions of the mountain. Havenith et al. (2003) studied the influence of topography as well as site-specific conditions on the Ananevo rockslide in the north-eastern Tien Shan Mountains and found that the strongest amplifications occur at the top of the slope and along the crest. Athanasopoulos et al. (1998) found that the concentration of damage due to the 1995 earthquake around the town of Egion, Greece, was in the elevated region adjacent to the crest of high and steep escarpments. Castro (1999) estimated the concentration of damage within a range of about 40 m from the slope edges for the earthquake of 1999 in Columbia. Geli et al. (1988) carried out the experimental and theoretical study to find the effect of topography on earthquake ground motion. They noted significant amplification of vibration at the hilltop with respect to the base for the frequencies corresponding to the wavelengths about equal to the mountain width. Bouchon (1973) also found amplification towards the top ridge and attenuation along the ridge flanks.

Under these circumstances, finite difference numerical modelling program FLAC (Fast Lagrangian Analysis of Continua) was used to investigate the response of slopes during dynamic loading so as to understand the change in

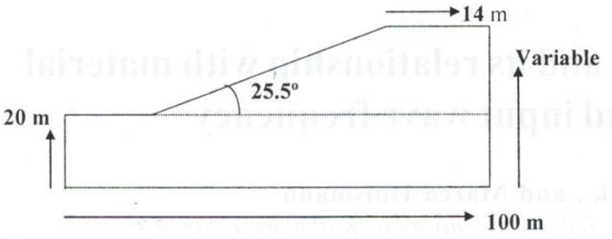


Fig. 1: General geometry of model slope used in numerical calculation

seismic amplification due to the change in properties of the ground. The variables of this study were shear modulus, frequency and wavelength of the input signals, and the height of slope.

NUMERICAL MODELLING PROGRAM, FLAC

FLAC was originally developed by Peter Cundall in 1986 and commercially released by ITASCA, C.G., and Inc. The capacity of performing large two-dimensional calculations without excessive memory requirements and the facility for dynamic modelling make it versatile in rock mechanics and earthquake engineering.

FLAC performs lagrangian analysis based on an explicit finite difference scheme. This means that every derivative in the set of governing equations is replaced directly by an algebraic expression written in terms of the field variables (e.g., stress or displacement) at discrete points in space. Unlike finite element programs, FLAC does not combine the element matrices into a large global stiffness matrix so that the memory requirements are reduced and it is efficient to regenerate the finite difference equations at each step.

Dynamic modelling can be executed in FLAC using either of two methods: The equivalent linear method or fully non-linear method. The equivalent linear method is common in the study of soil-structure interaction and earthquake engineering for modelling wave transmission in layered sequences. In the equivalent linear method (Seed and Idriss 1970), a linear analysis is performed with the same initial values assumed for the damping ratio and the shear modulus.

NUMERICAL SIMULATION FOR DYNAMIC LOADING IN FLAC

Numerical calculation for dynamic loading for a slope with simple geometry is carried out in order to investigate the influence of its geometry and material properties on slope motion amplification during dynamic loading by an earthquake. The variation pattern of the displacement, velocity, and acceleration due to the slope motion are studied. The study is more focussed towards the crest of the slope, as the publications suggest that the vibrations are significant amplified there. The methodology suggested in FLAC Manual is followed in this study.

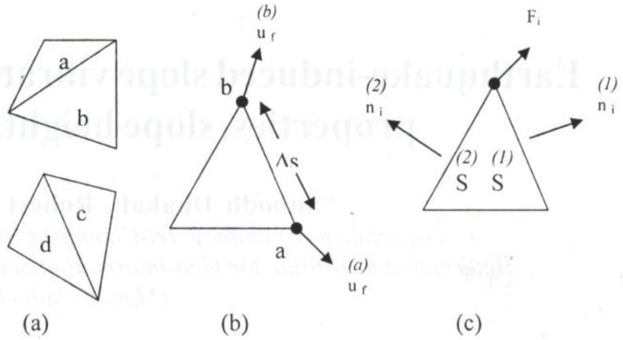


Fig. 2: (a) Overlayed quadrilateral elements used in FLAC; (b) typical triangular element with velocity vectors; (c) nodal force vector (ITASCA 2000)

Slope geometry

Present study is based solely on the numerical simulation of the slope response during dynamic loading with the variation in slope geometry, input wave frequencies, and material properties. Therefore, for the modelling purpose, some of the simple schematic slope profiles are used. The dimensions of the model are kept small to moderate in order to reduce the calculation time and the memory needed for the computer. An example of slope geometry used in this study is shown in Fig. 1.

Discritisation and generation of mesh

Like in other finite element programs, in FLAC, the structure is built out of finite amount of elements by discritisation to form a mesh. The mesh is composed of quadrilateral zones (or elements), internally subdivided in two sets of triangular elements of constant-strain (Fig. 2). The four triangular sub-elements are termed *a*, *b*, *c*, and *d*. The deviatoric stress components of each triangle are maintained independently. The zones or elements are organised in rows and columns. To refer to a particular zone, a pair of numbers representing the column and row are used.

Constitutive models

A constitutive model is a mathematically formulated expression to show a relationship between stress and strain, and is used to predict the mechanical behaviour of engineering materials during dynamic loading. When selecting a constitutive model for a particular analysis, two considerations are important:

- the known mechanical characteristics of the material being modelled, and
- the intended application of the results.

The Mohr–Coulomb plasticity model was used for this study. This type of model is used when the stress levels are such that the failure of intact material is expected. It represents a material that yields when subjected to shear loading.

Material properties

The material properties affect significantly the ground response during dynamic loading. Bulk modulus, shear modulus, density, friction angle, cohesion, and tensile

Table 1: Material properties used for the simulation

Parameters	Values
Slope angle (°)	25.5°
Slope height (m)	Variable
Density (kg/m^3)	2000
Shear modulus	Variable
Bulk modulus	Variable
Friction angle (°)	42.0
Cohesion (MPa)	6.7
Tensile strength (MPa)	1.58

strength are needed as material properties in the Mohr–Coulomb constitutive model. In this study, the material type was continuously changed to find the corresponding relationship with the amplification of vibration. The material properties of some particular limestones given in UDEC Manual (Table 1) were considered in the present study.

Initial equilibrium

The acceleration due to gravity, g = 9.8 m/s^2 was applied to the model and the program was run until an equilibrium state was obtained. Generally, it is possible to come up with an equilibrium state of the model slope for the recorded displacements and velocities at some of its selected zones (i.e., at the slope crest). A plot of the horizontal and vertical displacements against the unbalanced force becomes sub-horizontal when reaching the equilibrium state. The velocity of the model slope tends to zero after reaching the initial equilibrium state. Once a static equilibrium is reached, the displacements and velocities are reset to zero for the purpose of removing the kinetic energy that could affect the subsequent dynamic calculation.

Boundary conditions

Boundary conditions should be differentiated for static- and dynamic-stage calculations. The static calculation is done prior to the dynamic analysis. For the static analysis, the first and the last columns in the x-direction and the first row in the y-direction are fixed. For the dynamic calculation however, free-field boundaries are applied to the two vertical sides of the model to avoid distortions in the wave transmissions and to make the model close to reality, since the slopes are not isolated with the other topographic features in the lateral sides. The base is kept rigid with the introduction of quiet boundaries at the first row in the x- and y-directions.

Input parameters for dynamic loading

The input parameters for dynamic loading can be applied in the form of velocity, acceleration, or stress fields. Before applying the dynamic input parameters, the velocity, acceleration or stress fields are reset to zero of avoid the effect of static calculation. At the beginning of the dynamic stage, the dynamic mode is reactivated and the dynamic time is reset to zero, to avoid any calculation errors that could be induced from the previous stage (static equilibrium).

For this research, a sinusoidal wave function with acceleration, of 1 m/s^2 and duration of 0.25 sec was defined using the FISH (a programming language) function available in FLAC. The frequency of the input signals however is varied to investigate its effect on amplification. Later on, the defined input function was applied through the bottom of the model (i.e., the first row).

Dynamic damping

For the dynamic analysis, a Rayleigh damping value of 5% was applied to compensate for the energy dissipation through the medium. The value closely corresponds with that of the geological materials (ITASCA 2000; Paolucci 2002).

Output

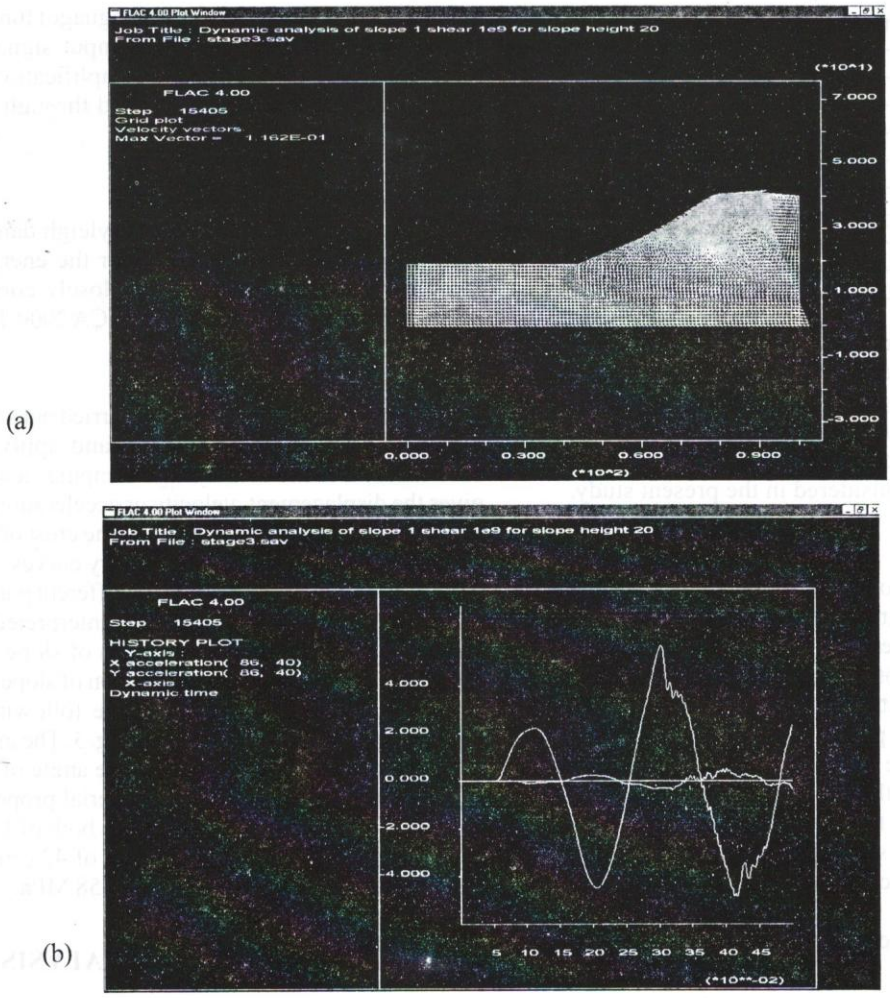
The numerical calculation was carried out after introducing parameters for dynamic loading and applying boundary conditions as well as the Rayleigh damping value. The analysis gives the displacement, velocity or acceleration path histories at the co-ordinates of interest (e.g., the crest of the slope with co-ordinates 40, 30). The path history curves are plotted and compared against dynamic time at different parts of the slope. The results obtained in this way are interpreted and analysed to find out the required relationship of slope geometry and material properties to the amplification of slope vibration. The details of the result are given in the following sections. A sample output in FLAC is shown in Fig. 3. The input parameters were the slope height of 20 m, slope angle of 25.5°, and the input frequency of 5 Hz. Other material properties included the shear modulus and bulk modulus both of 1 GPa, material cohesion of 6.7 MPa, friction angle of 42°, mass density of 2000 kg/m^3, and tensile strength of 1.58 MPa.

RESULTS OF ANALYSIS

The results of a total of 417 calculations are summarised below. Shear modulus, slope height, and the frequency of input signals were the input parameters of the analysis.

Shear modulus versus amplification

The input parameters used for the numerical calculation by varying shear modulus are listed in Table 2. In general, to make the energy of the system constant, the amplitude of wave should increase in the material with lower shear modulus and should decrease in the material with higher shear modulus. It clearly indicates that the amplitude of the wave is increased due to the material with a lower shear modulus. Simulation in FLAC reveals that In addition to this, the harmonic effect strongly controls the amplification of vibration in the slope due to the change in shear modulus. As it can be seen in Fig 4, the first peak of amplification is at around 10–300 MPa of shear modulus and the second peak at around 1000–3000 MPa of shear modulus depending on the height of the slope. These peaks are the result of the resonance of the slope in that particular combination of material properties and input signals. It is also seen that if the slope height is lower, the peak of amplification starts at a lower value of shear modulus. For the higher values of shear modulus, after certain limit the amplification remains constant as is seen in Fig. 4. For the shear modulus greater than 10000



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Job Title : Dynamic analysis of slope 1 shear 1e9 for slope height 20
From File : stage3.sav

FLAC 4.00
Step    15405
Grid plot
Velocity vectors
Max Vector =    1.162E-01
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Job Title : Dynamic analysis of slope 1 shear 1e9 for slope height 20
From File : stage3.sav

FLAC 4.00
Step    15405
HISTORY PLOT
  Y-axis :
X acceleration(  86,  40)
Y acceleration(  86,  40)
  X-axis :
Dynamic time
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Fig. 3: Sample example of the screen capture of FLAC calculation to show (a) the maximum velocity vectors and mesh generation. (b) the variation of X and Y accelerations

MPa, amplification remains constant in between 2 and 3. The maximum magnitude of seismic amplification in this study was found to be 6.5 for the slope height of 40 m with the material having shear modulus of 3000 MPa.

Slope height and input frequency versus amplification

The input parameters used for the numerical calculation to find the relationship of slope height and input frequency with amplification of vibration is listed in Table 3, however the slope height and frequency is varied continuously to find the variation pattern. Similarly, bulk modulus is also varied but for the analysis purpose, the value of bulk modulus that gives maximum amplification is considered. Present study reveals that amplification of vibration in the slope is strongly frequency dependent as some slopes are amplified by some input frequency while other slopes with different heights and material properties are amplified by another input frequency. As an example, for the input frequency of 10 Hz, a 40 m slope gives the maximum seismic amplification, whereas for the input frequency of 15 Hz, a 20 m slope gives maximum amplification (Fig. 5). It is noteworthy that the amplification of vibration as much as 17 (horizontal acceleration in the order of 1.7 g) is obtained for the slope height of 20 m at the input frequency of 15 Hz.

In general it is seen that the amplification for higher slope height is maximum with the lower input frequency whereas that for the lower slope height is maximum with the higher input frequency. The result revealed in general that the longer objects would be influenced more by lower frequencies whereas shorter objects are more affected by the higher frequency signals. This is because the taller objects will have smaller resonance frequency than the shorter ones. In addition, harmonic effect is strongly reflected as can be seen in the plot of the slope height versus maximum amplification with different input frequencies (Figs. 5 and 6). Generation of standing waves by harmonic frequencies that control the amplification of the slope might be the reason behind it.

The general nature of the relationship of the slope height and input frequency with the amplification can be generalised and shown schematically in qualitative terms

Table 2: Parameters for the slope geometry, input signal and material properties used to investigate the variation of amplification with the change in shear modulus.

Parameters	Values
Slope angle (°)	25.5°
Slope height (m)	Variable
Density (kg/m^3)	2000
Input wave frequency (Hz)	5
Bulk modulus (MPa)	200
Friction angle (°)	42.0
Cohesion (MPa)	6.7
Tensile strength (MPa)	1.58

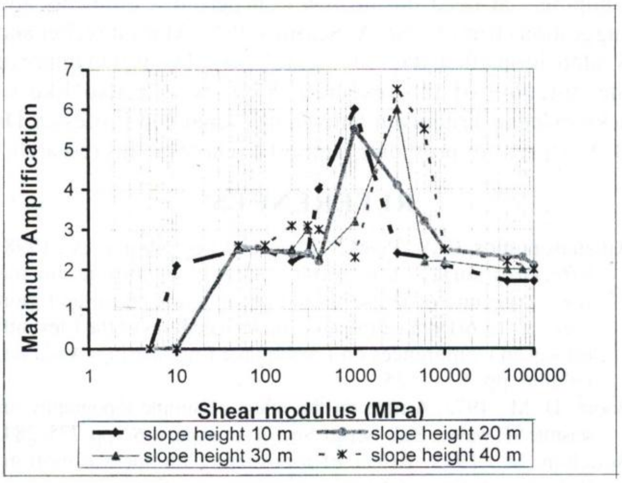

Fig. 4: Variation of amplification with shear modulus for different slope heights at slope angle of 25.5° and frequency 5 Hz.

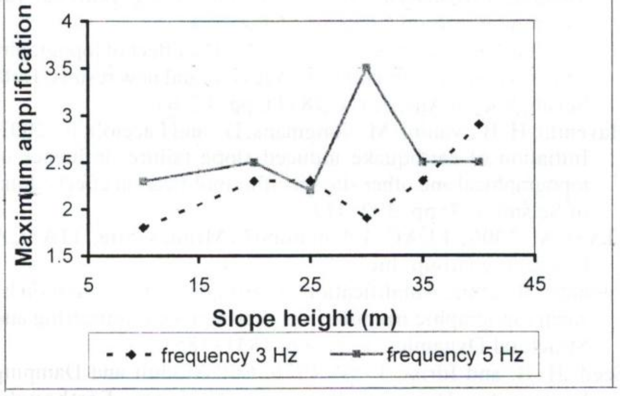

Fig. 6: Variation of amplification with slope height for different input frequencies at slope angle 25.5°

Table 3: The input parameters used in numerical simulation to show the variation of amplification with slope height and input frequency

Parameter	Values
Slope angle (°)	25.5°
Slope height (m)	Varied
Density (kg/m^3)	2000
Input wave frequency (Hz)	Varied
Shear modulus (MPa)	10000
Bulk modulus (MPa)	200
Friction angle (°)	42.0
Cohesion (Mpa)	6.7
Tensile strength (MPa)	1.58

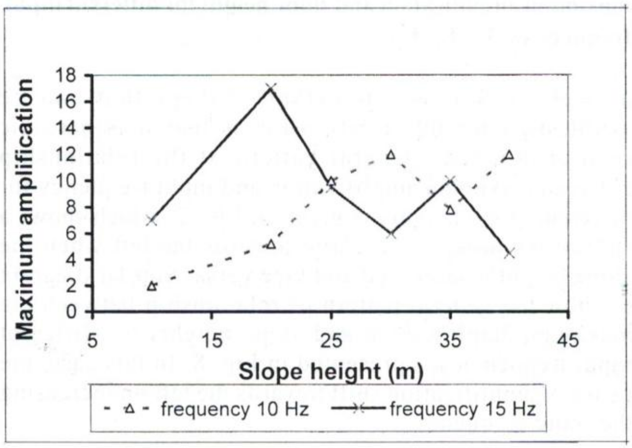

Fig. 5: Variation of amplification of vibration with slope height for different input frequencies at slope angle of 25.5°

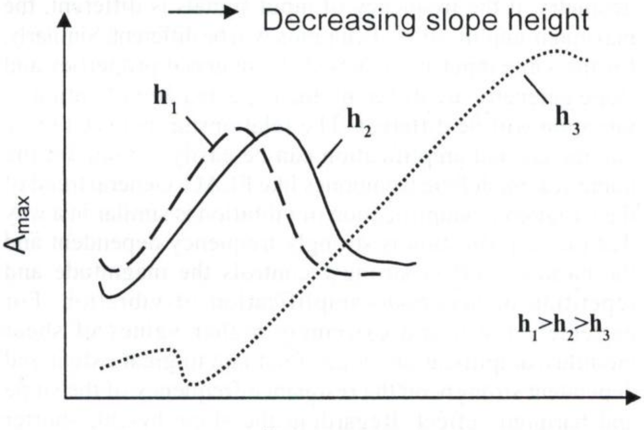

Fig. 7: General sketch of the relationship between the maximum amplification of vibration (A_{max}) and input frequency for different slope heights (h_1>h_2>h_3)

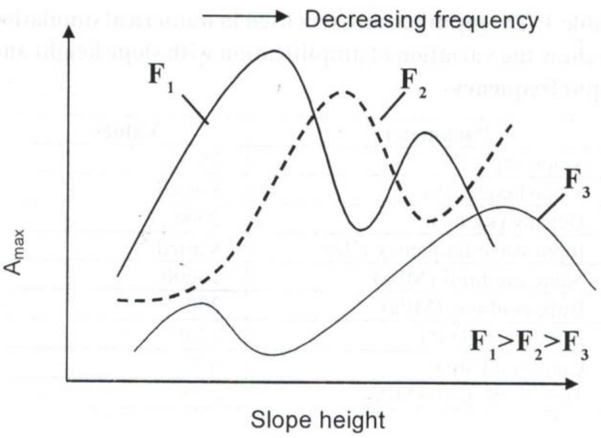

Fig. 8: General sketch showing the relationship between maximum amplification and slope height for different input frequencies ($F_1>F_2>F_3$)

as well. The most important concept that can be synthesised for this relationship is that it is periodic relationship. The general pattern of the relationship between maximum amplification and input frequency for different slope heights is given in Fig. 7, which shows a shift of the peak of the curve towards the left when the slope height is increased and vice versa. Similar diagram to show the general pattern of relationship between the maximum amplification and slope heights for different input frequencies is presented in Fig. 8. In this case, the peaks of amplification shift towards the left on increasing the input frequency.

CONCLUSIONS

The material properties, slope geometry and input wave signals will have great influence on the amplification of vibration of slope during earthquake. This means that even for the slope with same material properties and slope geometry, if the frequency of input signals is different, the maximum amplification of vibration will be different. Similarly, for the same input frequency if the material properties and slope geometry are different, the expected amplification of vibration will be different. The relationship between these parameters and amplification can be easily obtained using numerical modelling techniques like FLAC. General trend of the variation of amplification of vibration is similar in a way that the amplification is strongly frequency dependent and the harmonic effect strongly controls the magnitude and repetition of maximum amplification of vibration. For extremely lower and extremely higher values of shear modulus, amplification values fluctuate to great extent and dependent strongly on the resonance frequency of the slope and harmonic effect. Regarding the slope height, shorter slopes are amplified more by the application of higher input frequency whereas taller slopes are amplified more by smaller input frequencies. Maximum horizontal amplification of vibration of 17 (horizontal acceleration of 1.7 g) as obtained in this study indicates the probable scale of disaster if the material properties, slope geometry and earthquake input signals used for this simulation match in reality. Such type of relationship and probable scale of problem should be considered during the design of slope. The earthquake prone countries in particular should be aware about this fact and the design of slope should be done to prevent the damage due to anticipated earthquake in that particular area of interest.

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REFERENCES

Athanasopoulos, G. A., Pelekis, P. C., and Leonidou, E. A., 1998, Effects of surface topography and soil conditions on the seismic ground response-including liquefaction in the Egion (Greece) 15/6/1995 earthquake. *In*: Proceedings of the Eleventh European Conferences on Earthquake Engineering, Abstract. volume, pp. 525–525.

Boore, D. M., 1972, A note on the effect of simple topography on seismic SH waves. Bull. Seism. Soc. of America, v. 62, pp. 275–284.

Bouchon, M., 1973, Effect of topography on surface motion. Bull. Seism. Soc. of America, v. 63 (3), pp. 615–632.

Castro, E., 1999, Topographic site characteristics and damage pattern of the January 25th 1999 earthquake in Armenia-Colombia. M. Sc. Thesis submitted to ITC Delft, The Netherlands, 84 p.

Davis, L. L. and West, L. R., 1973, Observed effects of topography on ground motion. Bull. Seism. Soc. of America, v. 63(1), pp. 283–298

Day, R., 2002, *Geotechnical Earthquake Engineering Handbook*. New York, McGraw-Hill.

Desai, C. S., and Abel, J. F., 1972, *Introduction to the Finite Element Method: A Numerical Method for Engineering Analysis*. Van Nostrand Reinhold Company, New York.

Geli, L., Bard, P. Y., and Jullien, B., 1988, The effect of topography on earthquake ground motion: A review and new results. Bull. Seism. Soc. of America, v. 78 (1), pp. 42–63.

Havenith, H. B., Vanini, M., Jongmans, D., and Faccioli, E., 2003, Initiation of earthquake induced slope failure: influence of topographical and other site specific amplification effects. Jour. of Seism., v. 7, pp. 397–412.

ITASCA, 2000, FLAC 4.0 manuals. Minnessotta, ITASCA Consulting Group, Inc.

Paolucci, R., 2002, Amplification of earthquake ground motion by steep topographic irregularities. Earthquake Engineering and Structural Dynamics, v. 31, pp. 1831–1853.

Seed, H. B. and Idriss, I. M., 1970, Soil Moduli and Damping Factors for Dynamic Response Analyses. Earthquake Engineering Research Centre, College of Engineering, University of California - Berkeley California, Report No. EERC 70-10.