Validation of spatial autocorrelation (SPAC) method with L-shape array in Jyoso City, Japan

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ABSTRACT

This study is aimed to validate the efficiency of an L-shape array for SPAC method using microtremor in estimating the shear wave velocity (V_s) structure. The experiment for validation was conducted in the Toyota Community Baseball Ground, Jyoso City, Ibaraki Prefecture, Japan in March 2009 with an equilateral triangle array with side length of 40 m and in June 2010 with an equilateral triangle array with side length of 50 m, together with an L-shape array of the similar size. Multichannel Analysis of Surface Waves (MASW) was also performed simultaneously in June 2010. In the same lot PS-logging data are available from nearby IBRH10 station of the KIK-NET (NIED, Japan) that shows soft sediment of about 20 m thick with V_s of 110 m/s in the geological column of the site.

The comparison of the determined phase velocity and that calculated from PS logging data shows close matching of two sets of curves separately. One is between PS logging and the triangle array (40 m), and the other is between the triangle array (50 m) and the L-shape array (50 m). Former two are of the almost same place whereas other two arrays are deployed about 200 m away from the other set. Some discrepancy between two sets is shown. This seems due to lateral variation of underground velocity structure which is consistent with the result of MASW.

Based on the results of analysis one can say that a L-shape array can be applied to estimate shear wave velocity for shallow depth so it can be layout in urban areas to determine phase velocity information from microtremors. Therefore, it may be feasible to apply it in the Kathmandu Valley, Nepal that is based upon the soft soil with high possibility of liquefaction or earthquake hazard.

Keywords: SPAC, PS logging data, equilateral triangle array, L-shape array

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INTRODUCTION

Amplification and duration of the seismic wave field are highly dependent of the local ground conditions, including the local geology through its velocity structure and density. Indeed, subsurface low rigidity geological layers deposited on high rigidity ones tend to induce ground motion amplifications. Furthermore, the 3D geometry of the sedimentary deposits has also a large impact on the seismic solicitations, through basin effects, favoring trapping of the seismic waves and interferences. These site effects could play a major role in the destructive potential of the ground motion in Nepal. Actually, Nepal is subject to a very high level of seismic hazard and several major cities, including Kathmandu, Biratnagar and Pokhara among others are built on soft sediments, deposited over hard rocks.

To better estimate the potential aggressiveness of the ground motion and its spatial variations several geophysical methods have been developed to characterize the subsurface seismic site response including refraction and reflection seismic, boreholes with seismic wave loggers (PS-logging), etc. Most of these methods are costly and some of them inappropriate for use in dense urban environment. However, some passive seismic methods dealing with the recording of the ambient seismic noise, called microtremors methods have been developed; have a tool to estimate site response since the pioneering work of Kanai et al. (1954). Since the microzonation on the basis of shallow V_s (velocity of S-wave) structure is a basic step of seismic hazard assessment, the methods to obtain a V_s structure and site response from microtremor measurement are SPAC (Spatial Autocorrelation), H/V (Horizontal to Vertical spectral ratio and F-K (Frequency Wave number).

In this study SPAC method is applied since this method seems to be reliable, easy to handle, comparatively affordable and does not cause any environmental problems, thus suitable as a tool for seismic microzonation and earthquake disaster mitigation. There are various types of array shapes in SPAC method like circular, equilateral triangle, linear, L-shape or isosceles triangle and dodecagon etc., and among them equilateral triangle is the most popular one. Conventional circular or equilateral triangle arrays, however, are sometimes difficult to deploy in urban areas where only L-shape or linear array layout can be suitable to road pattern.

The purpose of this study is to understand the effectiveness, limitations and advantages of L-shape array for the SPAC method by applying and verifying the applicability and accuracy by comparison with standard equilateral triangle array in Jyoso City, Ibaraki, Japan.

METHODOLOGY

SPAC method

The spatial autocorrelation method (SPAC) is a successful method to determine the phase velocity information from surface waves contained in microtremor (Aki 1957, Okada 2003). This technique is based on the assumption that the complex wave motion of microtremor is a stationary stochastic process in time and space. Morikawa et al. (2004) gave a brief description of the SPAC method as follows, based on the theory of stochastic process developed by Aki (1957).

Let us consider a circular array with a radius r for observing microtremor. The harmonic waves at angular frequency ω of the vertical component of the microtremor are represented by u (t; ω, 0, 0) and u (t; ω, r, θ), which are observed at the center c (0, 0) of the array and a site x (r, θ) on the circle, respectively. It is thought that the vertical component of the microtremor mainly consists of the Rayleigh waves with the fundamental mode. The SPAC function is defined as,

$$\phi\,(\omega;r,\theta) = <\mathrm{u}\,(\mathrm{t};\omega,\,0,\,0)\ \mathrm{u}\,(\mathrm{t};\omega,r,\theta)> \quad ...(1)$$

The SPAC coefficients are defined as the average of SPAC function over all the observation sites on the circular array, in other words,

$$\rho(\omega,r) = \frac{1}{2\pi\phi\,(\omega;0,0)}\int_0^{2\pi}\phi(\omega;r,\theta)d\theta \quad ....\ (2)$$

where ϕ (ω, 0, 0) is the auto correlation function at center C (0, 0). After a mathematical reduction, the integral of the above equation is rewritten as,

$$\rho\,(\omega,r) = J_o\left(\frac{\omega r}{c(\omega)}\right) = \frac{1}{2\pi}\int_0^{2\pi}\frac{\mathrm{Re}[S_{\alpha}\,(\omega;r,\theta]}{\sqrt{S_c\,(\omega;0,0)S_x\,(\omega,r,\theta)}}d\theta \quad ....(3)$$

where, $J_o(\ )$ is the Bessel function of the first kind with zero order, c(ω) is the phase velocity (ω= $2\pi f$) (e. g., Okada 2003). The SPAC coefficients denoted by equation (2) can be directly calculated in a frequency domain using the Fourier transformation of the observed microtremor, as shown in right hand side of above equation.

Here Re [ ] stands for the real part of a complex value and S_{xx}(ω,r,θ) denotes the cross spectra between u(t; ω,0,0) at c(0,0) and u(t; ω,r,θ) at x(r,θ). S_{cc}(ω; 0, 0) and S_{xx}(ω; r,θ) denote the power spectra of the microtremor at c(0,0) and x(r, θ), respectively. The integrand is sometimes called the complex coherence function (CCF, Shiraishi et al. 2006). It is clear from the above theoretical consideration that the phase velocity at a certain frequency can be determined from the SPAC coefficient of the component wave of angular frequency ù by fitting J_0 (ωr/c(ω)).

New interpretation

Shiraishi et al. (2006) proposed the following formula using a mathematical relationship between $cos(xcos\theta)$ and the Bessel function of the first kind,

$$\mathrm{Re}(\gamma_{pq}) = J_0\left(\frac{\omega r}{c(\omega)}\right) + 2\sum_{n=1}^{\infty}\left\{(-1)^n J_{2n}\left(\frac{\omega r}{c(\omega)}\right)\sum_{l=1}^{L}\lambda_l\cos 2n\theta_l\right\} \quad ....\ (4)$$

where, $Re\ (\gamma_{pq})$ is equal to the integrand of the third member of equation (3), namely the coherency, L is the number of wave sources, λ_l is rate of the contribution of the l-th wave source to the power spectra at the observation point, θ_l is azimuth of the l-th wave source, $J_m()$ is m-th order Bessel function of first kind. Here $J_0(\omega r/c(\omega))$ is already included in the integrand. The application of L-shape array relies on this formula. L-shape array has error of the 4th and higher order Bessel functions as follows.

$$\rho_L(\omega,r_{01}) = \frac{\mathrm{Re}\left\{(\gamma_z)_{0,1}\right\} + \mathrm{Re}\left\{(\gamma_z)_{0,2}\right\}}{2} = J_0\left(\frac{\omega r_{01}}{c(\omega)}\right) + J_4\left(\frac{\omega r_{01}}{c(\omega)}\right)\left\{2\sum_{l=1}^{L}\lambda_l\cos 4\theta_l\right\} + O\left(J_8\left(\frac{\omega r_{01}}{c(\omega)}\right)\right) \quad ....(5)$$

Equilateral triangle array averages out the terms of $J_2(\ )$ and $J_4(\ )$, whereas L-shape array cancels out the term of $J_2(\ )$ only. Therefore, L-shape array seems weaker against undesirable azimuth dependent noise contained in microtremor. From equation (5), it is expected that the average over two pairs that form the short sides can reduce the influence of $J_2(\ )$ term drastically, whereas the oblique sides follow equation (4). Therefore, it is better to eliminate the oblique sides from the analysis.

DATA ACQUISITION

Observation site

The experiment was conducted in the Toyota Community Baseball Ground, Jyoso City, Ibaraki Prefecture, Japan in March 2009 and June 2010. IBRH10 of KIK-NET (NIED, Japan) is located at the south-east corner of the same lot where elastic waves velocity data by PS-logger are available. MASW (Multichannel Analysis of Surface Waves) (Hayashi and Suzuki 2004) was also conducted this year. Soft sediment as thick as about 20 m with velocity of S-wave (V_s) 110 m/s is observed in the geological column at IBRH10. There are national road R294 about 300 m west and prefectural road 24 about 100 m south and both of them have heavy traffic all day long (Fig. 1). The baseball ground itself, however, was almost quiet during the measurement (Yokoi and Hayashi 2009).

Array deployment and instruments

Triangle and L-shape arrays are set up to get the microtremor data (Fig. 2). Equilateral triangle array with side length of 40 m with 7 sensors was deployed in March 2009 nearby IBRH10 whereas 50 m of equilateral triangle with 10 sensors and side length of 50 m of L-shape array with 11 sensors were deployed in June 2010 about 200 m away from IBRH10. All sensors are vertical components. MASW with total length of 105 m was conducted in the same place. The seismometers used for 40 m array are VSE12-CC type from Tokyo Soku-shin Ltd. and for 50 m, L22D type from Sercel Company.

ANALYSIS AND RESULTS

Procedure of analysis

Firstly, multiplexing is done to sort the data that individually stored in single channel files into a multichannel file of the time-sequential format. Secondly, re-sampling is applied in order to reduce the size of data files and to reduce the load to computer processing. This process can cause the aliasing effect so the digital anti-aliasing filter (Saito 1978) was applied with high cut characteristics before thinning out. Next, SPAC coefficient is obtained and the screening is conducted in two steps, after this dispersion curve of Rayleigh wave is determined and finally velocity structure model is estimated by the heuristic search method using the very fast simulated annealing method (VFSA) combined with the down hill simplex method (DHSM) (VFSA-DHSM, Yokoi 2005).

SPAC coefficient calculation

Re-sampled and screened time block files are used to calculate SPAC coefficient, that is an azimuthal average of the coherency between microtremor records at two stations. The initial frequency range of analysis is set from 0.1 Hz to 10 Hz. Band width of Parzen window for smoothing power and cross spectra is set at 0.5 Hz. Fig. 3a shows the SPAC coefficients of station pairs that compose the 50 m triangle array and Fig. 3b shows SPAC coefficient with its standard deviation of the 50 m triangle array (inter-station distance 25 m). It is clear that for the shorter distances the first positive lobe of SPAC coefficient shows higher values and has wider range of frequencies. It is also clearly seen that SPAC coefficient is decreasing at low frequency side but theoretically, it should reach 1 at the frequency 0.0 Hz. This variation is due to the available frequency range of seismometer. For Fig. 3 (both 3a and 3b) the natural frequency of the used seismometers is 2 Hz.

Determination of dispersion curve

The dispersion curve of Rayleigh wave i. e., the phase velocity depending upon the frequency is determined from SPAC coefficient. First, SPAC coefficient $\rho(\omega r)$ is converted to the value of kr by applying the following fifth order polynomial equation that approximates the inverse function of $J_0(kr)$ from $kr = 0.0$ to the first trough of $J_0(kr)$. If $x = J_0(y)$,

$$y \approx -6.0803x^5 + 9.2477x^4 - 3.9322x^3 + 0.1815x^2 - 1.7079x + 2.4121 \quad \text{......................(6)}$$

The first maximum of $c(\omega)$ from the low frequency side is recognized as a lower limit of the available frequency range

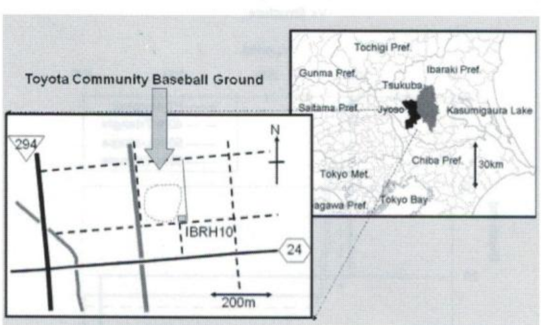

Fig. 1: Location map of measurement site (Yokoi and Hayashi 2009)

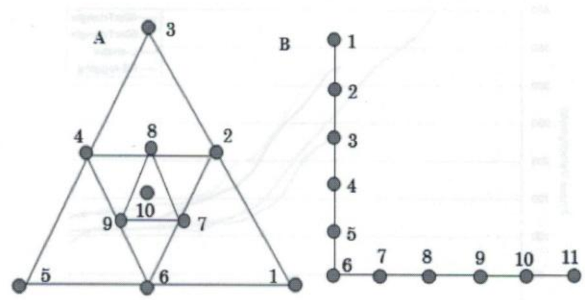

Fig. 2: Schematic figures of (A) equilateral triangle array and (B) L-shape array

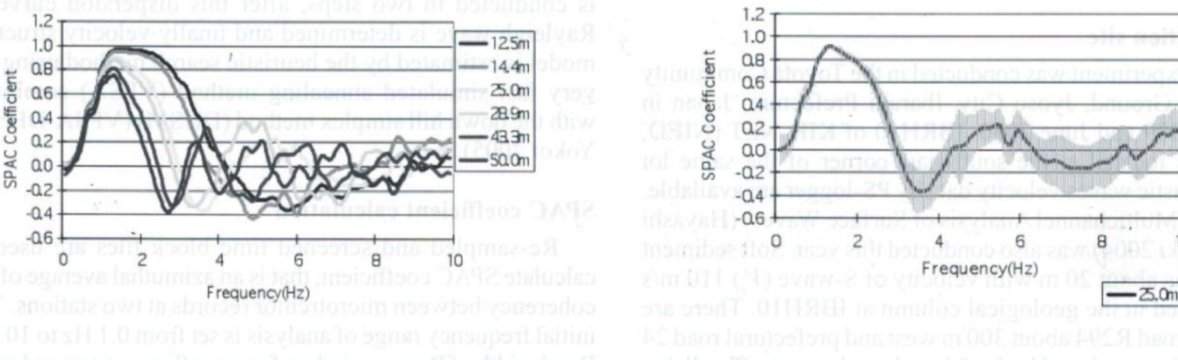

Fig. 3: SPAC coefficients, (a) those of the station pairs that compose the 50m triangular, (b) of the 50m triangular array (inter-station distance 25m).

for the respective inter-station distance. The maximum from this lower limit to the highest one is recognized as the high frequency limit of the available frequency range. Then these values again are averaged and converted to $c(\omega) = r\omega/(kr)$. The weight coefficient used at averaging is the reciprocal of the variance of SPAC coefficients at the respective inter-station distance and the frequency.

Estimation of velocity structure by Heuristic search

A Heuristic search method is conducted to obtain the optimum underground structure by fitting the theoretical phase velocity of Rayleigh wave to the observed dispersion curve. Five layer models are introduced with its search range. The method used is the downhill simplex method (DHSM) (Press et al. 2002) combined with the very fast simulated annealing (VFSA, Ingber 1989). Hereafter, the combined methods is called the DHSM-VFSA. The optimum, namely, the fastest schedule for the inversion of underground velocity structure from the dispersion curve of Rayleigh waves is used with the parameters t_0=1.0, a=0.6, and c=1.3 as given by Yokoi (2005).

DISCUSSIONS

Comparison of dispersion curves

Fig. 4 shows a comparison of phase velocity over frequency between 40 m triangle, 50 m triangle and 50 m L-shape arrays with the curve of PS logging data. This shows close matching of two sets of curves separately, one is between PS logging and 40 m triangle, and the other is 50 m triangle and L-shape. PS logging and 40 m triangle array are from the same place where other two arrays are also from the same place but about 200 m away from the other set. A discrepancy is seen between these two sets. This seems to be due to lateral variation of underground velocity structure as MASW result implies.

Comparison of velocity structure

Fig. 5 shows the comparison of V_s structure models determined by the data of the three arrays with the PS logging data. V_p is fixed to 1500 m/s for the 1st to 4th layer and 1956 m/s for the underlying half space. Density is calculated from V_p then fixed to 1.9 g/cm^3 and 2.13 g/cm^3 respectively (Ludwig et al. 1970). As seismometer's natural frequency is 2.0 Hz, the lower limit of the frequency range for analysis is set at 2.0 Hz and the upper one at 5.0 Hz by considering on the S/N of microtremor. The value of misfit is as small as

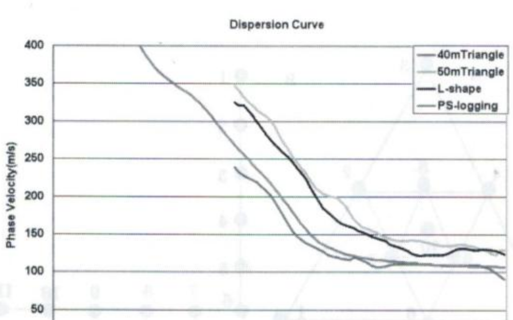

Fig. 4: Comparison of the dispersion curves with PS logging data

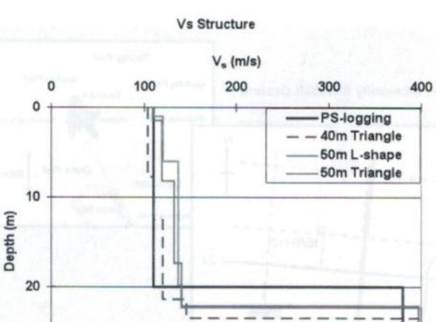

Fig. 5: Comparison of velocity structures with PS-logging data

3.0, 4.0 and 4.2 m/s for the triangular array (40 m), triangular array (50 m) and L-shape array, respectively.

From Fig. 5 it is clear that for the 20 m depth V_s of all arrays are similar to the PS logging data i.e., around 110 m/s to 150 m/s that implies soft soil. V_s structures of the triangular array (50 m) and L-shape array coincide to each other whereas 40 m triangle shows a clear deviation from them and lower V_s than PS logging data. This shows that the lateral variation of underground velocity structure implied by MASW is detected also using SPAC method.

CONCLUSIONS

In this study the analysis of the data of microtremor array observation was conducted in the Toyota Community Baseball Ground, Jyoso city, Ibaraki prefecture, Japan for shallow depth using L-shape and equilateral triangle array with the SPAC method for Rayleigh wave in comparison with PS logging data and MASW. The dispersion curves and V_s structures determined by the data of L-shape array coincide well to those of equilateral triangular array (50 m). These deviate together from V_s structure of PS logging data and the dispersion curve calculated from PS logging data, respectively. This discrepancy, however, is due to lateral variation of V_s structure as the result of the triangular array (40 m) shows and that of MASW implies. Based on the results of analysis L-shape array can be applied to estimate V_s structure for shallow depth, so it can be layout in Kathmandu to determine phase velocity information from surface wave in microtremor for seismic microzonation and seismic hazard assessment.

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