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Research Article

MASS TRANSFER EFFECTS ON MIXED CONVECTIVE MHD FLOW OF SECOND GRADE FLUID PAST A VERTICAL INFINITE PLATE WITH VISCOUS DISSIPATION AND JOULE HEATING

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ABSTRACT

An attempt is made to study an unsteady, two-dimensional, laminar, mixed convective Magnetohydrodynamic (MHD) flow of an incompressible visco-elastic fluid (Walters B' fluid model) past an infinite vertical plate. The reduced governing equations are solved analytically using two-term harmonic and non-harmonic functions. The effects of different pertinent parameters are discussed with the help of graphs and tables. The novelty of the present study is to account for the effects of viscous and joules dissipative heat and a linear first-order chemical reaction of diffusive species and mixed convective flow phenomena on an infinite vertical plate subjected to time-dependent suction velocity and a transverse magnetic field acting at a distance. The important findings reported herein are: increasing values of chemical reaction parameter cause low velocity and concentration, a decline in concentration profile is seen for the higher values of Schmidt number, Prandtl number contributes to more active convection. The application of the present study may be seen in combustion systems, nuclear reactors, and chemical processes. Before concluding the considered problem, our results are validated with previous results and are found to be in good agreement.

Keywords: Chemical reaction, Joules heating, MHD flow, mixed convection, viscous dissipation

INTRODUCTION

Viscous dissipation changes the temperature distributions by playing a role like an energy source, which leads to affect heat transfer rates. The merit of the effect of viscous dissipation depends on whether the plate is being cooled or heated. Apart from the viscous dissipation in viscous fluid flow, the joules dissipation also acts as a volumetric heat source in MHD heat transfer. Heat transfer analysis over porous surface is of much practical interest due to its abundant applications. To be more specific, heat treated materials travelling between a feed roll and wind-up roll or materials manufactured by extrusion, glass-fibre and paper production, cooling of metallic sheets or electronic chips, crystal growing just to name a few. In these cases, the final product of desired characteristics depends on the rate of cooling as well as the process of stretching. In view of these aspects, the present work deals with the effect of viscous dissipation and joules dissipation on MHD flow.

The mixed convection boundary layer flow of a non-Newtonian fluid in the presence of strong magnetic field has wide range of applications in nuclear engineering and industries. In astrophysical and geophysical studies, the MHD boundary layer flows of an electrically conducting fluid through porous media have also numerous applications for modelling and simulation. Many researchers have studied the transient laminar natural convection flow past a vertical porous plate for the application in the branch of science technology such as in the field of agriculture engineering and chemical engineering.

The problem of heat and mass transfer combined with chemical reaction is very important due to its industrial application. It has been the subject of many works in recent years. Heat and mass transfer occur simultaneously in processes such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler. Other examples of industrial applications are curing of plastic, cleaning and chemical processing of materials relevant to the manufacture of printed circuitry, manufacture of pump-insulated cables etc. Two types of chemical reaction can take place; homogeneous reaction which occurs uniformly throughout a given phase, while a heterogeneous reaction takes place in a restricted region or within the boundary of a phase. Heat and mass transfer with chemical reaction has attracted of many authors because of its applications.

An analysis of thermal boundary layer in an electrically conducting fluid over a linearly stretching sheet in the presence of a constant transverse magnetic field with suction or blowing at the sheet was carried out by Chaim (1977). The viscous and joules dissipation and internal heat generation was taken into account in the energy equation. A mathematical analysis has been carried out on momentum and heat transfer characteristics in an incompressible, electrically conducting viscoelastic boundary layer fluid flow over a linear stretching sheet (Abel et al., 2008). Chen (2004) examined the effect of combined heat and mass transfer on MHD free convection from a vertical surface with ohmic heating and viscous dissipation. The effect of viscous dissipation and joules heating on MHD free convection flow past a semi-infinite vertical flat plate in the presence of combined effect of Hall and non-slip currents for the case of power-law variation of the wall temperature is analyzed (Eldahab and Aziz, 2005). The flow and heat transfer in a second grade fluid have been studied by many researchers in different contexts. For example, Parida et al. (2011) have examined the magnetic effect on the flow and heat transfer of second grade fluid in a channel with porous wall and later Bhargava and Singh (2012) have numerically analysed the flow and heat transfer of a second grade fluid over an oscillatory stretching sheet including viscous dissipation and joule heating. A boundary layer analysis for Newtonian conducting fluid over an infinite vertical porous plate has been carried out (Singh and Gorla, 2009). In this investigation they studied the effect of viscous dissipation, joule heating, thermal diffusion and Hall current. The study of heat dissipation on the flow of visco-elastic fluid past an infinite vertical plate is studied by Uwanta et al. (2011). Mahanta and Choudhury (2012) reported the analysis of mixed convective MHD flow of second grade fluid past a vertical infinite plate with mass transfer. The MHD free convection and mass transfer flow of Newtonian fluid over an infinite vertical porous plate with viscous dissipation have been investigated by Poonia and Choudhary (2010). Barletta and Celli (2008) investigated the mixed convection MHD flow in a vertical channel with joules and viscous dissipation effects.

The effects of viscous and joules dissipation on MHD flow, heat and mass transfer past a stretching porous surface embedded in a porous medium has been studied by Anjali Devi and Ganga (2009). Barik et al.(2013) have been studied the MHD flow and heat transfer over a stretching porous sheet subject to power law heat flux in the presence of chemical reaction and viscous dissipation. The mass transfer effect on a free convective visco-elastic fluid over an infinite vertical porous plate with viscous dissipation has been investigated by Barik et al. (2012). Nayak and Panda (2013) studied the mixed convective MHD flow of second grade fluid with viscous dissipation and joule heating past a vertical infinite plate with mass transfer. Swain and Senapati (2015) inspected the aftermath of mass transfer on free convective flow set in a porous medium. Mabood and Shateyi (2019) studied radiative MHD unsteady flow with multiple slips. Sekhar et al. (2018) illustrated the multiple slips impacts on MHD flow through porous medium. Swain et al. (2020) studied viscous dissipation and joule heating effect on MHD flow and heat transfer past a stretching sheet embedded in a porous medium. Nadeem et al. (2022) analyzed the second grade fuzzy hybrid nanofluid stagnation point flow and heat transfer over a permeable stretching/ shrinking sheet incorporating viscous dissipation and nonlinear thermal radiation. Swain et al. (2021) reported the unsteady electrically conducting viscous fluid, flowing in permeable capillary, subjected to magnetic field and temporal deformation with radiative and dissipative heat, representing the blood flow incorporating the second-order slip at the capillary surface. Khan et al. (2022) observed the free convection flow of second grade dusty fluid between two parallel plates using Fick's and Fourier's laws as a fractional model taking heat and mass transfer into account. Biswal et al.(2022) elucidated the MHD boundary layer free convective stagnation-point flow toward an inclined nonlinearly stretching sheet embedded in a porous medium discussing the effect of dissipative heat, nonuniform space dependent volumetric heat power, and a linear first order chemical reaction of diffusive species.

Due to techonological applications of heat and mass transfer problems in hydrometallurgical and chemical industries, it is very important to investigate the thermodynamic behaviour of the chemical reaction process. In Practice, there are a large number of transport phenomena occurring in solar collectors, chemical engineering, nuclear reactors etc. are governed by the mutual action of buoyancy forces owing to both heat and mass diffusions under the action of chemical reaction effects. This motivates the present study to perform the fluid flow, heat and mass transfer analysis of a visco-elastic fluid with mixed MHD flow.

The novelty of the present study is to analyze the mixed convective MHD flow of second grade fluid incorporating the chemical reaction in the mass transfer phenomena along with a transverse magnetic field act at a distance. The effect of viscous dissipation is also taken into account. The analytical solution of the problem is in good agreement with the numerical solution obtained by Nayak and Panda (2013) in the absence of chemical reaction which is the main feature of the problem. The flow phenomena have been characterized with the help of flow parameters and their effects on the velocity field, temperature and concentration have been analysed and results are presented graphically.

MATHEMATICAL FORMULATION

We consider an unsteady two dimensional mixed convective boundary layer flow of an incompressible and electrically conducting non-Newtonian second grade visco-elastic fluid along an infinite vertical heated plate in the presence of thermal and solutal buoyancy effects. The x-axis is taken in the upward direction of the plate and y-axis is normal to it. A constant magnetic field of strength B_0 is applied in the direction perpendicular to the plate.

The following assumptions are made during the course of present analysis.

(i) An infinite vertical heated plate is subjected to space dependent transverse magnetic field in the presence of thermal and solutal buoyancy effects.
(ii) The magnetic number is so small that the induced magnetic field can be neglected in comparison with the appeared space dependent applied one.
(iii) The suction velocity is assumed to be time dependent.
(iv) It is also assumed that no polarization voltage exists. This then corresponds to the case when no energy is added to or extracted from the fluid by the electric field.
(v) The diffusive species is chemically reactive with variable concentration at the surface.

Now under the usual Boussinesq approximation, the governing boundary layer equations (cf.[15]) are
Equation of Continuity:

$$\frac{\partial v}{\partial y}=0 \tag{1}$$

Equation of Motion :

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}-\frac{k_0}{\rho}\left(\frac{\partial^3 u}{\partial t\partial y^2}+v\frac{\partial^3 u}{\partial y^3}\right)+g\beta_T(T-T_\infty)+g\beta_C(C-C_\infty)-\frac{\sigma B_0{}^2}{\rho}u \tag{2}$$

Equation of Energy:

$$\rho C_p\left(\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2+\sigma B_0^2 u^2 \tag{3}$$

Equation of Mass Transfer:

$$\frac{\partial C}{\partial t}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-k_c(C-C_\infty) \tag{4}$$

From Eq. (1), it is clear that v is a constant or function of time only, so we consider it in the following form:

$$v=-v_0\left(1+\varepsilon Ae^{i\omega t}\right), \tag{5}$$

where A is the suction parameter (real positive constant) , ω-the frequency of the suction velocity, v_0 is a suction velocity which is non-zero positive constant, $\in$ and $\in$A are small less than unity. The negative sign indicates that the suction is towards the plate.

The boundary conditions are given by [15]:

$$\left.\begin{array}{l} u=0,\ T=T_\infty+T_0(\mathrm{t})\left(T_\mathrm{w}-T_\infty\right),\ C=C_\infty+C_0(\mathrm{t})\left(C_\mathrm{w}-C_\infty\right),\quad \text{at } y=0 \\ u\to 0,\ \dfrac{\partial u}{\partial y}\to 0,\ T\to T_\infty,\ C\to C_\infty,\ \text{as } y\to\infty \end{array}\right\} \tag{6}$$

The subscripts w and ∞ refer to the conditions at wall and far away from the plate respectively. Here, $T_0=C_0=\left(1+\varepsilon e^{i\omega t}\right)$.

Non-dimensional quantities are defined as:

$$\left.\begin{array}{l} y=\left(\dfrac{\upsilon}{v_0}\right)\tilde{y},\ t=\left(\dfrac{4\upsilon}{v_0^2}\right)\tilde{t},\ u=v_0\tilde{u},\ w=\left(\dfrac{v_0^2}{4\upsilon}\right)\tilde{w} \\ T=T_\infty+\theta\left(T_w-T_\infty\right),\ C=C_\infty+\phi\left(C_w-C_\infty\right) \end{array}\right\} \tag{7}$$

$$P_r=\frac{\mu C_p}{K},\qquad S_c=\frac{\upsilon}{D},\qquad G_r=\frac{g\beta_T\upsilon}{v_0^3}(T_w-T_\infty),\ G_m=\frac{g\beta_C\upsilon(C_w-C_\infty)}{v_0^3},$$

$$E_c=\frac{v_0^2}{C_p(T_w-T_\infty)},\ M=\frac{\sigma B_0^2\upsilon}{\rho v_0^2},\ R_c=\frac{k_0 v_0^2}{\rho\upsilon^2},\ k_c=\frac{\tilde{k}_c v_0^2}{\upsilon} \tag{8}$$

In view of (7) and (8), equations (2) to (4) and after dropping the tilde (~), become

$$\frac{1}{4}\frac{\partial u}{\partial t}-(1+\varepsilon Ae^{i\omega t})\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}-R_c\left\{\frac{1}{4}\frac{\partial^3 u}{\partial t\partial y^2}-(1+\varepsilon Ae^{i\omega t})\frac{\partial^3 u}{\partial y^3}\right\}+G_r\theta+\mathrm{G_m}\phi-Mu \tag{9}$$

$$\frac{1}{4}\frac{\partial\theta}{\partial t}-\left(1+\varepsilon Ae^{i\omega t}\right)\frac{\partial\theta}{\partial y}=\frac{1}{\mathrm{P_r}}\frac{\partial^2\theta}{\partial y^2}+E_c\left(\frac{\partial u}{\partial y}\right)^2+E_c Mu^2 \tag{10}$$

$$\frac{1}{4}\frac{\partial\phi}{\partial t}-(1+\varepsilon Ae^{i\omega t})\frac{\partial\phi}{\partial y}=\frac{1}{S_c}\frac{\partial^2\phi}{\partial y^2}-k_c\varphi \tag{11}$$

The boundary conditions corresponding to Eq. (6) are

$$\left.\begin{array}{l} u=0,\ \theta=T_0,\ \phi=C_0\ \ at\ \ y=0 \\ u\to 0,\ \dfrac{\partial u}{\partial y}\to 0,\ \theta\to 0,\ \phi\to 0\ \ as\ y\to\infty \end{array}\right\} \tag{12}$$

SOLUTION PROCEDURE

In order to obtain solutions of eqn. (9)-(11) together with boundary conditions (12), The superimposed solutions following Poonia and Choudhary [10] are.

$$\left.\begin{array}{l} u(t,y)=u_0(y)+\varepsilon e^{i\omega t}u_1(y) \\ \theta(t,y)=(1-\theta_0(y))+\varepsilon e^{i\omega t}(1-\theta_1(y)) \\ \phi(t,y)=(1-\phi_0(y))+\varepsilon e^{i\omega t}(1-\phi_1(y)) \end{array}\right\} \tag{13}$$

Substituting Eq. (13) into Eqs.(9) -(11) and equating the coefficient of the powers of ε , we obtain

Zeroth Order:

$$R_c u_0'''+u_0''+u_0'-Mu_0+G_r(1-\theta_0)+G_m(1-\phi_0)=0 \tag{14}$$

$$\theta_0''+\mathrm{P_r}\,\theta_0'-E_c\,\mathrm{P_r}\left(u_0'^2+Mu_0^2\right)=0 \tag{15}$$

$$\phi_0''+S_c\phi_0'+S_ck_c(1-\phi_0)=0 \tag{16}$$

with boundary conditions:

$$\left.\begin{array}{l} u_0=0,\ \theta_0=0,\ \phi_0=0\ \ at\ \ y=0 \\ u_0\to 0,\ u_0'\to 0,\ \theta_0\to 1,\ \phi_0\to 1\ \ as\ \ y\to\infty \end{array}\right\} \tag{17}$$

First Order:

$$R_c u_1'''+\left(1-\frac{i\omega R_c}{4}\right)u_1''+u_1'-\left(M+\frac{i\omega}{4}\right)u_1=-G_r(1-\theta_1)-G_m(1-\phi_1)-Au_0'-R_cAu_0''' \tag{18}$$

$$\theta_1''+\mathrm{P_r}\,\theta_1'-\frac{i\omega\mathrm{P_r}}{4}\theta_1+\frac{i\omega}{4}\mathrm{P_r}+\mathrm{P_r}\,A\theta_0'-2E_c\,\mathrm{P_r}\left(u_0'u_1'+Mu_0u_1\right)=0 \tag{19}$$

$$\phi_1''+S_c\phi_1'-\left(\frac{i\omega}{4}+k_c\right)S_c\phi_1+\left(\frac{i\omega}{4}+k_c\right)S_c+S_cA\phi_0'=0 \tag{20}$$

with boundary conditions:

$$\left.\begin{array}{l} \mathrm{u_1=0},\ \theta_1=0,\ \phi_1=0,\qquad \mathrm{at\ y=0} \\ \mathrm{u_1}\to 0,\ \mathrm{u_1'}\to 0,\ \theta_1\to 1,\ \phi_1\to 1,\ as\ \ \mathrm{y}\to\infty \end{array}\right\} \tag{21}$$

Solving (16) and (20), under the boundary conditions (17) and (21) we get:

$$\phi_0(y)=1-e^{m_0y} \tag{22}$$

$$\phi_1(y)=\left(1-e^{m_1y}\right)+iI_0\left(e^{m_0y}-e^{m_1y}\right) \tag{23}$$

The equations (14), (15) and (18), (19) are still coupled non-linear, whose analytical solutions are not possible. So we apply perturbation method with E_c<<1 as the perturbation parameter and obtain

$$\left.\begin{aligned} u_0(y) &= u_{00} + E_c u_{01} + 0\left(E_c^2\right) \\ u_1(y) &= u_{10} + E_c u_{11} + 0\left(E_c^2\right) \\ \theta_0(y) &= \theta_{00} + E_c \theta_{01} + 0\left(E_c^2\right) \\ \theta_1(y) &= \theta_{10} + E_c \theta_{11} + 0\left(E_c^2\right) \end{aligned}\right\} \tag{24}$$

Introducing (24) into (14), (15) and (18), (19) we get

$$R_c u_{00}''' + u_{00}'' + u_{00}' - M u_{00} + G_r\left(1-\theta_{00}\right) + G_m\left(1-\phi_0\right) = 0 \tag{25}$$

$$R_c u_{01}''' + u_{01}'' + u_{01}' - M u_{01} - G_r \theta_{01} = 0 \tag{26}$$

$$R_c u_{10}''' + \left(1 - \frac{i\omega R_c}{4}\right) u_{10}'' + u_{10}' - \left(M + \frac{i\omega}{4}\right) u_{10} = -G_r\left(1-\theta_{10}\right) - G_m\left(1-\phi_1\right) - A u_{00}' - R_c A u_{00}''' \tag{27}$$

$$R_c u_{11}''' + \left(1 - \frac{i\omega R_c}{4}\right) u_{11}'' + u_{11}' - \left(M + \frac{i\omega}{4}\right) u_{11} = G_r \theta_{11} - A u_{01}' - R_c A u_{01}''' \tag{28}$$

$$\theta_{00}'' + P_r \theta_{00}' = 0 \tag{29}$$

$$\theta_{01}'' + P_r \theta_{01}' - P_r\left(u_{00}'^2 + M u_{00}^2\right) = 0 \tag{30}$$

$$\theta_{10}'' + P_r \theta_{10}' - \frac{i\omega P_r}{4}\theta_{10} = -\frac{i\omega}{4} P_r - P_r A \theta_{00}' \tag{31}$$

$$\theta_{11}'' + P_r \theta_{11}' - \frac{i\omega P_r}{4}\theta_{11} = -P_r A \theta_{01}' + 2P_r\left(u_{00}' u_{10}' + M u_{00} u_{10}\right) \tag{32}$$

The corresponding boundary conditions are:

$y = 0$: $u_{00} = 0$, $u_{01} = 0$, $u_{10} = 0$, $u_{11} = 0$, $\theta_{00} = 0$, $\theta_{01} = 0$, $\theta_{10} = 0$, $\theta_{11} = 0$

$y \to \infty$: $u_{00} \to 0$, $u_{01} \to 0$, $u_{10} \to 0$, $u_{11} \to 0$, $u_{00}' \to 0, u_{01}' \to 0, u_{10}' \to 0, u_{11}' \to 0$

$\theta_{00} \to 1$, $\theta_{01} \to 0$, $\theta_{10} \to 1$, $\theta_{11} \to 0$, (33)

Solving (29) and (31) under the boundary condition (33) , we get

$$\theta_{00} = 1 - e^{-P_r y} \tag{34}$$

$$\theta_{10} = I_6\left(e^{m_2 y} - e^{-P_r y}\right) + 1 - e^{m_2 y} \tag{35}$$

The equations (25)-(28), (30) and (32) are still coupled and nonlinear. Hence, we again apply perturbation method with elastic parameter $R_c << 1$ as the perturbation parameter. Our assumption is justified as we work with a viscoelastic model with short memory i.e., slightly elastic fluid. So we can expand u_{00}, u_{01}, u_{10}, u_{11}, in terms of elastic parameter (R_c) in the following form:

$u_{00} = u_{000} + R_c u_{001} + 0\left(R_c^2\right)$

$u_{01} = u_{010} + R_c u_{011} + 0\left(R_c^2\right)$

$u_{10} = u_{100} + R_c u_{101} + 0\left(R_c^2\right)$

$u_{11} = u_{110} + R_c u_{111} + 0\left(R_c^2\right)$ (36)

The corresponding boundary conditions:

$y = 0: \ u_{000} = 0, u_{001} = 0, u_{010} = 0, u_{011} = 0$

$u_{100} = 0, u_{101} = 0, u_{110} = 0, u_{111} = 0$

$y \to \infty: \ u_{000} \to 0, u_{001} \to 0, u_{010} \to 0, u_{011} \to 0$

$u_{000}' \to 0,\ u_{001}' \to 0, u_{010}' \to 0, u_{011}' \to 0,$

$u_{100} \to 0,\ u_{101} \to 0, u_{110} \to 0, u_{111} \to 0,$

$$u'_{100} \to 0,\ u'_{101} \to 0, u'_{110} \to 0, u'_{111} \to 0, \tag{37}$$

In view of (36) into equations (25) to (28) are expressed as the following system of equations:

$$u''_{000} + u'_{000} - Mu_{000} = -G_r\left(1-\theta_{00}\right) - G_m\left(1-\phi_0\right) \tag{38}$$

$$u''_{001} + u'_{001} - Mu_{001} = -u'''_{000} \tag{39}$$

$$u''_{010} + u'_{010} - Mu_{010} = G_r\theta_{01} \tag{40}$$

$$u''_{011} + u'_{011} - Mu_{011} = -u'''_{010} \tag{41}$$

$$u''_{100} + u'_{100} - \left(M + \frac{i\omega}{4}\right)u_{100} = -G_r\left(1-\theta_{10}\right) - G_m\left(1-\phi_1\right) - Au'_{000} \tag{42}$$

$$u''_{101} + u'_{101} - \left(M + \frac{i\omega}{4}\right)u_{101} = -u'''_{100} + \frac{i\omega}{4}u''_{100} - Au'_{001} - Au'''_{000} \tag{43}$$

$$u''_{110} + u'_{110} - \left(M + \frac{i\omega}{4}\right)u_{110} = G_r\theta_{11} - Au'_{010} \tag{44}$$

$$u''_{111} + u'_{111} - \left(M + \frac{i\omega}{4}\right)u_{111} = -u'''_{110} + \frac{i\omega}{4}u''_{110} - Au'_{011} - Au'''_{010} \tag{45}$$

Using boundary conditions (37), we obtain the following solutions of the equations (38)-(45) and (27), (28), (30) and (32):

$$u_{000}(y) = (I_3 + I_4)e^{m_3 y} - I_3 e^{-P_r y} - I_4 e^{m_0 y} \tag{46}$$

$$u_{001}(y) = (I_7 + I_8)e^{m_3 y} - I_7 e^{-P_r y} - I_8 e^{m_0 y} \tag{47}$$

$$\theta_{01}(y) = I_{18}e^{-P_r y} + I_{12}e^{2m_3 y} + I_{13}e^{-2P_r y} + I_{14}e^{2m_0 y} + I_{15}e^{-(P_r - m_3)y} + I_{16}e^{-(P_r - m_0)y} - I_{17}e^{(m_0+m_3)y} \tag{48}$$

$$u_{010}(y) = I_{32}e^{m_3 y} + I_{25}e^{-P_r y} + I_{26}e^{2m_3 y} + I_{27}e^{-2P_r y} + I_{28}e^{2m_0 y} + I_{29}e^{-(P_r - m_3)y} + I_{30}e^{-(P_r - m_0)y} - I_{31}e^{(m_0+m_3)y} \tag{49}$$

$$u_{011}(y) = I_{41}e^{m_3 y} + I_{34}e^{-P_r y} - I_{35}e^{2m_3 y} + I_{36}e^{-2P_r y} - I_{37}e^{2m_0 y} + I_{38}e^{-(P_r - m_3)y} + I_{39}e^{-(P_r - m_0)y} + I_{40}e^{(m_0+m_3)y} \tag{50}$$

$$u_{100}(y) = I_{55}e^{m_4 y} + I_{48}e^{m_2 y} - I_{56}e^{-P_r y} - I_{50}e^{m_1 y} + I_{57}e^{m_0 y} - I_{52}e^{m_3 y} \tag{51}$$

$$u_{101} = I_{83}e^{m_4 y} + I_{78}e^{m_3 y} + I_{79}e^{m_2 y} + I_{80}e^{m_1 y} + I_{81}e^{m_0 y} + I_{82}e^{-P_r y} \tag{52}$$

$$\begin{aligned}\theta_{11}(y) &= I_{145}e^{m_2 y} + I_{129}e^{-P_r y} + I_{130}e^{2m_3 y} + I_{131}e^{-2P_r y} + I_{132}e^{2m_0 y} + I_{133}e^{-(P_r - m_3)y}\\ &+ I_{134}e^{-(P_r - m_0)y} + I_{135}e^{(m_0+m_3)y} - I_{136}e^{(m_3+m_4)y} - I_{137}e^{(m_3+m_2)y} - I_{138}e^{(m_3+m_1)y}\\ &- I_{139}e^{-(P_r - m_4)y} - I_{140}e^{-(P_r - m_2)y} - I_{141}e^{-(P_r - m_1)y} + I_{142}e^{(m_0+m_4)y} + I_{143}e^{(m_0+m_2)y} + I_{144}e^{(m_0+m_1)y}\end{aligned} \tag{53}$$

$$\begin{aligned}u_{110}(y) &= A_1 e^{m_4 y} + I_{161}e^{m_2 y} + I_{186}e^{-P_r y} + I_{187}e^{2m_3 y} + I_{188}e^{-2P_r y} + I_{189}e^{2m_0 y}\\ &+ I_{190}e^{-(P_r - m_3)y} + I_{191}e^{-(P_r - m_0)y} + I_{192}e^{(m_0+m_3)y} - I_{169}e^{(m_3+m_4)y} - I_{170}e^{(m_3+m_2)y}\\ &- I_{171}e^{(m_3+m_1)y} - I_{172}e^{-(P_r - m_4)y} - I_{173}e^{-(P_r - m_2)y} - I_{174}e^{-(P_r - m_1)y}\\ &+ I_{175}e^{(m_0+m_4)y} + I_{176}e^{(m_0+m_2)y} + I_{177}e^{(m_0+m_1)y} - I_{195}e^{m_3 y}\end{aligned} \tag{54}$$

$$\begin{aligned}u_{111}(y) &= A_{34}e^{m_4 y} + A_2 e^{m_2 y} + A_{26}e^{-P_r y} + A_{27}e^{2m_3 y} + A_{28}e^{-2P_r y} + A_{29}e^{2m_0 y}\\ &+ A_{30}e^{-(P_r - m_3)y} + A_{31}e^{-(P_r - m_0)y} - A_{32}e^{(m_3+m_0)y} + I_{193}e^{(m_3+m_4)y} + I_{194}e^{(m_3+m_2)y}\\ &+ A_{10}e^{(m_3+m_1)y} - A_{11}e^{-(P_r - m_4)y} - A_{12}e^{-(P_r - m_2)y} - A_{13}e^{-(P_r - m_1)y}\\ &- A_{14}e^{(m_4+m_0)y} - A_{15}e^{(m_2+m_0)y} - A_{16}e^{(m_0+m_1)y} + A_{33}e^{m_3 y}\end{aligned} \tag{55}$$

$$\begin{aligned}u_{00} &= u_{000} + R_c u_{001}\\ &= I_9 e^{m_3 y} - I_{10}e^{-P_r y} - I_{11}e^{m_0 y}\end{aligned} \tag{56}$$

$$u_{01} = u_{010} + R_c u_{011}$$
$$= I_{42}e^{m_3 y} + I_{84}e^{-P_r y} + I_{85}e^{2m_3 y} + I_{86}e^{-2P_r y} + I_{87}e^{2m_0 y} + I_{88}e^{-(P_r - m_3)y}$$
$$+I_{89}e^{-(P_r - m_0)y} - I_{90}e^{(m_0 + m_3)y} \tag{57}$$
$$u_{10} = u_{100} + R_c u_{101}$$
$$= I_{91}e^{m_4 y} + I_{92}e^{m_2 y} + I_{93}e^{-P_r y} + I_{94}e^{m_1 y} + I_{95}e^{m_0 y} + I_{96}e^{m_3 y} \tag{58}$$
$$u_{11} = u_{110} + R_c u_{111}$$
$$= A_{35}e^{m_4 y} + A_{36}e^{m_2 y} + A_{37}e^{-P_r y} + A_{38}e^{2m_3 y} + A_{39}e^{-2P_r y} + A_{40}e^{2m_0 y}$$
$$+A_{41}e^{-(P_r - m_3)y} + A_{42}e^{-(P_r - m_0)y} + A_{43}e^{(m_0 + m_3)y} - A_{44}e^{m_3 y} - A_{45}e^{(m_3 + m_4)y} - A_{46}e^{(m_3 + m_2)y}$$
$$-A_{47}e^{(m_3 + m_1)y} - A_{48}e^{-(P_r - m_4)y} - A_{49}e^{-(P_r - m_2)y} - A_{50}e^{-(P_r - m_1)y}$$
$$+A_{51}e^{(m_0 + m_4)y} + A_{52}e^{(m_0 + m_2)y} + A_{53}e^{(m_0 + m_1)y} \tag{59}$$
$$u_0 = u_{00} + E_c u_{01}$$
$$= \left(I_9 e^{m_3 y} - I_{10}e^{-P_r y} - I_{11}e^{m_0 y}\right)$$
$$+E_c\Big[I_{42}e^{m_3 y} + I_{84}e^{-P_r y} + I_{85}e^{2m_3 y} + I_{86}e^{-2P_r y} + I_{87}e^{2m_0 y} + I_{88}e^{-(P_r - m_3)y}$$
$$+I_{89}e^{-(P_r - m_0)y} - I_{90}e^{(m_0 + m_3)y}\Big] \tag{60}$$
$$u_1 = u_{10} + E_c u_{11}$$
$$= \left(I_{91}e^{m_4 y} + I_{92}e^{m_2 y} + I_{93}e^{-P_r y} + I_{94}e^{m_1 y} + I_{95}e^{m_0 y} + I_{96}e^{m_3 y}\right)$$
$$+E_c\Big[A_{35}e^{m_4 y} + A_{36}e^{m_2 y} + A_{37}e^{-P_r y} + A_{38}e^{2m_3 y} + A_{39}e^{-2P_r y} + A_{40}e^{2m_0 y}$$
$$+A_{41}e^{-(P_r - m_3)y} + A_{42}e^{-(P_r - m_0)y} + A_{43}e^{(m_0 + m_3)y} - A_{44}e^{m_3 y} - A_{45}e^{(m_3 + m_4)y} - A_{46}e^{(m_3 + m_2)y}$$
$$-A_{47}e^{(m_3 + m_1)y} - A_{48}e^{-(P_r - m_4)y} - A_{49}e^{-(P_r - m_2)y} - A_{50}e^{-(P_r - m_1)y}$$
$$+A_{51}e^{(m_0 + m_4)y} + A_{52}e^{(m_0 + m_2)y} + A_{53}e^{(m_0 + m_1)y}\Big] \tag{61}$$
$$\theta_0(y) = \theta_{00} + E_c\theta_{01}$$
$$= \left(1 - e^{-P_r y}\right) + E_c\Big[I_{18}e^{-P_r y} + I_{12}e^{2m_3 y} + I_{13}e^{-2P_r y} + I_{14}e^{2m_0 y} + I_{15}e^{-(P_r - m_3)y} + I_{16}e^{-(P_r - m_0)y} - I_{17}e^{(m_0 + m_3)y}\Big] \tag{62}$$
$$\theta_1(y) = \theta_{10} + E_c\theta_{11}$$
$$= \left\{I_6\left(e^{m_2 y} - e^{-P_r y}\right) + \left(1 - e^{m_2 y}\right)\right\}$$
$$+E_c\Big[\ I_{145}e^{m_2 y} + I_{129}e^{-P_r y} + I_{130}e^{2m_3 y} + I_{131}e^{-2P_r y} + I_{132}e^{2m_0 y} + I_{133}e^{-(P_r - m_3)y}$$
$$+I_{134}e^{-(P_r - m_0)y} + I_{135}e^{(m_0 + m_3)y} - I_{136}e^{(m_3 + m_4)y} - I_{137}e^{(m_3 + m_2)y} - I_{138}e^{(m_3 + m_1)y}$$
$$-I_{139}e^{-(P_r - m_4)y} - I_{140}e^{-(P_r - m_2)y} - I_{141}e^{-(P_r - m_1)y} + I_{142}e^{(m_0 + m_4)y} + I_{143}e^{(m_0 + m_2)y} + I_{144}e^{(m_0 + m_1)y}\ \Big] \tag{63}$$

The physical quantities of second grade fluid which are of more interest:

(i) Nusselt number (Nu) $= -\left(\frac{\partial \theta}{\partial y}\right)_{y=0}$

(ii) Sherwood number $= -\left(\frac{\partial \phi}{\partial y}\right)_{y=0}$

RESULTS AND DISCUSSION

The effect of different parameters like viscous, viscoelastic and magnetic strength on the velocity of the fluid, the effects of viscous dissipation and Joule effects on temperature field as well as the effect of chemical reaction on mass transfer are analysed.

The influence of Eckert number is shown in Fig. 1. It is observed that the velocity is the increasing function of E_c. The higher Eckert number implies greater viscous dissipative heat and causes an increase in the velocity. The presence of frictional heating forces in the second grade fluid is converted into heat energy and therefore, the velocity profile increases in the boundary layer region.

Figure 2 depicts the effect of chemical reaction parameter on velocity profiles. It is observed that the velocity of the fluid reduces with an increase of chemical reaction parameter. Higher chemical reaction leads to lower concentration which results large buoyancy force producing lower velocity.

The effect of magnetic strength on the motion of the fluid is analysed in Fig. 3. It is seen that the magnetic field acts like drag force (Lorentz force) and decelerates the motion of the fluid on the boundary layer and then finally it approaches to zero.

The effects of Schmidt number on velocity and concentration field are explained in Fig. 4 and Fig. 7, respectively. Concentration reduces with an increase of Schmidt number (Fig. 7) and as a result a large buoyancy force produces and it decreases the fluid velocity (Fig. 4).

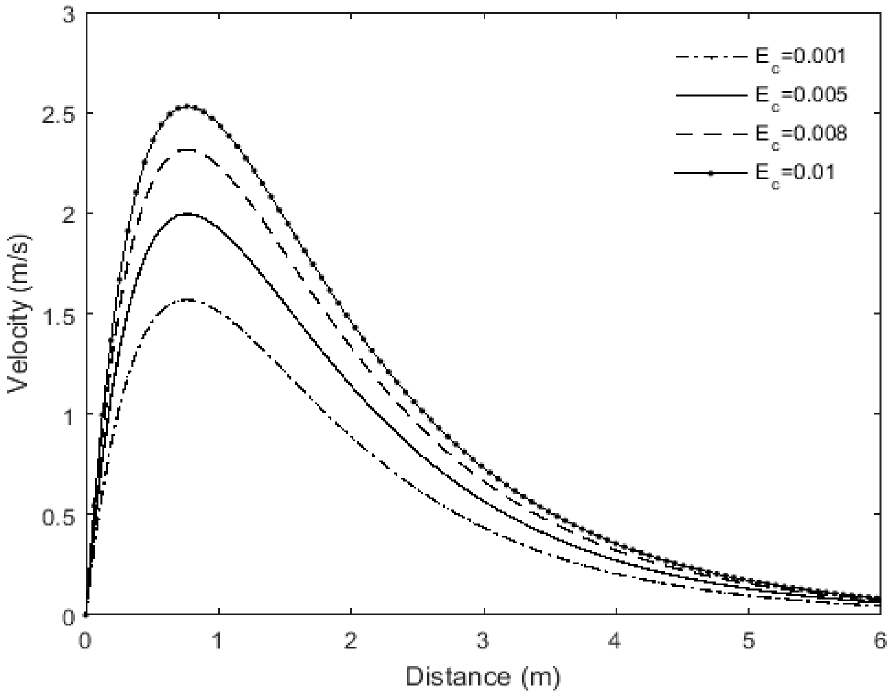

Figure 1 Velocity profiles for E_c when $\epsilon = 0.01, \omega = 10, \omega t = 0$, S_c=0.3, G_r=5, G_m=5, R_c=0.1, K_c=2, P_r=0.7, M=2, A=0.5

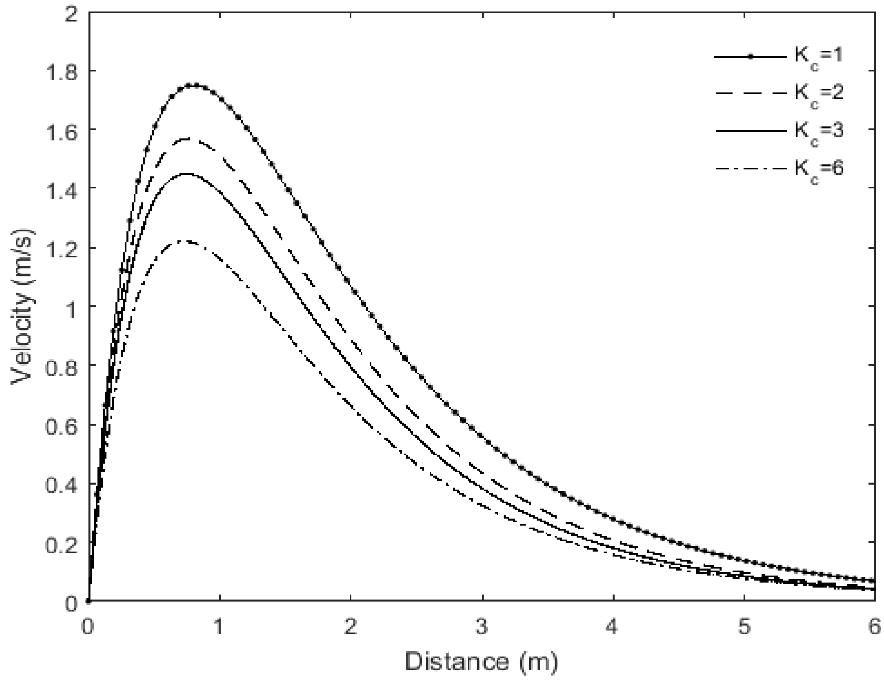

Figure 2 Velocity profiles for K_c when $\in = 0.01, \omega = 10, \omega t = 0$, S_c=0.3, G_r=5, G_m=5, R_c=0.1, E_c=0.001, P_r=0.7, M=2, A=0.5

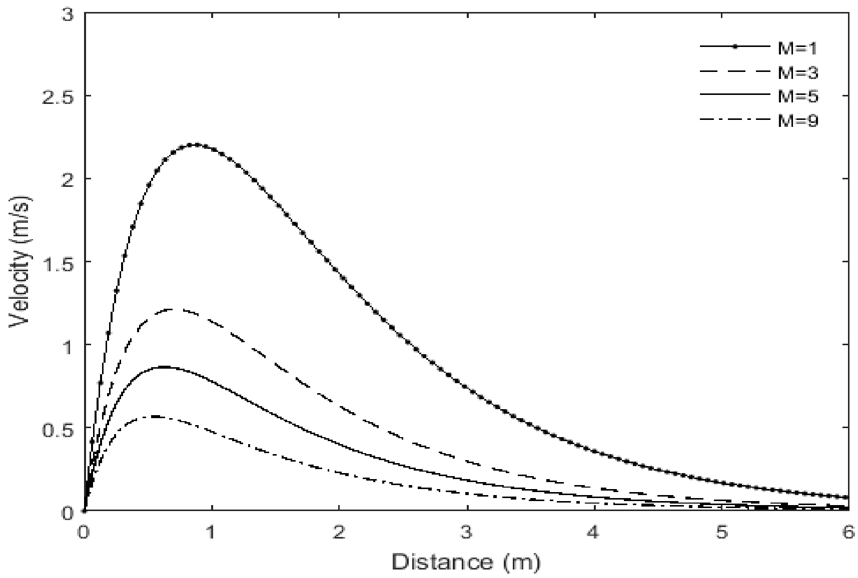

Figure 3 Velocity profiles for M when $\in = 0.01, \omega = 10, \omega t = 0$, S_c=0.3, G_r=5, G_m=5, R_c=0.1, E_c=0.001, P_r=0.7, K_c=2, A=0.5

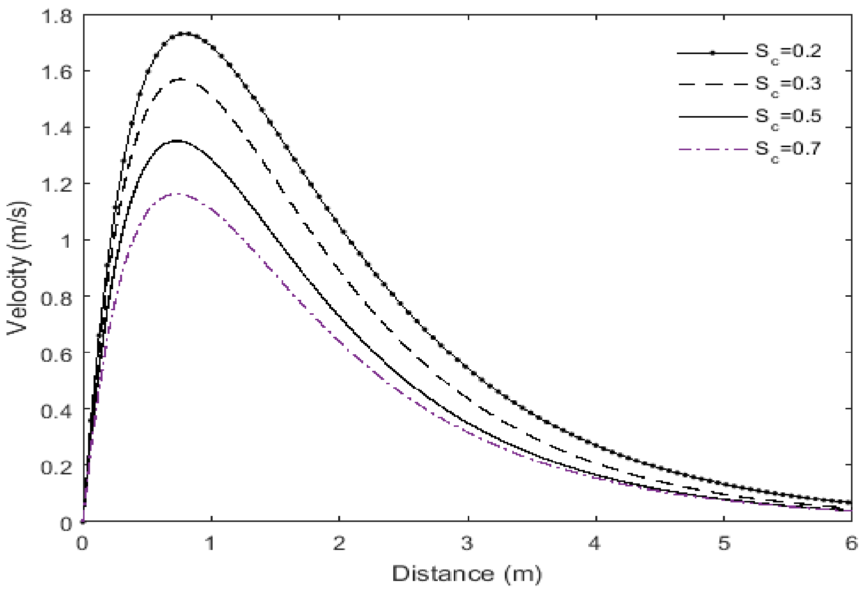

Figure 4 Velocity profiles for S_c when $\in = 0.01, \omega = 10, \omega t = 0$, M=2, G_r=5, G_m=5, R_c=0.1, E_c=0.001, P_r=0.7, K_c=2, A=0.5

The Prandtl number defines the ratio of momentum diffusivity to thermal diffusivity. It is observed from fig-5 that an increase in the Prandtl number results a decrease of the thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that increasing values of Prandtl number equivalent to increase the thermal conductivities and therefore heat is able to diffuse away from the heated plate more rapidly. Thus, it is concluded that in case of smaller Prandtl number as the thermal boundary layer is thicker, the rate of heat transfer is reduced.

Fig-6 shows the effect of chemical reaction parameter on concentration field. It is seen that as the chemical reaction parameter increases, the concentration decreases. Again concentration is higher at the plate and gradually decreases farther away from the plate. The reactants consume during chemical reaction proceeds. By increasing chemical reaction parameter, the reaction proceeds more efficiently in the presence of porous media. Hence, a decline in concentration profile is seen.

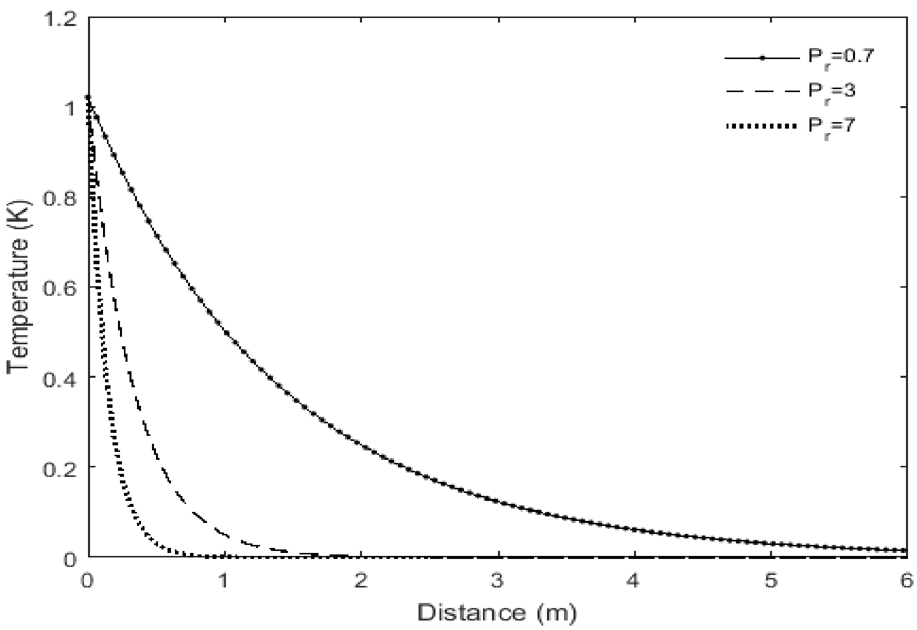

Figure 5 Temperature profiles for P_r when $\in = 0.01, \omega = 10, \omega t = 0$, M=2, G_r=5, G_m=5, R_c=0.1, E_c=0.0001, K_c=2, A=0.5, S_c=0.3

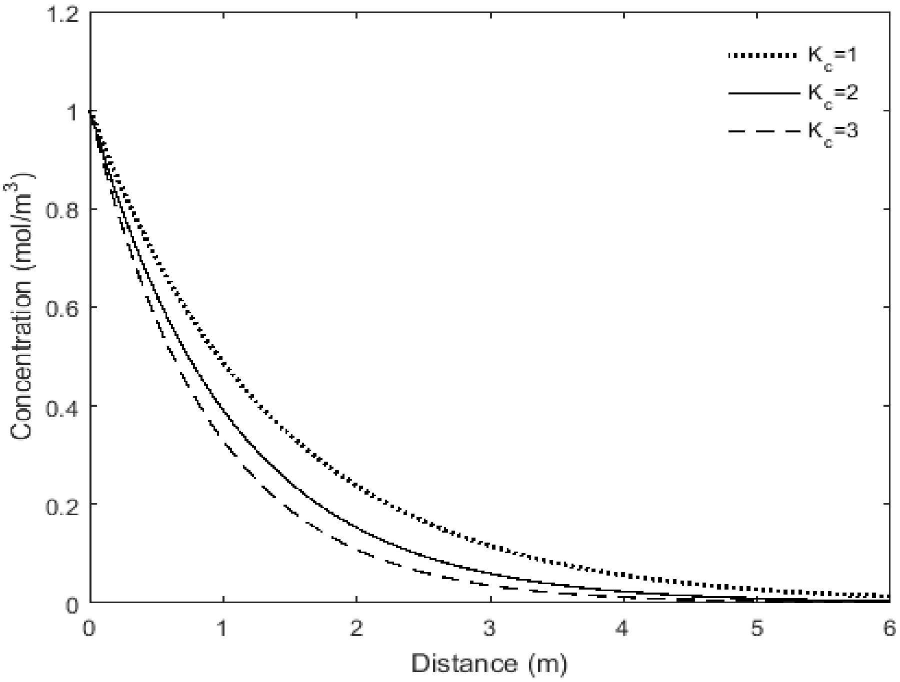

Figure 6 Concentration profiles for K_c when $\in = 0.001, \omega = 10, \omega t = 0$, S_c=0.3

Figure 7 indicates the influence of Schimdt number on concentration distribution. It is elucidated that higher values of Schimdt number reduces the concentration. The reason is that Schimdt number is the ratio of kinematic viscosity to the molecular diffusivity and higher Schimdt number means lower molecular diffusion rate which leads lower concentration.

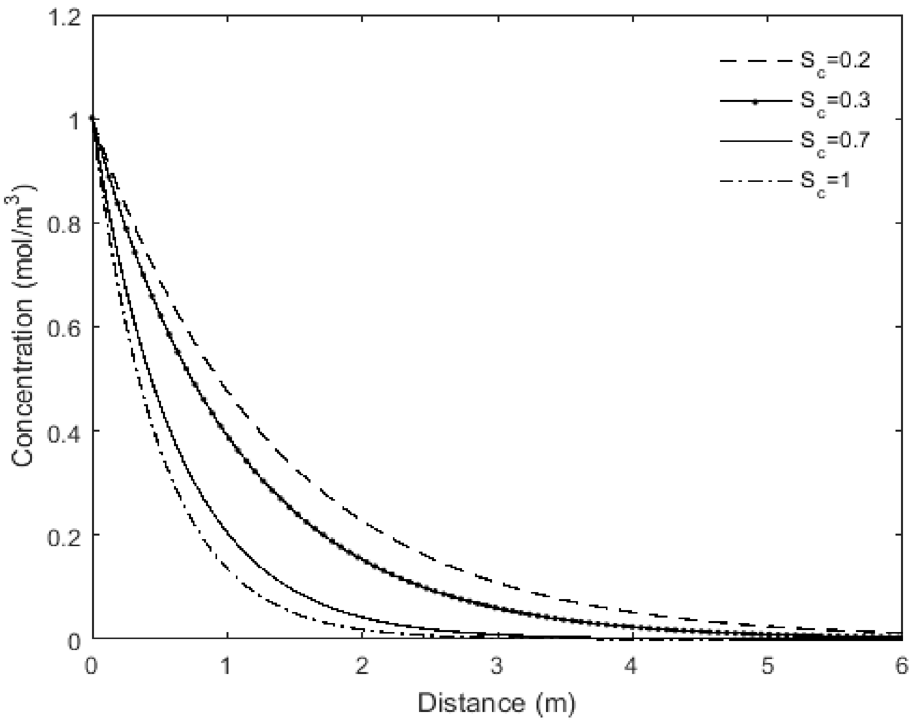

Figure 7 Concentration profiles for S_c when $\in = 0.001, \omega = 10, \omega t = 0$, K_c=2

Table 1 Nusselt number data for $\in = 0.001, \omega = 10, \omega t = 0$, G_r=5, G_m=5, R_c=0.1, A=0.5, Sc=0.3

M	Pr	Ec	Nu (Nayak and Panda[15])	Nu (present)
1	0.7	0.001	0.116963	0.11575
2	0.7	0.001	0.118819	0.11781
3	0.7	0.001	0.119624	0.11862
3	1	0.001	0.166763	0.16685
3	2	0.001	0.239854	0.23976
3	5	0.001	0.367016	0.36702
3	5	0.005	0.352350	0.35244
3	5	0.008	0.340166	0.34025
3	5	0.010	0.331399	0.33148

Table 2 Sherwood number data for $\in = 0.001, \omega = 10, \omega t = 0$, G_r=5, G_m=5, R_c=0.1, A=0.5

K_c	Sc	Sh
1	0.2	0.5626
2	0.2	0.7446
3	0.2	0.8853
2	0.3	0.9440
2	0.7	1.5921

Table 1 shows the Nusselt number for different pertinent parameters at phase $\omega t = 0$. It is observed that the Nusselt number increases with an increase of magnetic parameter. This is due to the fact that the applied magnetic field induces a Lorentz force and a Joule heating which are responsible for the increase of the rate of heat transfer and the decrease of the thermal boundary layer thickness. Further, it is observed that an increase in the Prandtl number results an increase in the Nusselt number that corresponds to more active convection but the reverse effect is marked when the Eckert number increases. Furthermore, Table-1 represents a comparison of present study with the results of Nayak and Panda [15] which shows a good agreement.

Sherwood numbers (Sh) are given in Table-2. It is observed that Sherwood number increases with increase in both chemical reaction parameter and Schmidt number. Sherwood number represents the ratio of the convective mass transfer to the rate of diffusive mass transport. So for the higher values of chemical reaction parameter and Schimdt number, convective mass transfer is dominant which may be of more interest due to application in drying, evaporation or dissolution.

CONCLUSIONS

An unsteady two dimensions mixed convective boundary layer flow of an incompressible and electrically conducting non-Newtonian second grade visco-elastic fluid along an infinite vertical heated plate in the presence of thermal and solutal buoyancy effects. The dimensionless governing equations for this investigation are solved analytically using two-term harmonic and non-harmonic functions. All the calculations and graphs have been accomplished using MATLAB. Some of the important conclusions are given below.

- The higher Eckert number implies greater viscous dissipative heat and causes an increase in the velocity.
- Increasing values of chemical reaction parameter lead to low velocity and concentration.
- Concentration is reduced by the increasing values of Schmidt number.
- An increase in the Prandtl number results a decrease of the thermal boundary layer thickness.
- It is observed that an increase in the Prandtl number results an increase in the Nusselt number but the reverse effect is marked when the Eckert number increases.
- Higher values of chemical reaction parameter and Schimdt number enhance the mass transfer rate.

AUTHOR CONTRIBUTIONS

Both authors contributed equally.

CONFLICT OF INTERESTS

The authors declare no conflict of interests.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author, upon reasonable request.

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APPENDIX

$$\alpha=\left(\frac{i\omega}{4}+k_c\right)S_c,\quad \mathrm{m}_0=-\left(\frac{S_c+\sqrt{S_c^2+4S_ck_c}}{2}\right),\ \mathrm{m}_1=-\left(\frac{S_c+\sqrt{S_c^2+4\alpha}}{2}\right)$$

$$\mathrm{m}_2=-\left(\frac{P_r+\sqrt{P_r^2+i\omega P_r}}{2}\right),\mathrm{m}_3=-\left(\frac{1+\sqrt{1+4M}}{2}\right),\mathrm{m}_4=-\left(\frac{1+\sqrt{1+4\left(M+\frac{i\omega}{4}\right)}}{2}\right),$$

$$I_0=\frac{4Am_0}{\omega},\quad I_1=P_r^2-P_r-M,\quad I_2=m_0^2+m_0-M,\ I_3=\frac{G_r}{I_1},\ I_4=\frac{G_m}{I_2},\quad I_5=m_3^2+m_3-M$$

$$I_6=\frac{4iP_rA}{\omega},\quad I_7=\frac{P_r^3I_3}{I_1},\quad I_8=\frac{-m_0^3I_4}{I_2},$$

$I_9=(I_3+I_4)+R_c(I_7+I_8)$, $I_{10}=I_3+R_cI_7$, $I_{11}=I_4+R_cI_8$

$$I_{12}=\frac{P_rI_9^2(m_3^2+M)}{2m_3(2m_3+P_r)},\quad I_{13}=\frac{I_{10}^2P_r(P_r^2+M)}{2P_r^2},\ I_{14}=\frac{I_{11}^2P_r(m_0^2+M)}{2m_0(2m_0+P_r)}$$

$$I_{15}=\frac{2I_9I_{10}P_r\left(m_3P_r-M\right)}{m_3^2-P_rm_3},\ I_{16}=\frac{2P_rI_{10}I_{11}\left(M-m_0P_r\right)}{m_0^2-m_0P_r},\ I_{17}=\frac{2P_rI_9I_{11}\left(M+m_0m_3\right)}{\left(m_3+m_0\right)\left(P_r+m_3+m_0\right)}$$

$I_{18}=I_{17}-I_{12}-I_{13}-I_{14}-I_{15}-I_{16}$, $I_{19}=4m_3^2+2m_3-M$, $I_{20}=4P_r^2-2P_r-M$,

$$I_{21}=4m_0^2+2m_0-M,\quad I_{22}=\left(P_r-m_3\right)^2-\left(P_r-m_3\right)-M$$

$I_{23}=(P_r-m_0)^2-(P_r-m_0)-m_0$, $I_{24}=(m_3+m_0)^2+(m_3+m_0)-M$,

$$I_{25}=\frac{G_rI_{18}}{I_1},\ I_{26}=\frac{G_rI_{12}}{I_{19}},\ I_{27}=\frac{G_rI_{13}}{I_{20}},\ I_{28}=\frac{G_rI_{14}}{I_{21}},\ I_{29}=\frac{G_rI_{15}}{I_{22}},\ I_{30}=\frac{G_rI_{16}}{I_{23}},\ I_{31}=\frac{G_rI_{17}}{I_{24}},$$

$I_{32}=I_{31}-I_{25}-I_{26}-I_{27}-I_{28}-I_{29}-I_{30}$, $I_{34}=\frac{I_{25}P_r^3}{I_1}$, $I_{35}=\frac{8m_3^3I_{26}}{I_{19}}$, $I_{36}=\frac{8P_r^3I_{27}}{I_{20}}$, $I_{37}=\frac{8m_0^3I_{28}}{I_{21}}$,

$I_{38}=\frac{I_{29}\left(P_r-m_3\right)^3}{I_{22}}$, $I_{39}=\frac{I_{30}\left(P_r-m_0\right)^3}{I_{23}}$, $I_{40}=\frac{I_{31}\left(m_3+m_0\right)^3}{I_{24}}$, $I_{41}=I_{35}+I_{37}-I_{40}-I_{39}-I_{38}-I_{36}-I_{34}$,

$I_{42}=I_{32}+R_cI_{41}$, $I_{43}=m_2^2+m_2-\left(M+\frac{i\omega}{4}\right)$, $I_{44}=P_r^2-P_r-\left(M+\frac{i\omega}{4}\right)$

$$I_{45}=m_1^2+m_1-\left(M+\frac{i\omega}{4}\right),\ I_{46}=m_0^2+m_0-\left(M+\frac{i\omega}{4}\right),I_{47}=m_3^2+m_3-\left(M+\frac{i\omega}{4}\right),$$

$$I_{48}=\frac{G_r\left(\frac{4iP_rA}{\omega}-1\right)}{I_{43}},\ I_{49}=\frac{4iG_rP_rA}{\omega I_{44}},I_{50}=\frac{G_m\left(1+\frac{4im_0A}{\omega}\right)}{I_{45}},\ I_{51}=\frac{4iG_mm_0A}{\omega I_{46}}$$

$I_{52}=\frac{A\left(I_3+I_4\right)m_3}{I_{47}}$, $I_{53}=\frac{AP_rI_3}{I_{44}}$, $I_{54}=\frac{AI_4m_0}{I_{46}}$, $I_{55}=I_{49}+I_{50}+I_{52}+I_{53}-I_{48}-I_{51}-I_{54}$,

$I_{56}=I_{49}+I_{53}$, $I_{57}=I_{51}+I_{54}$, $I_{58}=m_4^2+m_4-\left(M+\frac{i\omega}{4}\right)$

$$I_{59}=\frac{I_{55}m_4^3}{I_{58}},I_{60}=\frac{I_{48}m_2^3}{I_{43}},I_{61}=\frac{I_{56}P_r^3}{I_{44}},I_{62}=\frac{m_1^3I_{50}}{I_{45}},I_{63}=\frac{m_0^3I_{57}}{I_{46}},I_{64}=\frac{m_3^3I_{52}}{I_{47}},$$

$$I_{65} = \frac{i\omega}{4}\frac{I_{55}m_4^2}{I_{58}}, I_{66} = \frac{i\omega}{4}\frac{I_{48}m_2^2}{I_{43}}, I_{67} = \frac{I_{56}P_r^2}{I_{44}}, I_{68} = \frac{I_{50}m_1^2}{I_{45}}, I_{69} = \frac{I_{57}m_0^2}{I_{46}},$$

$$I_{70} = \frac{I_{52}m_3^2}{I_{47}}, I_{71} = \frac{(I_7 + I_8)m_3}{I_{47}}, I_{72} = \frac{I_7 P_r}{I_{44}}, I_{73} = \frac{I_8 m_0}{I_{46}}, I_{74} = \frac{(I_3 + I_4)m_3^3}{I_{47}}, I_{75} = \frac{I_3 P_r^3}{I_{44}}, I_{76} = \frac{m_0^3 I_4}{I_{46}}$$

,

$I_{77} = I_{65} - I_{59}$, $I_{78} = I_{64} - I_{70} - AI_{71} - AI_{74}$, $I_{79} = I_{66} - I_{60}$, $I_{80} = I_{62} - I_{68}$, $I_{81} = I_{69} - I_{63} + AI_{73} + AI_{76}$,
$I_{82} = -I_{61} - I_{67} - AI_{72} - AI_{75}$, $I_{83} = -I_{78} - I_{79} - I_{80} - I_{81} - I_{82}$, $I_{84} = I_{25} + R_c I_{34}$, $I_{85} = I_{26} - R_c I_{35}$,
$I_{86} = I_{27} + R_c I_{36}$, $I_{87} = I_{28} - R_c I_{37}$, $I_{88} = I_{29} + R_c I_{38}$, $I_{89} = I_{30} + R_c I_{39}$, $I_{90} = I_{31} - R_c I_{40}$,
$I_{91} = I_{55} + R_c I_{83}$, $I_{92} = I_{48} + R_c I_{79}$, $I_{93} = R_c I_{82} - I_{56}$

$I_{94} = R_c I_{80} - I_{50}$, $I_{95} = R_c I_{81} + I_{57}$, $I_{96} = R_c I_{78} - I_{52}$, $I_{97} = P_r^2 A I_{18}$

$I_{98} = -2P_r A I_{12} m_3 - 2 P_r I_9 I_{96} m_3^2 - 2P_r M I_9 I_{96}$

$I_{99} = 2 P_r^2 A I_{13} + 2 P_r^3 I_{10} I_{93} + 2 P_r M I_{10} I_{93}$

$I_{100} = -2\, m_0 P_r A I_{14} + 2 P_r m_0^2 I_{11} I_{95} + 2 P_r M I_{11} I_{95}$

$I_{101} = P_r A I_{15} (P_r - m_3) + 2 P_r^2 I_9 I_{93} m_3 - 2 P_r^2 I_{10} I_{96} m_3 - 2MP_r I_9 I_{93} + 2MP_r I_{10} I_{96}$

$I_{102} = P_r A (P_r - m_0) I_{16} - 2 P_r^2 I_{10} I_{95} m_0 + 2P_r M I_{10} I_{95} - 2 P_r^2 m_0 I_{11} I_{93} + 2MP_r I_{11} I_{93}$

$I_{103} = P_r A (m_3 + m_0) I_{17} - 2P_r m_0 m_3 I_9 I_{95} - 2MP_r I_9 I_{95} + 2 P_r m_0 I_{11} I_{95} m_3 + 2MP_r I_{11} I_{96}$

$I_{104} = 2P_r I_9 (I_{92} m_3 m_4 + M I_{91})$, $I_{105} = 2 P_r I_9 I_{92} (m_2 m_3 + M)$, $I_{106} = 2 P_r I_9 I_{94} (m_3 m_1 + M)$
$I_{107} = 2 P_r I_{10} (P_r m_4 I_{92} - I_{91} M)$, $I_{108} = 2 P_r I_{10} I_{92} (P_r m_2 + M)$, $I_{109} = 2P_r I_{10} I_{94} (P_r m_1 - M)$

$I_{110} = 2 P_r I_{11} (M I_{91} + m_0 m_4 I_{92})$, $I_{111} = 2 P_r (M + m_0 m_2) I_{11} I_{92}$, $I_{112} = 2 P_r I_{11} I_{94} (M + m_0 m_1)$

$$I_{113} = \frac{4i}{\omega P_r},\ I_{114} = 4m_3^2 + 2m_3 P_r - \frac{i\omega P_r}{4}, I_{115} = 2P_r^2 - \frac{i\omega P_r}{4},\ I_{116} = 4m_0^2 + 2m_0 P_r - \frac{i\omega P_r}{4}$$

$$I_{117} = (P_r - m_3)^2 - P_r(P_r - m_3) - \frac{i\omega P_r}{4},\ I_{118} = (P_r - m_0)^2 - P_r(P_r - m_0) - \frac{i\omega P_r}{4}$$

$$I_{119} = (m_3 + m_0)^2 + P_r(m_3 + m_0) - \frac{i\omega P_r}{4},\ I_{120} = (m_3 + m_4)^2 + P_r(m_3 + m_4) - \frac{i\omega P_r}{4}$$

$$I_{121} = (m_3 + m_2)^2 + P_r(m_3 + m_2) - \frac{i\omega P_r}{4},\ I_{122} = (m_3 + m_1)^2 + P_r(m_3 + m_1) - \frac{i\omega P_r}{4}$$

$$I_{123} = (P_r - m_4)^2 - P_r(P_r - m_4) - \frac{i\omega P_r}{4},\ I_{124} = (P_r - m_2)^2 - P_r(P_r - m_2) - \frac{i\omega P_r}{4}$$

$$I_{125} = (P_r - m_1)^2 - P_r(P_r - m_1) - \frac{i\omega P_r}{4},\ I_{126} = (m_4 + m_0)^2 + P_r(m_4 + m_0) - \frac{i\omega P_r}{4}$$

$$I_{127} = (m_2 + m_0)^2 + (m_2 + m_0)P_r - \frac{i\omega P_r}{4},\ I_{128} = (m_1 + m_0)^2 + (m_1 + m_0)P_r - \frac{i\omega P_r}{4}$$

$$I_{129} = \frac{I_{97}}{I_{113}}, I_{130} = \frac{I_{98}}{I_{114}}, I_{131} = \frac{I_{99}}{I_{115}}, I_{132} = \frac{I_{100}}{I_{116}}, I_{133} = \frac{I_{101}}{I_{117}}, I_{134} = \frac{I_{102}}{I_{118}},$$

$$I_{135} = \frac{I_{103}}{I_{119}}, I_{136} = \frac{I_{104}}{I_{120}}, I_{137} = \frac{I_{105}}{I_{121}}, I_{138} = \frac{I_{106}}{I_{122}}, I_{139} = \frac{I_{107}}{I_{123}}, I_{140} = \frac{I_{108}}{I_{124}},$$

$$I_{141} = \frac{I_{109}}{I_{125}}, I_{142} = \frac{I_{110}}{I_{126}}, I_{143} = \frac{I_{111}}{I_{127}}, I_{144} = \frac{I_{112}}{I_{128}},$$

$I_{145} = I_{136} + I_{137} + I_{138} + I_{139} + I_{140} + I_{141} - I_{129} - I_{130} - I_{131} - I_{132} - I_{133} - I_{134} - I_{135} - I_{142} - I_{143} - I_{144}$

$$I_{146} = 4m_3^2 + 2m_3 - \left(M + \frac{i\omega}{4}\right), I_{147} = 4P_r^2 - 2P_r - \left(M + \frac{i\omega}{4}\right)$$

$$I_{148} = 4m_0^2 + 2m_0 - \left(M + \frac{i\omega}{4}\right), I_{149} = (P_r - m_3)^2 - (P_r - m_3) - \left(M + \frac{i\omega}{4}\right)$$

$$I_{150}=\left(P_r-m_0\right)^2-\left(P_r-m_0\right)-\left(M+\frac{i\omega}{4}\right), I_{151}=\left(m_3+m_0\right)^2+\left(m_3+m_0\right)-\left(M+\frac{i\omega}{4}\right)$$

$$I_{152}=\left(m_3+m_4\right)^2+\left(m_3+m_4\right)-\left(M+\frac{i\omega}{4}\right), I_{153}=\left(m_3+m_2\right)^2+\left(m_3+m_2\right)-\left(M+\frac{i\omega}{4}\right)$$

$$I_{154}=\left(m_3+m_1\right)^2+\left(m_3+m_1\right)-\left(M+\frac{i\omega}{4}\right), I_{155}=\left(P_r-m_4\right)^2-\left(P_r-m_4\right)-\left(M+\frac{i\omega}{4}\right)$$

$$I_{156}=\left(P_r-m_2\right)^2-\left(P_r-m_2\right)-\left(M+\frac{i\omega}{4}\right), I_{157}=\left(P_r-m_1\right)^2-\left(P_r-m_1\right)-\left(M+\frac{i\omega}{4}\right)$$

$$I_{158}=\left(m_4+m_0\right)^2+\left(m_4+m_0\right)-\left(M+\frac{i\omega}{4}\right), I_{159}=\left(m_2+m_0\right)^2+\left(m_2+m_0\right)-\left(M+\frac{i\omega}{4}\right)$$

$$I_{160}=\left(m_1+m_0\right)^2+\left(m_1+m_0\right)-\left(M+\frac{i\omega}{4}\right)$$

$$I_{161}=\frac{G_rI_{145}}{I_{43}}, I_{162}=\frac{G_rI_{129}}{I_{44}}, I_{163}=\frac{G_rI_{130}}{I_{146}}, I_{164}=\frac{G_rI_{131}}{I_{147}}, I_{165}=\frac{G_rI_{132}}{I_{148}},$$

$$I_{166}=\frac{G_rI_{133}}{I_{149}}, I_{167}=\frac{G_rI_{134}}{I_{150}}, I_{168}=\frac{G_rI_{135}}{I_{151}}, I_{169}=\frac{G_rI_{136}}{I_{152}}, I_{170}=\frac{G_rI_{137}}{I_{153}},$$

$$I_{171}=\frac{G_rI_{138}}{I_{154}}, I_{172}=\frac{G_rI_{139}}{I_{155}}, I_{173}=\frac{G_rI_{140}}{I_{156}}, I_{174}=\frac{I_{141}G_r}{I_{157}}, I_{175}=\frac{I_{142}G_r}{I_{158}}, I_{176}=\frac{G_rI_{143}}{I_{159}},$$

$$I_{177}=\frac{I_{144}G_r}{I_{160}}, I_{178}=\frac{I_{32}m_3}{I_{47}}, I_{179}=\frac{I_{25}P_r}{I_{44}}, I_{180}=\frac{2m_3I_{26}}{I_{146}}, I_{181}=\frac{2P_rI_{27}}{I_{147}}, I_{182}=\frac{2m_0I_{28}}{I_{148}},$$

$$I_{183}=\frac{I_{29}\left(P_r-m_3\right)}{I_{149}}, I_{184}=\frac{I_{30}\left(P_r-m_0\right)}{I_{150}}, I_{185}=\frac{\left(m_3+m_0\right)I_{31}}{I_{151}},$$

$I_{186} = I_{162} + A\, I_{179}$, $I_{187} = I_{163} - AI_{180}$, $I_{188} = I_{164} + A\, I_{181}$, $I_{189} = I_{165} - AI_{182}$,
$I_{190} = I_{166} + AI_{183}$, $I_{191} = I_{167} + AI_{184}$, $I_{192} = I_{168} + AI_{185}$,

$$I_{193}=\frac{I_{169}\left\{\left(m_3+m_4\right)^3-\frac{i\omega}{4}\left(m_3+m_4\right)^2\right\}}{I_{152}}$$

$$I_{194}=\frac{I_{170}\left\{\left(m_3+m_2\right)^3-\frac{i\omega}{4}\left(m_3+m_2\right)^2\right\}}{I_{153}}$$

$I_{195} = AI_{178}$,
$A_1 = I_{169} + I_{170} + I_{171} + I_{172} + I_{173} + I_{174} + I_{195} - I_{175} - I_{176} - I_{177}$
$- I_{161} - I_{186} - I_{187} - I_{188} - I_{189} - I_{190} - I_{191} - I_{192}$

$$A_2=\frac{I_{161}m_2^2\left(\frac{i\omega}{4}-m_2\right)}{I_{43}}, A_3=\frac{I_{186}P_r^2\left(P_r+\frac{i\omega}{4}\right)}{I_{44}}, A_4=\frac{4I_{187}m_3^2\left(\frac{i\omega}{4}-2m_3\right)}{I_{146}}$$

$$A_5=\frac{4I_{188}P_r^2\left(2P_r+\frac{i\omega}{4}\right)}{I_{147}}, A_6=\frac{4I_{189}m_0^2\left(\frac{i\omega}{4}-2m_0\right)}{I_{148}}, A_7=\frac{I_{190}\left(P_r-m_3\right)^2\left(P_r-m_3+\frac{i\omega}{4}\right)}{I_{149}},$$

$$A_8=\frac{I_{191}\left(P_r-m_0\right)^2\left(P_r-m_0+\frac{i\omega}{4}\right)}{I_{150}}, A_9=\frac{I_{192}\left(m_3+m_0\right)^2\left(m_3+m_0-\frac{i\omega}{4}\right)}{I_{151}},$$

$$A_{10}=\frac{I_{171}\left(m_3+m_1\right)^2\left(m_3+m_1-\frac{i\omega}{4}\right)}{I_{154}},\ A_{11}=\frac{I_{172}\left(P_r-m_4\right)^2\left(P_r-m_4+\frac{i\omega}{4}\right)}{I_{155}},$$

$$A_{12}=\frac{I_{173}\left(P_r-m_2\right)^2\left(P_r-m_2+\frac{i\omega}{4}\right)}{I_{156}},\ A_{13}=\frac{I_{174}\left(P_r-m_1\right)^2\left(P_r-m_1+\frac{i\omega}{4}\right)}{I_{157}},$$

$$A_{14}=\frac{I_{175}\left(m_4+m_0\right)^2\left(m_4+m_0-\frac{i\omega}{4}\right)}{I_{158}},\ A_{15}=\frac{I_{176}\left(m_2+m_0\right)^2\left(m_2+m_0-\frac{i\omega}{4}\right)}{I_{159}},$$

$$A_{16}=\frac{I_{177}\left(m_1+m_0\right)^2\left(m_1+m_0-\frac{i\omega}{4}\right)}{I_{160}},\ A_{17}=\frac{I_{195}m_3^2\left(m_3-\frac{i\omega}{4}\right)}{I_{47}},$$

$$A_{18}=\frac{Am_3\left(I_{32}m_3^2+I_{41}\right)}{I_{47}},\ A_{19}=\frac{AP_r\left(I_{25}P_r^2+I_{34}\right)}{I_{44}},$$

$$A_{20}=\frac{A\left(2I_{35}m_3-I_{26}8m_3^3\right)}{I_{146}},\ A_{21}=\frac{A\left(2I_{36}P_r+8I_{27}P_r^3\right)}{I_{147}},$$

$$A_{22}=\frac{A\left[\left(2I_{37}m_0-8I_{28}\left(m_0\right)^3\right)\right]}{I_{148}},\ A_{23}=\frac{A\left(P_r-m_3\right)\left\{\left(P_r-m_3\right)^2I_{29}+I_{38}\right\}}{I_{149}},$$

$$A_{24}=\frac{A\left(P_r-m_0\right)\left\{\left(P_r-m_0\right)^2I_{30}+I_{39}\right\}}{I_{150}},\ A_{25}=\frac{A\left(m_3+m_0\right)\left\{I_{40}-\left(m_3+m_0\right)^2I_{31}\right\}}{I_{151}},$$

$A_{26}=A_{19}+A_3$, $A_{27}=A_{20}+A_4$, $A_{28}=A_{21}+A_5$, $A_{29}=A_{22}+A_6$, $A_{30}=A_{23}+A_7$, $A_{31}=A_{24}+A_8$, $A_{32}=A_{25}+A_9$, $A_{33}=A_{17}-A_{18}$, $A_{34}=A_{32}+A_{11}+A_{12}+A_{13}+A_{14}+A_{15}+A_{16}-A_{26}-A_{27}-A_{28}-A_{29}-A_{30}-A_{31}-A_2-A_{10}-A_{33}-I_{193}-I_{194}$, $A_{35}=A_1+R_cA_{34}$, $A_{36}=I_{161}+R_cA_2$, $A_{37}=I_{186}+R_cA_{26}$, $A_{38}=I_{187}+R_cA_{27}$, $A_{39}=I_{188}+R_cA_{28}$, $A_{40}=I_{189}+R_cA_{29}$, $A_{41}=I_{190}+R_cA_{30}$, $A_{42}=I_{191}+R_cA_{31}$, $A_{43}=I_{192}-R_cA_{32}$, $A_{44}=I_{195}-R_cA_{33}$, $A_{45}=I_{169}-R_cI_{193}$, $A_{46}=I_{170}-R_cI_{194}$, $A_{47}=I_{171}-R_cA_{10}$, $A_{48}=I_{172}+R_cA_{11}$, $A_{49}=I_{173}+R_cA_{12}$, $A_{50}=I_{174}+R_cA_{13}$, $A_{51}=I_{175}-R_cA_{14}$, $A_{52}=I_{176}-R_cA_{15}$, $A_{53}=I_{177}-R_cA_{16}$

Nomenclature

B_0	Magnetic field of constant strength	G_r	thermal Grashof number
C	Concentration of the fluid	k	thermal conductivity
C_p	Specific heat	k_c	chemical reaction parameter
C_∞	Ambient concentration	k_0	elasticity parameter
D	Thermal diffusivity	K_p	Porosity parameter
E_c	Eckert number	k^*	Mean absorption coefficient
g	acceleration due to gravity	M	Magnetic field parameter
G_m	modified Grashof number	Nu	Nusselt number
P_r	Prandtl number	t	time
Q	Heat source parameter	T_∞	Ambient temperature
R_c	visco-elastic parameter	T_w	temperature of the wall
S_c	Schmidt number	u	x component of velocity
Sh	Sherwood number	v	y component of velocity
T	Temperature	x, y	Coorinates

Greek Symbols

θ	Non-dimensional temperature	ρ	Density
σ^*	Stefan-Boltzmann constant	β_T	Thermal expansion coefficient
β_C	Concentration expansion coefficient	α	Thermal diffusivity
ν	Kinematic viscosity	μ	coefficient of viscosity
η	Similarity variable	σ	electrical conductivity,
ϕ	Nondimensional concentration		