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Comparison of Energy of the Sitnikov's Restricted three Body Problem if the Primaries are Sources of Radiation, Oblate Spheroids and Triaxial Rigid Bodies

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ABSTRACT

The paper establishes the energy of Stinikov's restricted three body problem when the primaries are source of radiations, oblate spheroids and triaxial rigid bodies. The effect of source of radiations, oblate spheroids and triaxial rigid bodies with their corresponding equation of motions has been studied. The equation of motions of third body is used to calculate the required energy of the infinitesimal body.

Keywords: Infinitesimal mass; oblateness; radiation; triaxial rigid bodies.

INTRODUCTION

The three body problem has two primaries of equal masses and these two move in circular orbits and elliptic orbits around their centre of mass studied by Sitnikov (1960). The third body, infinitesimal mass is moving along a line which is perpendicular to the plane of primaries and passing through centre of mass of the primaries.

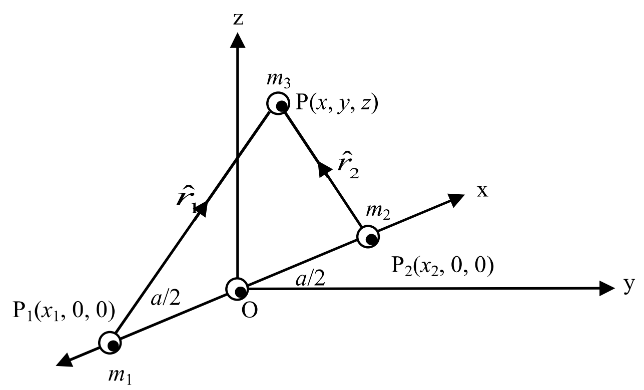
If two primaries are taken to be the sources of radiation then due to which the distance between the primaries will be increased and which is supposed to 'a' when $a > 1$. Thus both the primaries are at equal distances $a/2$ from the centre of mass.

But if the primaries are oblate spheroid in shape then due to their oblateness the primaries will not be equidistant from their center of mass. To keep the primaries equidistant from the centre of mass of the primaries, the following conditions are to be imposed:

(i) Principal axes of the oblate bodies should be parallel to the synodic axes.

(ii) The masses of both the primaries should be equal.

If we consider both the primaries are of masses m_1 and m_2 to be axis symmetric bodies with one of its axes as the axis of symmetry then equatorial plane coincident with the plane of motion. Let (O, XYZ) be the inertial frame of reference. Here, the line joining m_1 and m_2 is taken as the x-axis and 'O' their centre of mass as origin. The line passing through 'O' and perpendicular to the line joining m_1 and m_2 is taken as the y-axis and z-axis is taken according to right handed coordinate system. Let us consider a synodic system of coordinates (O, xyz) initially coincident (at $t = 0$) with the inertial system (O, XYZ) and rotating with angular velocity 'n' about z-axis (the Z-axis coincides with the z-axis). Let the coordinate of the mass m_3 be (x, y, z) at time t referred to the rotating frame of reference (O, xyz). We also assume that initially the principal axis of the body m_1 is parallel to the synodic axis (O, xyz) and one of the axes of symmetry be perpendicular to the plane of motion. Since the body m_1 is moving around 'O' without rotation with the same angular velocity 'n' as that of the synodic axes, therefore, the principal axes of the body with mass m_1 at P_1 will remain parallel to the synodic axis throughout the motion.



It is applied to compare the energy of Sitnikov's restricted problem of three bodies when primaries are source of radiations, oblate spheroids and triaxial rigid bodies.

Equations of Motion and Energy

Now following Thapa and Hassan (2013), the equation of motion of the third particle m_3 if primaries are sources of radiation can be written as

$$m_3 \ddot{z} = -\frac{G m_1 m_3 (1-p)}{r_1^2}\hat{r}_1 - \frac{G m_2 m_3 (1-p)}{r_2^2}\hat{r}_2 \quad (1)$$

$$\ddot{z} = -\frac{G m_1 (1-p)}{r_1^2}\hat{r}_1 - \frac{G m_2 (1-p)}{r_2^2}\hat{r}_2$$

The potential Energy of the system is given by

$$v(z) = -\frac{2Gm(1-p)}{\sqrt{z^2 + \frac{a^2}{4}}}$$

where $m_1 = m_2 = m$ and G is the constant of gravitation.

The equation of motion of the infinitesimal mass m_3, is

$$\frac{d^2z}{dt^2} = -\frac{\partial v}{\partial z}$$

$$= 2Gm(1-p)\left(-\frac{1}{2}\right)\left(z^2 + \frac{a^2}{4}\right)^{-\frac{1}{2}-1} \cdot 2z$$

$$\therefore \frac{d^2z}{dt^2} = \frac{-2Gm(1-p)z}{\left(z^2 + \frac{a^2}{4}\right)^{\frac{3}{2}}}$$

Now we fix the time in such a way that $G = 1$ and we fix the unit of mass such that $m_1 + m_2 = 2m = 1$.

Thus the equation reduces to

$$\frac{d^2z}{dt^2} + \frac{(1-p)z}{\left(z^2 + \frac{a^2}{4}\right)^{\frac{3}{2}}} = 0 \quad (2)$$

This is the cartesian equation of motion of m_3 in the Sitnikov circular restricted problem of three bodies.

$$\frac{dz}{dt}\cdot\frac{d^2z}{dt^2} + \frac{(1-p)z}{\left(z^2 + \frac{a^2}{4}\right)^{\frac{3}{2}}}\frac{dz}{dt} = 0$$

$$\Rightarrow \frac{1}{2}\frac{d}{dt}\left(\frac{dz}{dt}\right)^2 + \frac{(1-p)}{\left(z^2 + \frac{a^2}{4}\right)^{\frac{3}{2}}} z \frac{dz}{dt} = 0$$

Integration with respect to t, we get

$$\frac{1}{2}\left(\frac{dz}{dt}\right)^2 - \frac{(l-p)}{\sqrt{z^2 + a^2/4}} = \text{constant}$$

$$\Rightarrow \frac{1}{2}\left(\frac{dz}{dt}\right)^2 + v(z) = \text{constant}$$

Evidently the equation of motion is one dimensional.

The total energy of the system is

$$E = \frac{1}{2}\left(\frac{dz}{dt}\right)^2 - \frac{1-p}{\sqrt{z^2 + a^2/4}} = \text{constant} = \frac{A}{2} \quad (3)$$

Time period

We have from the energy equation

$$\frac{1}{2}\left(\frac{dz}{dt}\right)^2 - \frac{q}{\sqrt{z^2 + \frac{a^2}{4}}} = -\frac{A}{2} = E \text{ (Total Energy)} \quad (4)$$

$$-A = \left(\frac{dz}{dt}\right)^2 - \frac{2q}{\sqrt{z^2 + \frac{a^2}{4}}} = 2E$$

We get

$$\left(\frac{r}{z}\right)^2\left(\frac{dr}{dt}\right)^2 - \frac{2q}{r} = -A$$

$$\left(\frac{r}{z}\right)^2\left(\frac{dr}{dt}\right)^2 = \frac{2q}{r} - A$$

$$\left(\frac{dr}{dt}\right)^2 = \frac{z^2}{r^2}.2q\left(\frac{1}{r} - \frac{A}{2q}\right)$$

$$= \frac{2q}{r^2}\left(r^2 - \frac{a^2}{4}\right)\left(\frac{1}{r} - \frac{A}{2q}\right)$$

$$\left(\frac{dr}{dt}\right)^2 = 2q\left(1 - \frac{a^2}{4r^2}\right)\left(\frac{1}{r} - \frac{A}{2q}\right)$$

Putting $\quad r = \frac{1}{u^*} \Rightarrow \frac{dr}{dt} = -\frac{1}{u^{*2}}\frac{du^*}{dt}$

$$\Rightarrow \left(-\frac{1}{u^{*2}}\frac{du^*}{dt}\right)^2 = 2q\left(1 - \frac{a^2u^{*2}}{4}\right)\left(u^* - \frac{A}{2q}\right)$$

$$\Rightarrow \left(\frac{du^*}{dt}\right)^2 = 2qu^{*4}\left(1 - \left(\frac{au^*}{2}\right)^2\right)\left(u^* - \frac{A}{2q}\right)$$

$$= 2qu^{*4}\left[1 - (u)^2\right]\left(u^* - \frac{A}{2q}\right)$$

Putting $\quad \frac{au^*}{2} = u$

or, $\quad au^* = 2u$

$$\frac{du^*}{dt} = \frac{2}{a}\frac{du}{dt}$$

$$\therefore \left(\frac{2}{a}\frac{du}{dt}\right)^2 = 2q\left(\frac{2u}{a}\right)^4\left(1-u^2\right)\left(\frac{2u}{a} - \frac{A}{2q}\right)$$

$$\left(\frac{du}{dt}\right)^2 = \frac{16q}{a^3}u^4\left(1-u^2\right)\left(u - A^*\right)$$

Putting $v^2 = \frac{1-u}{1-A^*}$ and $k^2 = \frac{1-A^*}{2}$

or, $A^* = 1 - 2k^2, 2k^2v^2 = 1-u, u = 1 - 2k^2v^2$

$$-4k^2v\frac{dv}{dt} = \frac{du}{dt}$$

we get

$$\Rightarrow 16k^4v^2\left(\frac{dv}{dt}\right)^2 = \frac{16q}{a^3}\left(1-2k^2v^2\right)^4\cdot 4k^2v^2\left(1-k^2v^2\right)\cdot 2k^2\left(1-v^2\right)$$

$$\Rightarrow \left(\frac{dv}{dt}\right)^2 = \frac{8q}{a^3}\left(1-2k^2v^2\right)^4\left(1-k^2v^2\right)\left(1-v^2\right)$$

$$\frac{dv}{\left(1-2k^2v^2\right)^2\sqrt{\left(1-v^2\right)\left(1-k^2v^2\right)}} = \frac{2\sqrt{2q}}{a^{\frac{3}{2}}}dt$$

$$\int_0^t dt = \frac{a^{\frac{3}{2}}}{2\sqrt{2q}}\int_0^v \frac{dv}{\left(1-2k^2v^2\right)^2\sqrt{\left(1-v^2\right)\left(1-k^2v^2\right)}}$$

Putting $v = \sin\theta \Rightarrow dv = \cos\theta\, d\theta$

$$t = \frac{a^{\frac{3}{2}}}{2\sqrt{2q}}\int_0^{\frac{\pi}{2}} \frac{\cos\theta\, d\theta}{\left(1-2k^2\sin^2\theta\right)^2\sqrt{\left(1-\sin^2\theta\right)\left(1-k^2\sin^2\theta\right)}}$$

$$= \frac{a^{\frac{3}{2}}}{2\sqrt{2q}}\int_0^{\frac{\pi}{2}} \frac{d\theta}{\left(1-2k^2\sin^2\theta\right)^2\left(1-k^2\sin^2\theta\right)^{\frac{1}{2}}}$$

$$T = \frac{\pi a}{\sqrt{2E}}\left(\begin{array}{l} 6.771396637\left(\frac{aE}{1-p}\right)^{\frac{1}{2}} + 7.730049133\left(\frac{aE}{1-p}\right)^{\frac{3}{2}} \\ +4.118642807\left(\frac{aE}{1-p}\right)^{\frac{5}{2}} + 1.066179276\left(\frac{aE}{1-p}\right)^{\frac{7}{2}} \\ +0.1089727879\left(\frac{aE}{1-p}\right)^{\frac{9}{2}} + \ldots \end{array}\right) \tag{5}$$

Which gives periodicity of third body if primaries are sources of radiation.

Equations of Motion and Energy

Now following Thapa and Hassan (2014), the potential between two bodies m_1 and m_2 is given by

$$\begin{aligned} -v &= \frac{Gm_1m_2}{r} + \frac{Gm_1}{r^3}\left[\frac{A_1+B_1+C_1}{2} - \frac{3}{2}\left(A_1l_1^2 + B_1m_1^2 + C_1n_1^2\right)\right] \\ &\quad + \frac{Gm_2}{r^3}\left[\frac{A_2+B_2+C_2}{2} - \frac{3}{2}\left(A_2l_2^2 + B_2m_2^2 + C_2n_2^2\right)\right] \\ &= \frac{Gm_1m_2}{r} + \frac{Gm_1}{r^3}\left[\frac{1}{2}\left(\frac{a^2+c^2}{5} + \frac{a^2+c^2}{5} + \frac{2a^2}{5}\right) - \frac{3}{2}\left(\frac{a^2+c^2}{5}\cdot\frac{a^2}{r^2} + 0 + 0\right)\right] \\ &\quad + \frac{Gm_2}{r^3}\left[\frac{1}{2}\left(\frac{a^2+c^2}{5} + \frac{a^2+c^2}{5} + \frac{2a^2}{5}\right) - \frac{3}{2}\left(\frac{a^2+c^2}{5}\cdot\frac{a^2}{r^2} + 0 + 0\right)\right] \end{aligned} \tag{6}$$

$$\begin{aligned} &= \frac{Gm_1m_2}{r} + \frac{G(m_1+m_2)}{r^3}\left[\frac{1}{2}\left(\frac{a^2+c^2}{5} + \frac{a^2+c^2}{5} + \frac{2a^2}{5}\right) - \frac{3}{2}\left(\frac{a^2+c^2}{5}\cdot\frac{a^2}{r^2}\right)\right] \\ &= \frac{Gm_1m_2}{r} + \frac{G(m_1+m_2)}{2r^3}\left[2\left(\frac{a^2+c^2}{5}\right) + \frac{2a^2}{5} - \frac{3}{5}\left(a^2+c^2\right)\cdot\frac{a^2}{r^2}\right] \\ &= \frac{Gm_1m_2}{r} + \frac{G(m_1+m_2)}{10r^3}\left[2a^2 + 2c^2 + 2a^2 - 3\left(a^2+c^2\right)\right] \\ &= \frac{Gm_1m_2}{r} + \frac{G(m_1+m_2)}{10r^3}\left[a^2 - c^2\right] \end{aligned}$$

$$\Rightarrow -v = \frac{Gm_1m_2}{r} + \frac{GA(m_1+m_2)}{r^3}$$

where $A = \dfrac{a^2-c^2}{10}$ = oblateness of the primaries

$$-n^2 r_1\hat{u}_1 m_1 = \left[\frac{Gm_1m_2}{r^3} + \frac{2AG\left(m_1+m_2\right)}{r^5}\right]\left(r_2\hat{u}_2 - r_1\hat{u}_1\right)$$

$$-n^2 r_1\hat{u}_1\frac{1}{2} = \left[\frac{G}{4r^3} + \frac{2AG}{r^5}\right]\left(r_2\hat{u}_2 - r_1\hat{u}_1\right) \text{ as } m_1 = m_2 = \frac{1}{2} \text{ and } G = 1$$

$$\Rightarrow -n^2 r_1\hat{u}_1 = \left(\frac{1}{2r^3} + \frac{4A}{r^5}\right)\left(r_2\hat{u}_2 - r_1\hat{u}_1\right)$$

$$\Rightarrow -n^2 r_1\hat{u}_1 + \left(\frac{1}{2r^3} + \frac{4A}{r^5}\right)r_1\hat{u}_1 - \left(\frac{1}{2r^3} + \frac{4A}{r^5}\right)r_2\hat{u}_2 = 0$$

$$\Rightarrow \left(-n^2 + \frac{1}{2r^3} + \frac{4A}{r^5}\right)r_1\hat{u}_1 - \left(\frac{1}{2r^3} + \frac{4A}{r^5}\right)r_2\hat{u}_2 = 0 \tag{7}$$

Similarly from equation of motion of m_2, we get

$$-\left(\frac{1}{2r^3} + \frac{4A}{r^5}\right)r_1\hat{u}_1 + \left(-n^2 + \frac{1}{2r^3} + \frac{4A}{r^5}\right)r_2\hat{u}_2 = 0 \tag{8}$$

But when the primaries are spherical then $n^2 = \dfrac{1}{a^3}$

$$\frac{1}{a^3} = \frac{1}{a^3} + \frac{8A}{a^5} - \frac{3\lambda}{a^4}$$

$$\Rightarrow \frac{3\lambda}{a^4} = \frac{8A}{a^5}$$

$$\Rightarrow \lambda = \frac{8}{3}\left(\frac{A}{a}\right)$$

$$\therefore r = a + \frac{8}{3}\left(\frac{A}{a}\right)$$

Taking the unit of length such that $a = 1$, in the Sitnikov restricted three body problem, the oblateness of the primaries of equal masses is given by $r = 1 + \dfrac{8A}{3}$

Now following Suraj and Hassan (2013) we can find the equation of motion of the third body as

$$\frac{d^2z}{dt^2} = \frac{-z}{\left(z^2+r^2\right)^{3/2}} - \frac{9Az}{\left(z^2+r^2\right)^{5/2}} + \frac{15Az^2}{\left(z^2+r^2\right)^{7/2}} \tag{9}$$

$$\Rightarrow \frac{d^2z}{dt^2} = \frac{-z}{\left[z^2 + \left(1+\frac{8A}{3}\right)^2\right]^{3/2}} - \frac{9Az}{\left[z^2 + \left(1+\frac{8A}{3}\right)^2\right]^{5/2}} + \frac{15Az^2}{\left[z^2 + \left(1+\frac{8A}{3}\right)^2\right]^{7/2}}$$

But $\left(1+\frac{8A}{3}\right)^2 = 1+\frac{16A}{3}+\frac{64A^2}{9}$

$= 1+\frac{16A}{3}$, neglecting A^2 containing term

$\Rightarrow \left(1+\frac{8A}{3}\right)^2 = 1+\frac{16A}{3} = b$, say

the equation of motion becomes

$$\frac{d^2z}{dt^2} = \frac{-z}{(z^2+b)^{3/2}} - \frac{9Az}{(z^2+b)^{5/2}} + \frac{15Az^2}{(z^2+b)^{7/2}}$$

$$\Rightarrow \frac{d^2z}{dt^2} + \frac{z}{(z^2+b)^{3/2}} + \frac{9Az}{(z^2+b)^{5/2}} - \frac{15Az^2}{(z^2+b)^{7/2}} = 0$$

$$\Rightarrow 2\frac{dz}{dt}\cdot\frac{d^2z}{dt^2} + \frac{2z}{(z^2+b)^{3/2}}\frac{dz}{dt} + \frac{18Az}{(z^2+b)^{5/2}}\frac{dz}{dt} - \frac{30Az^2}{(z^2+b)^{7/2}}\frac{dz}{dt} = 0 \qquad (10)$$

We get,

$$\frac{1}{2}\left(\frac{dz}{dt}\right)^2 - \frac{1}{\sqrt{z^2+b}} - \frac{3A}{\left(z^2+b\right)^{3/2}} - 15A\left[\frac{2z^3}{15b^2\left(z^2+b\right)^{3/2}} + \frac{z^3}{5b\left(z^2+b\right)^{5/2}}\right] = \frac{c}{2}$$

$$\Rightarrow \frac{1}{2}\left(\frac{dz}{dt}\right)^2 - \frac{1}{\sqrt{z^2+b}} - \frac{3A}{\left(z^2+b\right)^{3/2}} - \frac{2Az^3}{b^2\left(z^2+b\right)^{3/2}} - \frac{3Az^3}{b\left(z^2+b\right)^{5/2}} = \frac{c}{2}$$

$$\Rightarrow \frac{1}{2}\left(\frac{dz}{dt}\right)^2 - \frac{1}{\sqrt{z^2+b}} - A\left[\frac{3}{\left(z^2+b\right)^{3/2}} + \frac{2z^3}{b^2\left(z^2+b\right)^{3/2}} + \frac{3z^3}{b\left(z^2+b\right)^{5/2}}\right] = \frac{c}{2}$$

but $b = 1+\frac{16A}{3}$

$$b^2 = 1+\frac{32A}{3}+\frac{256A^2}{9}$$

$$= 1+\frac{32A}{3}$$

$$\Rightarrow \frac{1}{2}\left(\frac{dz}{dt}\right)^2 - \frac{1}{\sqrt{z^2+b}} - A\left[\frac{3}{\left(z^2+b\right)^{3/2}} + \frac{2z^3}{\left(1+\frac{32A}{3}\right)\left(z^2+b\right)^{3/2}} + \frac{3z^3}{\left(1+\frac{16A}{3}\right)\left(z^2+b\right)^{5/2}}\right] = \frac{c}{2}$$

$$\Rightarrow \frac{1}{2}\left(\frac{dz}{dt}\right)^2 - \frac{1}{\sqrt{z^2+b}} - \frac{3A}{\left(z^2+b\right)^{3/2}} - \frac{2Az^3}{\left(z^2+b\right)^{3/2}} - \frac{3Az^3}{\left(z^2+b\right)^{5/2}} = \frac{c}{2}$$

$$\Rightarrow \frac{1}{2}\left(\frac{dz}{dt}\right)^2 - \frac{1}{\sqrt{z^2+b}} - \frac{A}{\left(z^2+b\right)^{3/2}}\left[3+2z^3+\frac{3z^3}{\left(z^2+b\right)}\right] = \frac{c}{2}$$

$$\Rightarrow \frac{1}{2}\left(\frac{dz}{dt}\right)^2 + v(z) = \frac{c}{2} \qquad (11)$$

Where potential energy $v(z)$ can be written as,

$$v(z) = -\left[\frac{1}{\left(z^2+b\right)^{1/2}} + \frac{A}{\left(z^2+b\right)^{3/2}}\left(3+2z^3+\frac{3z^3}{\left(z^2+b\right)}\right)\right] = \frac{c}{2}$$

This is an expression for the potential energy of the system when $b = 1+\frac{16A}{3}$. Thus the total energy of the system is given by

$$E = \frac{1}{2}\left(\frac{dz}{dt}\right)^2 - \left[\frac{1}{\left(z^2+1+\frac{16A}{3}\right)^{1/2}} + \frac{A}{\left(z^2+1+\frac{16A}{3}\right)^{3/2}}\left(3+2z^3+\frac{3z^3}{z^2+1+\frac{16A}{3}}\right)\right] = \frac{c}{2} \qquad (12)$$

= Constant

Which gives total energy of the system if primaries are oblate spheroids.

Equations of Motion and Energy

Now following Thapa (2015), we have the equation of motion of the third body of infinitesimal mass is

$$\ddot{z} = \frac{\partial\Omega}{\partial z} = \frac{-(1-p)z}{\left(z^2+\frac{a^2}{4}\right)^{\frac{3}{2}}} - \frac{3(2\sigma_1-\sigma_2+2\sigma_1'-\sigma_2')z}{4\left(z^2+\frac{a^2}{4}\right)^{\frac{5}{2}}} + \frac{15(\sigma_1+\sigma_1')z^3}{4\left(z^2+\frac{a^2}{4}\right)^{\frac{7}{2}}}$$

or, $$\frac{d^2z}{dt^2} = \frac{-(1-p)z}{\left(z^2+\frac{a^2}{4}\right)^{\frac{3}{2}}} - \frac{3\alpha z}{4\left(z^2+\frac{a^2}{4}\right)^{\frac{5}{2}}} + \frac{15}{4}\frac{(\sigma_1+\sigma_1')z^3}{\left(z^2+\frac{a^2}{4}\right)^{\frac{7}{2}}} \qquad (13)$$

when $\alpha = 2\sigma_1-\sigma_2+2\sigma_1'-\sigma_2'$.

If we multiply (13) by $2\frac{dz}{dt}$,

We get,

$$2\frac{dz}{dt}\frac{d^2z}{dt^2} = -\frac{2(1-p)z}{\left(z^2+\frac{a^2}{4}\right)^{\frac{3}{2}}}\frac{dz}{dt} - \frac{3\alpha}{4\left(z^2+\frac{a^2}{4}\right)^{\frac{5}{2}}}\times 2z\frac{dz}{dt} + \frac{15(\sigma_1+\sigma_1')z^3}{4\left(z^2+\frac{a^2}{4}\right)^{\frac{7}{2}}}\frac{2dz}{dt}$$

$$\text{or, } \frac{1}{2}\frac{d}{dt}\left(\frac{dz}{dt}\right)^2 = -\frac{(1-p)z}{\left(z^2+\frac{a^2}{4}\right)^{\frac{3}{2}}}\frac{dz}{dt} - \frac{3\alpha z}{4\left(z^2+\frac{a^2}{4}\right)^{\frac{5}{2}}}\times\frac{dz}{dt} + \frac{15(\sigma_1+\sigma_1')}{4}\frac{\left(z^2+\frac{a^2}{4}-\frac{a^2}{4}\right)z}{\left(z^2+\frac{a^2}{4}\right)^{\frac{7}{2}}}\frac{dz}{dt}$$

If we substitute $u^2 = z^2+\frac{a^2}{4}$ and $u\frac{du}{dt} = z\frac{dz}{dt}$ in the equation

We get,

$$\frac{1}{2}\int \frac{d}{dt}\left(\frac{dz}{dt}\right)^2 dt = (1-p)\int \frac{du^{-1}}{dt}dt + \frac{\alpha}{4}\int \frac{du^{-3}}{dt}dt - \frac{5(\sigma_1+\sigma_1')}{4}\int \frac{du^{-3}}{dt}dt + \frac{3(\sigma_1+\sigma_1')a^2}{16}\int \frac{du^{-5}}{dt}dt$$

$$\Rightarrow \frac{1}{2}\left(\frac{dz}{dt}\right)^2 = (1-p)u^{-1} + \frac{\alpha}{4}u^{-3} - \frac{5}{4}(\sigma_1+\sigma_1')u^{-3} + \frac{3(\sigma_1+\sigma_1')a^2}{16}u^{-5} + c$$

$$\Rightarrow \frac{1}{2}\left(\frac{dz}{dt}\right)^2 = \frac{(1-p)}{u} + \frac{\alpha}{4u^3} - \frac{5(\sigma_1+\sigma_1')}{4u^3} + \frac{3(\sigma_1+\sigma_1')a^2}{16}\frac{1}{u^5} + c$$

$$\Rightarrow \frac{1}{2}\left(\frac{dz}{dt}\right)^2 = \frac{(1-p)}{\sqrt{z^2+\frac{a^2}{4}}} + \frac{\alpha}{4\left(z^2+\frac{a^2}{4}\right)^{\frac{3}{2}}} - \frac{5(\sigma_1+\sigma_1')}{4\left(z^2+\frac{a^2}{4}\right)^{\frac{3}{2}}} + \frac{3(\sigma_1+\sigma_1')a^2}{16\left(z^2+\frac{a^2}{4}\right)^{\frac{5}{2}}} + c$$

$$\Rightarrow \frac{1}{2}\left(\frac{dz}{dt}\right)^2 - \frac{(1-p)}{\sqrt{z^2+\frac{a^2}{4}}} - \frac{\alpha}{4\left(z^2+\frac{a^2}{4}\right)^{\frac{3}{2}}} + \frac{5(\sigma_1+\sigma_1')}{4\left(z^2+\frac{a^2}{4}\right)^{\frac{3}{2}}} - \frac{3(\sigma_1+\sigma_1')a^2}{16\left(z^2+\frac{a^2}{4}\right)^{\frac{5}{2}}} = c$$

$$\Rightarrow \frac{1}{2}\left(\frac{dz}{dt}\right)^2 - \frac{(1-p)}{\sqrt{z^2+\frac{a^2}{4}}} + \frac{\left(5\sigma_1+5\sigma_1'-2\sigma_1+\sigma_2-2\sigma_1'+\sigma_2'\right)}{4\left(z^2+\frac{a^2}{4}\right)^{\frac{3}{2}}} - \frac{3(\sigma_1+\sigma_1')a^2}{16\left(z^2+\frac{a^2}{4}\right)^{\frac{5}{2}}} = c$$

$$\Rightarrow \frac{1}{2}\left(\frac{dz}{dt}\right)^2 - \frac{(1-p)}{\sqrt{z^2+\frac{a^2}{4}}} + \frac{\left(3\sigma_1+\sigma_2+3\sigma_1'+\sigma_2'\right)}{4\left(z^2+\frac{a^2}{4}\right)^{\frac{3}{2}}} - \frac{3(\sigma_1+\sigma_1')a^2}{16\left(z^2+\frac{a^2}{4}\right)^{\frac{5}{2}}} = c$$

$$\Rightarrow \frac{1}{2}\left(\frac{dz}{dt}\right)^2 - \frac{(1-p)}{\left(z^2+\frac{a^2}{4}\right)^{\frac{1}{2}}} + \frac{p}{4\left(z^2+\frac{a^2}{4}\right)^{\frac{3}{2}}} - \frac{3(\sigma_1+\sigma_1')a^2}{16\left(z^2+\frac{a^2}{4}\right)^{\frac{5}{2}}} = \text{constant}$$

i.e.

$$E = \frac{1}{2}\left(\frac{dz}{dt}\right)^2 - \frac{(1-p)}{\sqrt{z^2+\frac{a^2}{4}}} + \frac{p}{4\left(z^2+\frac{a^2}{4}\right)^{\frac{3}{2}}} - \frac{3(\sigma_1+\sigma_1')}{16\left(z^2+\frac{a^2}{4}\right)^{\frac{5}{2}}} = \text{constant.} \qquad (14)$$

Which is the required energy equation of the third body if primaries are source of radiation and triaxial rigid bodies.

RESULT AND DISCUSSION

If primaries are source of radiation and triaxial rigid bodies then the energy of third body depends upon radiation parameter p; σ_1 , σ_1^1 and distance 'a' between the primaries where

σ_1 = a_1^2-a_3^2/ $5R^2$ and σ_1^1 = b_1^2-b_3^2/ $5R^2$.

a_1, a_3 and b_1, b_3 be the length of semi axes of primaries parallel to x and z axes respectively and R be the dimensional distance between them but if primaries are oblate spheroids then it depends on oblateness parameter A= a^2-c^2/10, which is taken from the equation

$$-v = \frac{Gm_1m_2}{r} + \frac{GA(m_1+m_2)}{r^3}$$

a and c be length of semi-axes of primaries parallel to x and z axes respectively.

CONCLUSION

The energy of third body depending on main variable z, which is a constant quantity. It is a combination of both potential and kinetic energy. Due to the effect of the energy the third body can travel in its own path.

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