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Development of Polynomial Mode Shape Functions for Continuous Shafts with Different End Conditions

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Abstract:
Common methods used to determine the solutions for vibration response of continuous systems are assumed mode method, Rayleigh-Ritz method, Galerkin Method, finite element method, etc. Each of these methods requires the shape functions which satisfy the boundary conditions. Shape functions derived in most of the classical textbooks are simple trigonometric functions for some end conditions but are very complex transcendental functions for many end conditions. It is very difficult to determine the vibration response of a continuous system analytically by using such transcendental shape functions. Hence this paper presents a method to develop polynomial shape functions required to solve the vibration of continuous shafts with different end conditions. The natural frequencies obtained from the developed polynomial shape functions are compared to those obtained from the classical transcendental shape functions and found very close for the first three modes.

Keywords: Continuous shaft, Polynomial Shape Function, Transcendental Shape Function, Natural Frequencies

1. Introduction

Vibration analysis of any dynamic system begins with the mathematical modelling of the physical system under consideration. To develop the mathematical model of the system, its governing equation of motion is derived. In case of a continuous system, the governing equation will be in the form of a partial differential equation or a system of partial differential equations. To get the vibration response of any continuous system, this partial differential equation or a system of partial differential equations should be solved by using the associated boundary conditions for the natural frequencies and the corresponding mode shapes.

When the governing equation of the system is in the standard form of the partial differential equation (wave equation, Laplace equation, etc.), we can get the closed form solutions by using classical mathematical procedures such as separation of variables, D'Alembert's solution, etc. But when the governing equation is not in the standard form of the partial differential equation, we should use an appropriate approximate method or a numerical method. Most common approximates methods are assumed mode method, Rayleigh-Ritz method, Galerkin Method, finite element method, etc. To use these methods, we should start with the assumption of mode shapes which satisfies the boundary conditions of the given problem.

Many researchers have analyzed such problems by taking different types of functions for the assumed mode shapes. These assumed mode shapes are simple trigonometric functions in some cases but are also very complex functions in many cases.

Different aspects of dynamic behavior of continuous shafts have been studied by many investigators [1-14] by using trigonomteric shape functions by modelling the shaft as a simply supported shaft. Zhu and Chung [15] has studied nonlinear lateral vibrations of spinning shaft by using trancendental shape function by the shaft as a cantilever end conditions. Similalry, Luintel [16] has compared the critical frequecies of a simply supported shaft and a shaft fixed at both ends by using polynomial shape functions.

Using the assumed mode method with the transcendental functions for the mode shapes is relatively complex. Hence this paper presents a method to develop the polynomial functions for the mode shapes which can be used in a relatively easier way to analyze the vibration of a continuous shafts with different end conditions. To validate the developed polynomial functions, the natural frequencies obtained for the different modes are also compared with those obtained from the classical transcendental mode shape functions.

2. Mathematical Model for a Continuous Shaft Rotating at a Constant Speed

2.1 Equation of Motion

Consider a flexible shaft of length L as shown in Figure 1. The axes x, y and z are chosen such that x is along longitudinal direction of the shaft, y is along transverse direction of shaft on the horizontal plane and z is along the transverse direction of the shaft on the vertical plane. The shaft is rotating about x axis at a constant speed of Ω. Similarly, transverse displacements of any point of the shaft along horizontal and vertical directions are respectively v(x,t) and w(x,t).

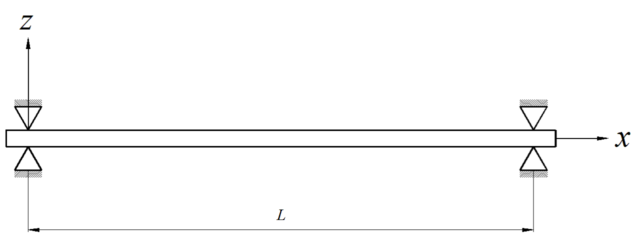


Figure 1: Flexible shaft supported by bearings

Then equations of motion of for transvers vibrations along y and z are given by [16] as:

$$\rho A\ddot{v} - \rho I_s\ddot{v}'' - 2\rho A\Omega\dot{w} + \rho J_{ps}\Omega\dot{w}'' + 2\rho I_s\Omega\dot{w}'' - \rho A\Omega^2 v + \rho I_s\Omega^2 v'' + EI_s v^{iv} = 0 \tag{1}$$

$$\rho A\ddot{w} - \rho I_s\ddot{w}'' + 2\rho A\Omega\dot{v} - \rho J_{ps}\Omega\dot{v}'' - 2\rho I_s\Omega\dot{v}'' - \rho A\Omega^2 w + \rho I_s\Omega^2 w'' + EI_s w^{iv} = 0 \tag{2}$$

where ρ is the density of the shaft material, I_s is the second moment of area of the shaft section, A is the cross sectional area of the shaft and J_{ps} is the polar moment of inertia of the shaft section.

2.2 Boundary Conditions

Boundary conditions associate with the continuous shaft systems for different end conditions are given below:

(a) Simply supported shaft

$$v(0,t) = 0;\ v''(0,t) = 0;\ v(L,t) = 0;\ v''(L,t) = 0 \tag{3}$$

$$w(0,t) = 0;\ w''(0,t) = 0;\ w(L,t) = 0;\ w''(L,t) = 0 \tag{4}$$

(b) Shaft fixed at both ends

$$v(0,t) = 0;\ v'(0,t) = 0;\ v(L,t) = 0;\ v'(L,t) = 0 \tag{5}$$

$$w(0,t) = 0;\ w'(0,t) = 0;\ w(L,t) = 0;\ w'(L,t) = 0 \tag{6}$$

(c) Shaft fixed at one end and free at the other end

$$v(0,t) = 0;\ v'(0,t) = 0;\ v''(L,t) = 0;\ v'''(L,t) = 0 \tag{7}$$

$$w(0,t) = 0;\ w'(0,t) = 0;\ w''(L,t) = 0;\ w'''(L,t) = 0 \tag{8}$$

2.3 Frequency Equations

Frequency equations for the continuous shaft systems for different end conditions [17] are given below:

(a) Simply supported shaft

$$\sin(\beta_i L) = 0 \tag{9}$$

(b)Shaft fixed at both ends

$$\cos(\beta_i L)\cosh(\beta_i L) = 1 \tag{10}$$

(c)Shaft fixed at one end and free at the other end

$$\cos(\beta_i L)\cosh(\beta_i L) = -1 \tag{11}$$

2.4 Trigonometric and Transcendental Mode Shape Functions

Using the boundary conditions mentioned above, mode shape functions derived from classical methods [17] for each end conditions mentioned above are given below:
(a) Simply supported shaft

$$\phi_i(x) = \sin(\beta_i x) \tag{12}$$

(b) Shaft fixed at both ends

$$\phi_i(x) = \Bigg[\{\sinh(\beta_i x) - \sin(\beta_i x)\} + \frac{\sinh(\beta_i L) - \sin(\beta_i L)}{\cos(\beta_i L) - \cosh(\beta_i L)}\{\cos(\beta_i x) - \cosh(\beta_i x)\}\Bigg] \quad (13)$$

(c) Shaft fixed at one end and free at the other end

$$\phi_i(x) = \Bigg[\{\sin(\beta_i x) - \sinh(\beta_i x)\} + \frac{\sin(\beta_i L) - \sinh(\beta_i L)}{\cos(\beta_i L) - \cosh(\beta_i L)}\{\cos(\beta_i x) - \cosh(\beta_i x)\}\Bigg] \quad (14)$$

It can be noticed from Equations (9) and (12) that the frequency equation and mode shape functions for a simply supported shaft are simple trigonometric functions but in case of shaft fixed at both ends and cantilevered shaft, both the frequency equation and mode shape functions for a simply supported shaft are complex transcendental functions as seen in Equations (10) & (13) and (11) & (14) respectively. Appropriate numerical method can be used to determine the approximate solutions of these transcendental frequency equations. Hence this paper presents a method to develop polynomial mode shape functions which can be used to perform vibration analysis of continuous shafts through semi-analytical approach.

3. Development of Polynomial Shape Functions

Equations of motion given by Equations (1) and (2) can be solved by using the Galerkin method or the assumed mode method. For this, the displacement variables $v(x,t)$ and $w(x,t)$ can be assumed as:

$$v = \{\phi(x)\}^T\{V(t)\} = \{\phi\}^T\{V\} \quad (15)$$

$$w = \{\phi(x)\}^T\{W(t)\} = \{\phi\}^T\{W\} \quad (16)$$

where $\{\phi(x)\}$ is the vector of mode shape functions. Each elements of $\{\phi(x)\}$ should satisfy the boundary conditions associated with the end conditions of the shaft and they should be orthogonal to each other.

Since the highest order of derivative in the governing equation is four, the assumed polynomial mode shape function should have the order equal to or greater than 4. Hence for the first three modes, the mode shape functions can be assumed as:

$$\phi_1 = x^4 + A_3x^3 + A_2x^2 + A_1x + A_0 \quad (17)$$

$$\phi_2 = x^5 + B_4x^4 + B_3x^3 + B_2x^2 + B_1x + B_0 \quad (18)$$

$$\phi_3 = x^6 + C_5x^5 + C_4x^4 + C_3x^3 + C_2x^2 + C_1x + C_0 \quad (19)$$

The constants A_i, B_i and C_i are determined by using the boundary conditions and orthogonal relationships of the mode shape functions, i.e.,

$$\int_0^L \phi_i\phi_j\, dx = 0 \quad (20)$$

3.1 Mode Shape Functions for a Shaft with Simply Supported Conditions at both Ends

Using boundary conditions given by Equation (3) or (4), we get boundary conditions for each mode shape functions as:

$$\phi_i(0) = 0;\ \phi_i''(0) = 0;\ \phi_i(L) = 0;\ \phi_i''(L) = 0 \quad (21)$$

Using boundary condition defined by Equation (21) for ϕ_1, ϕ_2 and ϕ_3, twelve equations are obtained and using orthogonal conditions defined by Equation (20) for ϕ_1 & ϕ_2, ϕ_2 & ϕ_3 and ϕ_3 & ϕ_1, additional five equations are obtained. Then solving these simultaneous equations, seventeen coefficients (A_i, B_i and C_i) can be determined to get the polynomial mode shape functions.

The coefficients of polynomial mode shape functions for a simply supported shaft are determined as:

$$A_0 = 0;\ A_1 = L^3;\ A_2 = 0;\ A_3 = -2L \quad (22)$$

$$B_0 = 0;\quad B_1 = -\frac{1}{6}L^4;\quad B_2 = 0;\quad B_3 = \frac{5}{3}L^2;\quad B_4 = -\frac{5}{2}L \quad (23)$$

$$C_0 = 0;\quad C_1 = \frac{27}{682}L^5;\quad C_2 = 0;\quad C_3 = -\frac{368}{341}L^3;\quad C_4 = \frac{2073}{682}L^2;\quad C_5 = -3L \quad (24)$$

Substituting these coefficients into Equations (17), (18) and (19), the expressions for the first three mode shape functions for a shaft with simply supported condition at both ends are obtained as:

$$\phi_1 = x^4 - 2Lx^3 + L^3x \tag{25}$$

$$\phi_2 = x^5 - \frac{5}{2}Lx^4 + \frac{5}{3}L^2x^3 - \frac{1}{6}L^4x \tag{26}$$

$$\phi_3 = x^6 - 3Lx^5 + \frac{2073}{682}L^2x^4 - \frac{368}{341}L^3x^3 + \frac{27}{682}L^5x \tag{27}$$

3.2 Mode Shape Functions for a Shaft Fixed at both Ends

Using boundary conditions given by Equation (3) or (4), the boundary conditions for each mode shape functions are obtained as:

$$\phi_i(0) = 0;\ \phi_i'(0) = 0;\ \phi_i(L) = 0;\ \phi_i'(L) = 0 \tag{28}$$

Following the similar procedure, the coefficients of polynomial mode shape functions for a shaft fixed at both ends are determined as:

$$A_0 = 0;\ A_1 = 0;\ A_2 = L^2;\ A_3 = -2L \tag{29}$$

$$\begin{aligned} &B_0 = 0;\ B_1 = 0;\ B_2 = -\frac{1}{6}L^3; \\ &B_3 = 2L^2;\ B_4 = -\frac{5}{2}L \end{aligned} \tag{30}$$

$$\begin{aligned} &C_0 = 0; \quad C_1 = 0; \quad C_2 = \frac{5}{22}L^4; \\ &C_3 = -\frac{16}{11}L^3; \quad C_4 = \frac{71}{22}L^2; \quad C_5 = -3L \end{aligned} \tag{31}$$

Substituting these coefficients into Equations (17), (18) and (19), the expressions for the first three mode shape functions a shaft fixed at both ends are obtained as:

$$\phi_1 = x^4 - 2Lx^3 + L^2x \tag{32}$$

$$\phi_2 = x^5 - \frac{5}{2}Lx^4 + 2L^2x^3 - \frac{1}{6}L^3x \tag{33}$$

$$\phi_3 = x^6 - 3Lx^5 + \frac{71}{22}L^2x^4 - \frac{16}{11}L^3x^3 + \frac{5}{22}L^4x \tag{34}$$

3.3 Mode Shape Functions for a Shaft Fixed at one End and Free at other End

Using boundary conditions given by Equation (3) or (4), the boundary conditions for each mode shape functions are obtained as:

$$\phi_i(0) = 0;\ \phi_i'(0) = 0;\ \phi_i''(L) = 0;\ \phi_i'''(L) = 0 \tag{35}$$

Following the similar procedure, the coefficients of polynomial mode shape functions for a shaft fixed at one end and free at the other end are determined as:

$$A_0 = 0;\ A_1 = 0;\ A_2 = 6L^2;\ A_3 = -4L \tag{36}$$

$$\begin{aligned} &B_0 = 0;\ B_1 = 0;\ B_2 = -\frac{163}{91}L^3; \\ &B_3 = \frac{412}{91}L^2;\ B_4 = -661182L \end{aligned} \tag{37}$$

$$\begin{aligned} &C_0 = 0;\ C_1 = 0;\ C_2 = \frac{115}{176}L^4; \\ &C_3 = -\frac{5560}{1793}L^3;\ C_4 = \frac{305815}{57376}L^2;\ C_5 = -\frac{9953}{2608}L \end{aligned} \tag{38}$$

Substituting these coefficients into Equations (17), (18) and (19), the expressions for the first three mode shape functions a shaft fixed at one end and free at the other end are obtained as:

$$\phi_1 = x^4 - 4Lx^3 + 6L^2x \tag{39}$$

$$\phi_2 = x^5 - \frac{661}{182}Lx^4 + \frac{412}{91}L^2x^3 - \frac{163}{91}L^3x \tag{40}$$

$$\phi_3 = x^6 - \frac{9953}{2608}Lx^5 + \frac{305815}{57376}L^2x^4 - \frac{5560}{1793}L^3x^3 + \frac{115}{176}L^4x \tag{41}$$

4. Equivalent System Parameters

Substituting Eqs. (15) and (16) into Eqs. (1) and (2) and applying orthogonality principle, ordinary differential equations of motion for i_{th} mode for $V_i(t)$ and $W_i(t)$ are obtained as:

$$M_i\ddot{V}_i(t) - C_i\dot{w}_i(t) + K_iV_i(t) = 0 \tag{42}$$

$$M_i\ddot{w}_i(t)+C_i\dot{v}_i(t)+K_iW_i(t)=0 \quad (43)$$

where M_i, C_i and K_i are respectively modal mass, modal damping and modal stiffness, are given by

$$M_i=\int_0^L \rho A\phi_i(x)\phi_i(x)dx-\int_0^L \rho I_s\phi_i''(x)\phi_i(x)dx \quad (44)$$

$$C_i=\int_0^L 2\rho A\Omega\phi_i(x)\phi_i(x)dx-\int_0^L \rho J_{ps}\Omega\phi_i''(x)\phi_i(x)dx \\ -\int_0^L 2\rho I_s\Omega\phi_i''(x)\phi_i(x)dx \quad (45)$$

$$K_i=-\int_0^L \rho A\Omega^2\phi_i(x)\phi_i(x)dx+\int_0^L \rho I_s\Omega^2\phi_i''(x)\phi_i(x)dx \\ +\int_0^L EI_s\phi_i^{iv}(x)\phi_i(x)dx \quad (46)$$

Using these equivalent parameters, the natural frequencies corresponding to backward whirl and forward whirl are respectively given by [16] as:

$$(\lambda_i)_1=\sqrt{\frac{1}{2}\left[\left\{\left(\frac{C_i}{M_i}\right)^2+2\frac{K_i}{M_i}\right\}-\sqrt{\left(\frac{C_i}{M_i}\right)^4+4\left(\frac{C_i}{M_i}\right)^2\frac{K_i}{M_i}}\right]} \quad (47)$$

$$(\lambda_i)_2=\sqrt{\frac{1}{2}\left[\left\{\left(\frac{C_i}{M_i}\right)^2+2\frac{K_i}{M_i}\right\}+\sqrt{\left(\frac{C_i}{M_i}\right)^4+4\left(\frac{C_i}{M_i}\right)^2\frac{K_i}{M_i}}\right]} \quad (48)$$

4.1 Equivalent Parameters for a Shaft with Simply Supported Conditions at both Ends

Substituting polynomial shape functions for a shaft with simply supported conditions at both ends given by Equations (25) to (27) into Equations (44) to (46), modal parameters for the first three modes are obtained as:

$$M_1=\frac{1}{630}(31AL^2+306I_s)\rho L^7 \quad (49)$$

$$C_1=\frac{1}{315}(31AL^2+306I_s+153J_{ps})\rho L^7\Omega \quad (50)$$

$$K_1=\frac{1}{630}(3024EI_s-31AL^4\Omega^2\rho-306I_sL^2\Omega^2\rho)L^5 \quad (51)$$

$$M_2=\frac{1}{16632}(5AL^2+198I_s)\rho L^9 \quad (52)$$

$$C_2=\frac{1}{8316}(5AL^2+198I_s+99J_{ps})\rho L^9\Omega \quad (53)$$

$$K_2=\frac{1}{16632}(7920EI_s-5AL^4\Omega^2\rho-198I_sL^2\Omega^2\rho)L^7 \quad (54)$$

$$M_3=\frac{1}{1269788520}(7781AL^2+700674I_s)\rho L^11 \quad (55)$$

$$C_3=\frac{1}{1269788520}(15562AL^2+1401348I_s \\ +700674J_{ps})\rho L^11\Omega \quad (56)$$

$$K_3=\frac{1}{1269788520}(64876656EI_s-7781AL^4\Omega^2\rho \\ -700674I_sL^2\Omega^2\rho)L^9 \quad (57)$$

4.2 Equivalent Parameters for a Shaft Fixed at both Ends

Substituting polynomial shape functions for a shaft with simply supported conditions at both ends given by Equations (32) to (34) into Equations (44) to (46), modal parameters for the first three modes are obtained as:

$$M_1=\frac{1}{630}\left(AL^2+12I_s\right)\rho L^7 \quad (58)$$

$$C_1=\frac{1}{315}(AL^2+12I_s+6J_{ps})\rho L^7\Omega \quad (59)$$

$$K_1 = \frac{1}{630}(504EI_s - AL^4\Omega^2\rho - 12I_sL^2\Omega^2\rho)L^5 \quad (60)$$

$$M_2 = \frac{1}{27720}(AL^2 + 44I_s)\rho L^9 \quad (61)$$

$$C_2 = \frac{1}{13860}(AL^2 + 44I_s + 22J_{ps})\rho L^9\Omega \quad (62)$$

$$K_2 = \frac{1}{27720}(3960EI_s - AL^4\Omega^2\rho - 44I_sL^2\Omega^2\rho)L^7 \quad (63)$$

$$M_3 = \frac{1}{3963960}(5AL^2 + 468I_s)\rho L^1 1 \quad (64)$$

$$C_3 = \frac{1}{3963960}(10AL^2 + 936I_s + 468J_{ps})\rho L^1 1\Omega \quad (65)$$

$$K_3 = \frac{1}{3963960}(81432EI_s - 5AL^4\Omega^2\rho - 468I_sL^2\Omega^2\rho)L^9 \quad (66)$$

4.3 Equivalent Parameters for a Shaft Fixed at one End and Free at other End

Substituting polynomial shape functions for a shaft with simply supported conditions at both ends given by Equations (39) to (41) into Equations (44) to (46), modal parameters for the first three modes are obtained as:

$$M_1 = \frac{1}{315}(728AL^2 - 540I_s)\rho L^7 \quad (67)$$

$$C_1 = \frac{1}{315}(1456AL^2 - 1080I_s - 540J_{ps})\rho L^7\Omega \quad (68)$$

$$K_1 = \frac{1}{315}(9072EI_s - 728AL^4\Omega^2\rho + 540I_sL^2\Omega^2\rho)L^5 \quad (69)$$

$$M_2 = \frac{1}{28693665}(88998AL^2 + 1191685I_s)\rho L^9 \quad (70)$$

$$C_2 = \frac{1}{28693665}(177996AL^2 + 2383370I_s + 1191685J_{ps})\rho L^9\Omega \quad (71)$$

$$K_2 = \frac{1}{28693665}(45901548EI_s - 88998AL^4\Omega^2\rho - 1191685I_sL^2\Omega^2\rho)L^7 \quad (72)$$

$$M_3 = \frac{1}{674038100736}(20806950AL^2 + 943633795I_s)\rho L^1 1 \quad (73)$$

$$C_3 = \frac{1}{674038100736}(41613900AL^2 + 1887267590I_s + 943633795J_{ps})\rho L^1 1\Omega \quad (74)$$

$$K_3 = \frac{1}{674038100736}(90709550100EI_s - 20806950AL^4\Omega^2\rho - 943633795I_sL^2\Omega^2\rho)L^9 \quad (75)$$

5. Numerical Results and Discussion

To have comparison of polynomial shape functions and the resulting critical frequencies with those for the classical transcendental shape functions for the shafts with different end conditions, different material and geometric properties of the shaft are taken as: shown in Table 1.

Table 1: Parameters of the System

Parameters	Value
Density of shaft material, ρ	7860 kg/m^3
Cross-sectional area of the shaft, A	0.8042×10^{-3} m^2
Length of the shaft, L	0.52 m
Modulus of Elasticity of shaft material, E	202×109 GPa
Area moment of inertia of the shaft section, I_s	5.1472×10^{-8} m^4
Polar moment of area of the shaft section, J_{ps}	1.0294×10^{-7} m^4

5.1 Results for a Shaft with Simply Supported Conditions at both Ends

Substituting the system parameters into Equation (9), parameter β_i for the first three modes are determined and the corresponding trimetric functions for the mode shape aree determined by using Equation (12). Substituting system parameters, polynomial shape functions for the first three modes are directly determined from Equations (25), (26) and (27). The plots of these mode shapes are shown in Figure 2.

Similarly using Equations (44), (45) and (46), equivalent mass, damping constant and stiffness of the system are determined separately for trigonometric and polynomial shape functions are determined. Then using these equivalent mass, damping constant and stiffness into Equations (47) and (48), Campbell diagram is plotted for both type of shape functions as shown in Figure 3.

5.2 Results for a Shaft Fixed at both Ends

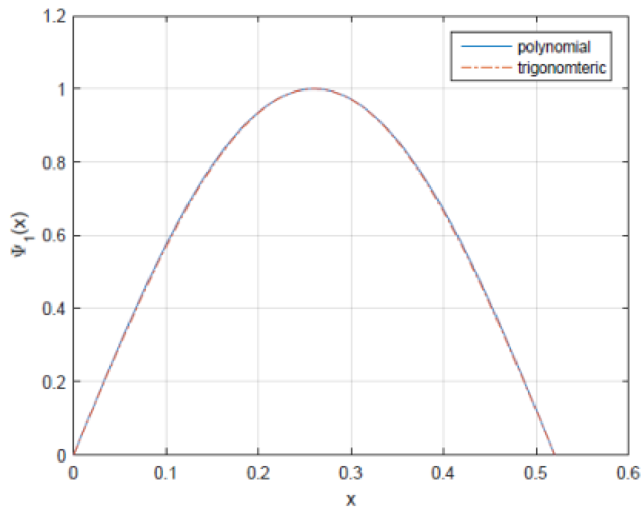
Substituting the system parameters into Equation (10), parameter β_i for the first three modes are determined and the corresponding trimetric functions for the mode shape are determined by using Equation (13). Substituting system parameters, polynomial shape functions for the first three modes can be directly determine from Equations (32), (33) and (34). The plots of these mode shapes are shown in Figure 4.

Similarly going through the same procedure as used for the simply supported end conditions, Campbell diagram is obtained for both type of shape functions as shown in Figure 5.

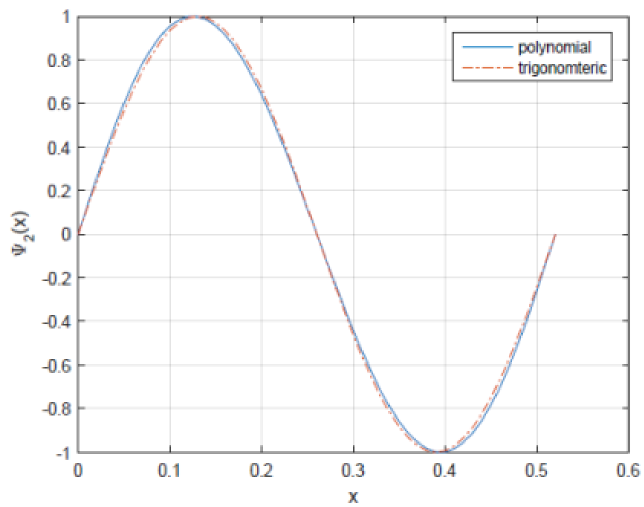
5.3 Results for a Shaft Fixed at one End and Free at other End

Substituting the system parameters into Equation (11), parameter β_i for the first three modes are determined and the corresponding trigonometric functions for the mode shape are determined by using Equation (14). Substituting system parameters, polynomial shape functions for the first three modes are directly determine from Equations (39), (40) and (41). The plots of these mode shapes are shown in Figure 6.

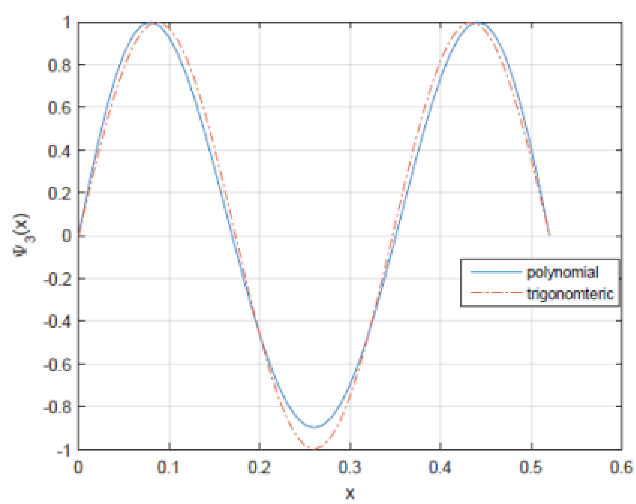
Similarly going through the same procedure as used for the simply supported end conditions, Campbell diagram is obtained for both type of shape functions as shown in Figure 7.



(a) First Mode

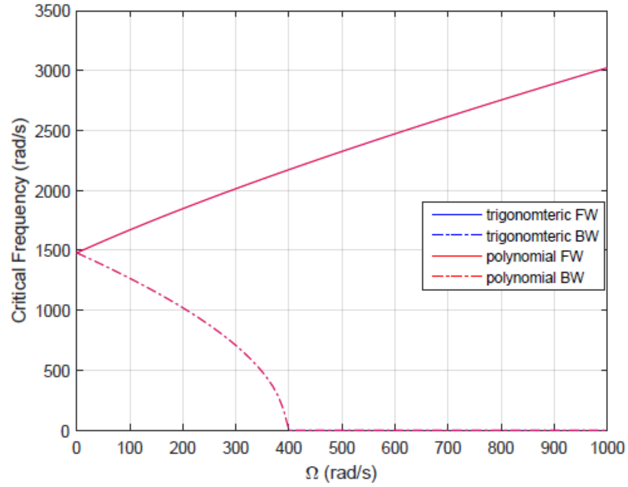


(b) Second Mode

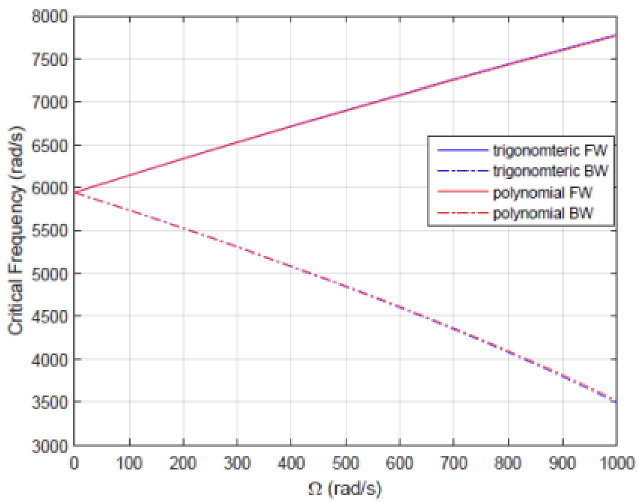


(c) Third Mode

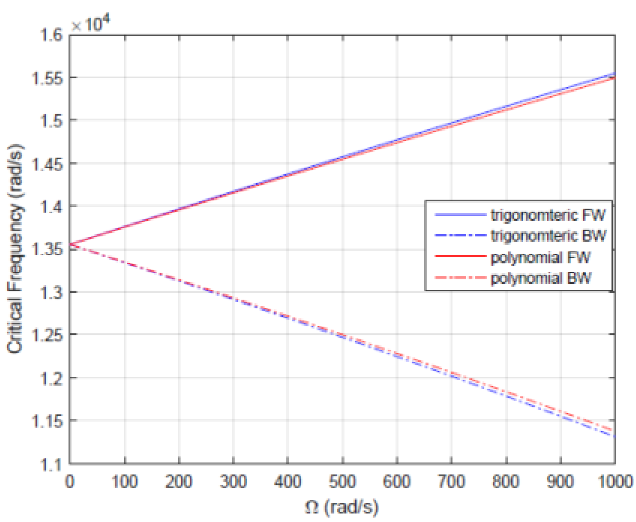
Figure 2: Mode Shapes for a Shaft with Simply Supported Conditions at both Ends



(a) First Mode

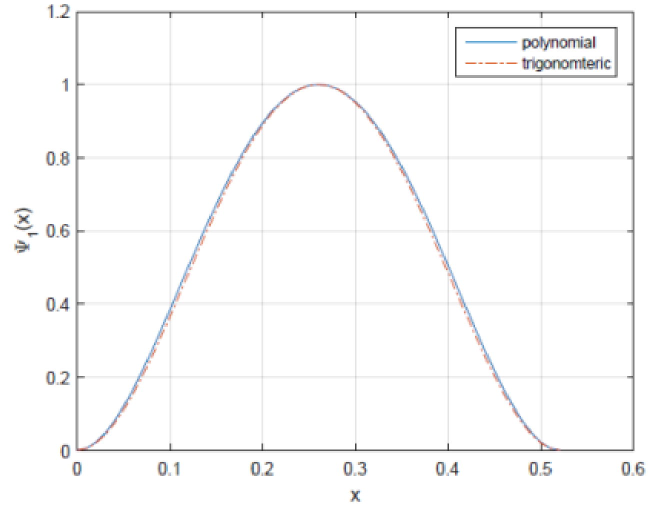


(b) Second Mode

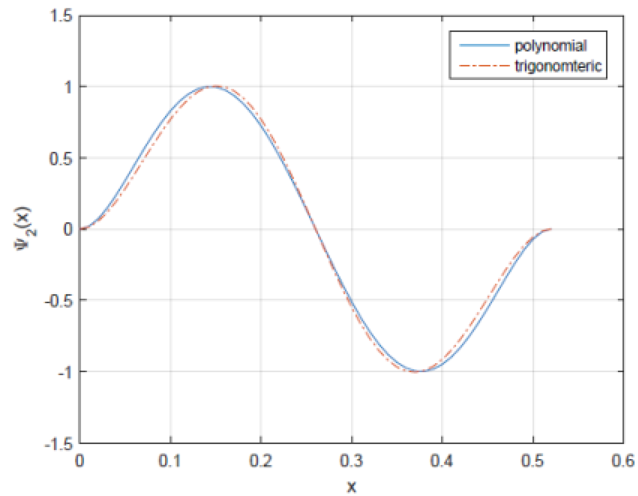


(c) Third Mode

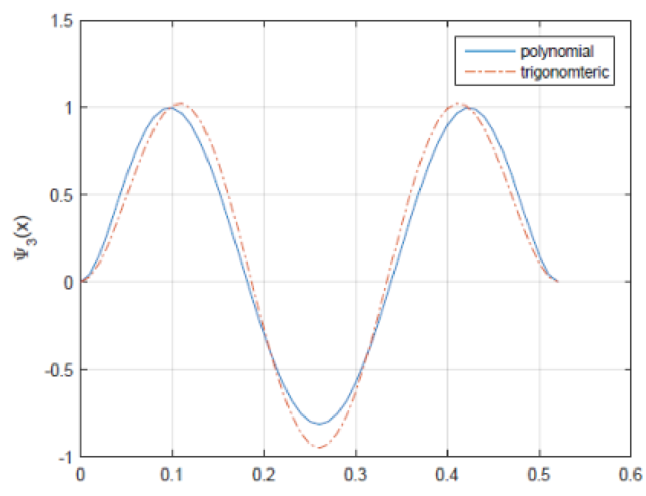
Figure 3: Campbell Diagram for a Shaft with Simply Supported Conditions at both Ends



(a) First Mode

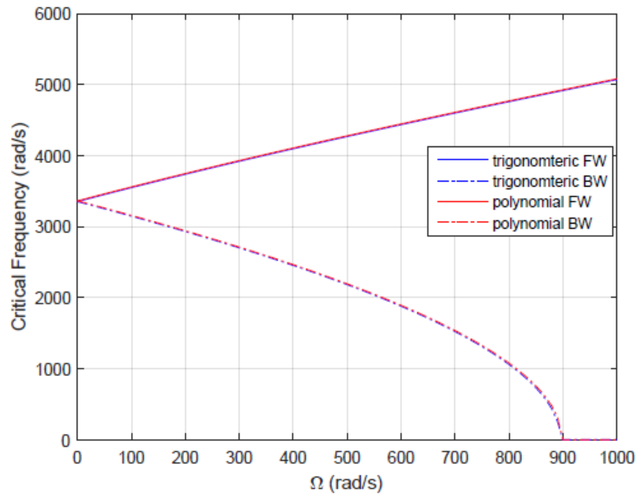


(b) Second Mode

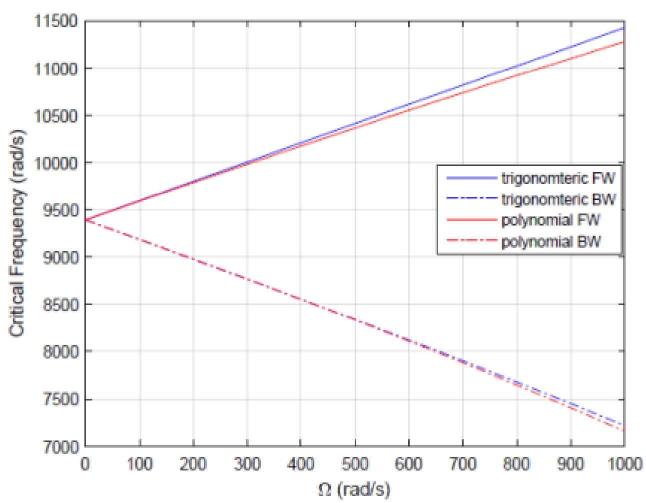


(c) Third Mode

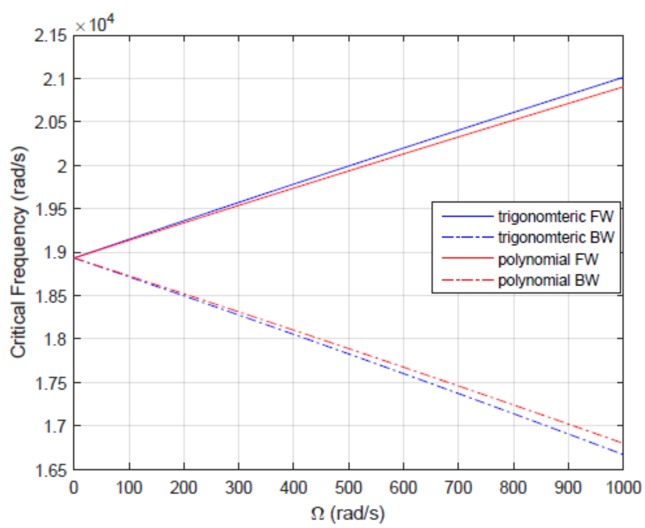
Figure 4: Mode Shapes for a Shaft Fixed at both Ends



(a) First Mode

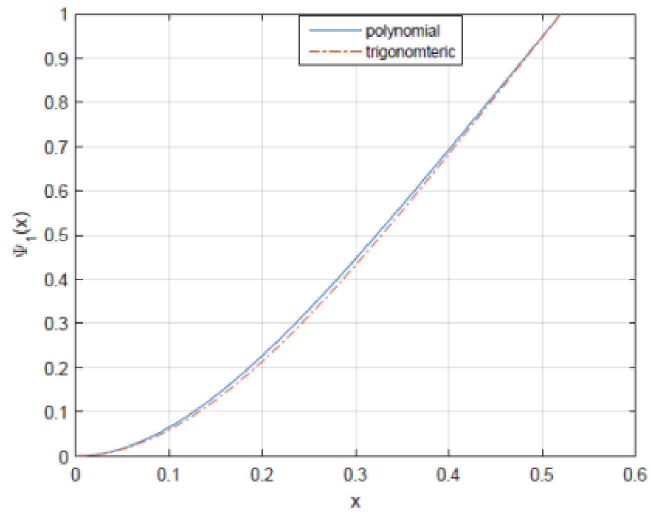


(b) Second Mode

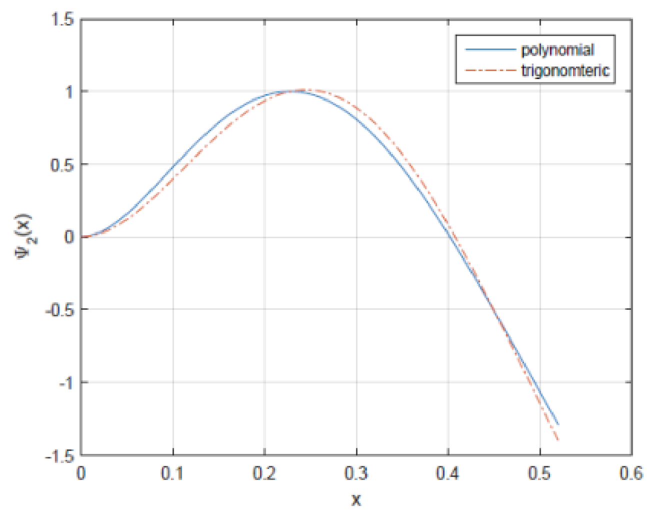


(c) Third Mode

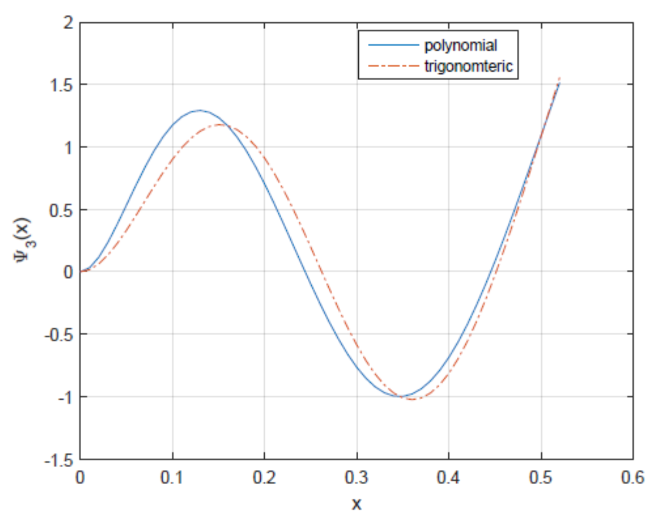
Figure 5: Campbell Diagram for a Shaft Fixed at both Ends



(a) First Mode

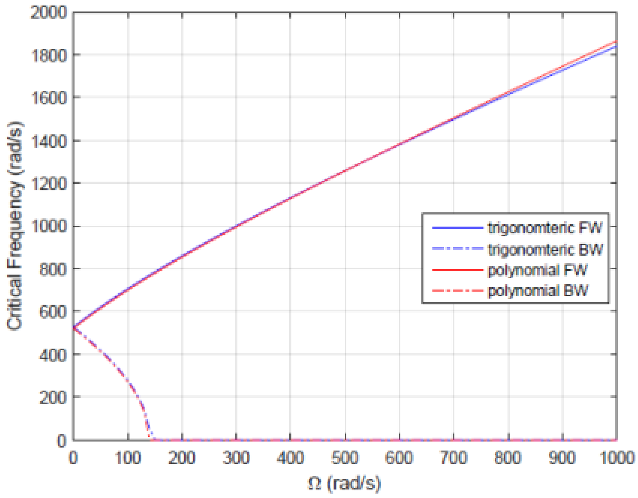


(b) Second Mode

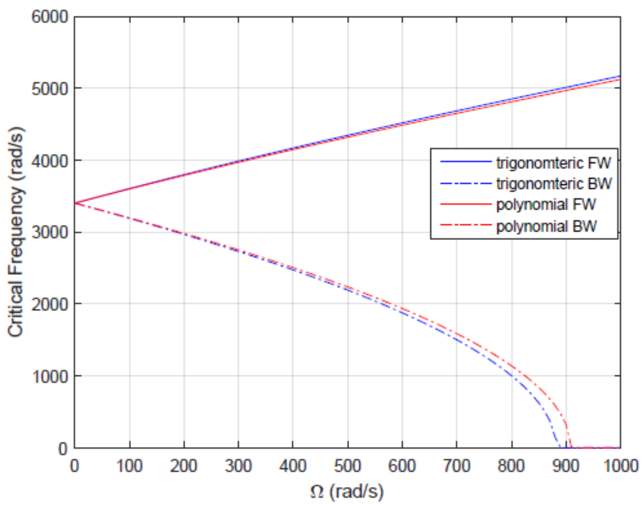


(c) Third Mode

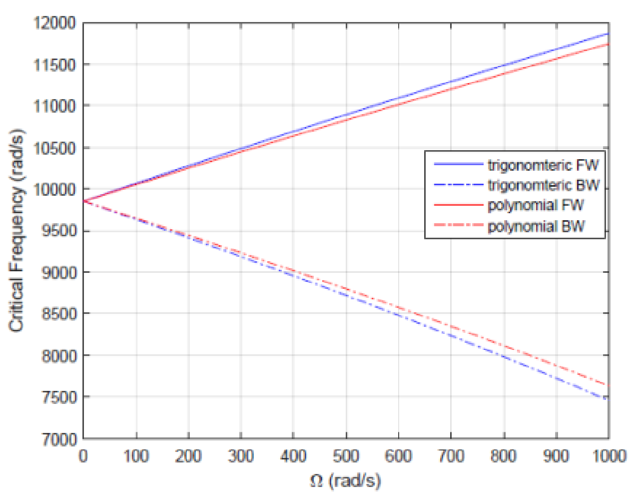
Figure 6: Mode Shapes for a Shaft Fixed at one End and Free at other End



(a) First Mode



(b) Second Mode



(c) Third Mode

Figure 7: Campbell Diagram for a Shaft Fixed at one End and Free at other End

It can be noted from Figures 2, 4 and 6 that the polynomial shape functions obtained from the proposed method match closely with the classical trigonometric and transcendental shape functions. The difference between theses shape functions are negligible for lower modes and it increases for the higher modes.

Similarly, the critical frequency of the system determined by using either of the shape function is same for the stationary shaft. The difference between critical frequencies for any mode is less for lower speed and this difference increases with the increase in speed of the shaft as shown in Figures 3, 5 and 7. Similarly, the difference between critical frequencies for any given speed is less for lower mode and this difference increases for the higher modes.

6. Conclusion

In this paper a method to develop polynomial shape functions, required to carry out vibration analysis, is presented. To validate the developed polynomial shapes functions, they are compared graphically with the classical transcendental shape function and the resulting critical frequencies are also compared with those obtained from the classical transcendental shape functions for the first three modes. The compassion shows that critical frequencies are convincingly close at lower speed. Hence the proposed method can be used to get dynamic behavior of the system running at low and moderate speeds.

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