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Study of the Expansion of Cylindrical Elastic Tube with Uneven Opening under Constant Internal Pressure

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Abstract: Our work is development of simplified model of elastic cylindrical tube by making several assumptions. We approximate tube walls to be thin but we do not neglect the stress exerted by the wall on the fluid inside. We also study expansion of such tubes under internal pressure and develop balance and kinematic equations of the model and then study kinematic properties of its deformation. We then modify Reynold's transport theorem and test its agreement with obtained results from the model. After that we perform further analysis for some of the results obtained.

Keywords: Modeling, Elasticity, Kinematics, Deformation, Reynold's transport theorem.

1. Introduction

Expansion of an elastic tube under internal pressure is a phenomenon which occurs due to deformation of solid walls under influence of some properties of fluid. Such entire class of problems fall under the study of fluid-structure interaction. Fluid-structure interaction (FSI) is a multiphysics coupling between the laws that describe fluid dynamics and structural mechanics. This phenomenon is characterized by interactions which can be stable or oscillatory between a deformable or moving structure and a surrounding or internal fluid flow [3]. When a fluid flow encounters a structure, stresses and strains are exerted on the solid object forces that can lead to deformations. We limit our study of FSI problems to the mathematical modeling of pressurized flows in pipes. Study of this kind is very important in understanding the nature of deformation of materials under external force. Study of FSI of pressurized flow in pipes seems to be very limited in terms of numbers of different aspects of FSI problems that can be studied but it has wide range of engineering and bio-medical applications along with increasing of our understanding of one of the most ubiquitous phenomenon of nature. It can be used to simulate the pressurized flows in supply pipes in hydroelectric installations, blood flow in artherosclerotic arteries and so on. If accurate mathematical and numerical models for such flow can be produced, then it will have significant impact on human life.

2. Theoretical Background

Reynold's Transport Theorem: Reynold's tranport theorem is 3 dimensional generalization of Leibniz integral rule. RTT is used to convert system analysis to control volume analysis and it can be applied to all conservation laws in classical mechanics. The formula of RTT differs on whether the control volume is fixed, moving or deformable.

2.1 For Fixed Control Volume

Let B be any property of the fluid (mass, energy, momentum, enthalpy etc.) and let $\beta = \frac{dB}{dm}$ be B per unit mass in any small element of the control volume of the fluid. Then, RTT can be written as, $\frac{\mathrm{d}B_{sys}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\int_{CV} \beta\rho dV\right) + \int_{CS} \beta\rho u cos\theta dA_{out} - \int_{CS} \beta\rho u cos\theta dA_{in}$

where u is the velocity of fluid flow across CS making angle θ with the surface.

2.2 For Control Volume Moving at Constant Velocity

If a control volume is moving with fixed velocity u_s and an observer fixed at CV will see fluid moving through CV at a relative velocity u_r such that $u_r = u - u_s$

where u is the fluid velocity measured at the same coordinate system in which u_s is measured. Then, RTT can be written as $\frac{\mathrm{d}B_{sys}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\int_{CV} \beta\rho dV\right) + \int_{CS} \beta\rho(u_r.n)dA$

where **n** is the outward normal unit vector everywhere on the control surface.

This formula reduces to RTT for fixed CV when $u_s = 0$.

2.3 For Control Volume Moving at Variable Velocity

If non-deformable control volume moving with variable velocity $u_s(t)$, then the boundary relative velocity [4] $u_r = u_{(r,t)} - u_s(t)$. Then, RTT can be written as

$$\frac{\mathrm{d}B_{sys}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\int_{CV} \beta\rho dV\right) + \int_{CS} \beta\rho(u_r.n)dA$$

where **n** is the outward normal unit vector everywhere on the control surface.

2.3 For Arbitrarily Moving and Deforming Control Volume

The control surface has a deformation, so its velocity $u_s = u_s(r,t)$, so that the relative velocity $u_r = u(r,t) - u_s(r,t)$. Then, RTT can be written as

$$\frac{\mathrm{d}B_{sys}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\int_{CV} \beta\rho dV\right) + \int_{CS} \beta\rho(u_r.n)dA$$

where **n** is the outward normal unit vector everywhere on the control surface.

This is the most general case, which can be compared with the equivalent form for a fixed control volume as $$\frac{dB_{sys}}{dt} = \int_{CV} \frac{\partial(\beta\rho)}{\partial t} dV + \int_{CS} \beta\rho(u.n)dA$$

The moving and deforming control volume, contains only two complications [4]:

(1) The time derivative of the first integral on the right must be taken outside and

(2) The second integral involves the relative velocity u_r between the fluid system and the control surface.

3. Model Description

We model the expansion of cylindrical elastic tube of length L with uneven opening under constant internal pressure ($\bar{P}$). Eventhough the outlet opening of the tube Γ_{out} has diameter smaller than the diameter of the inlet opening Γ_{in}, we ignore any sort of tapering and bulging near the openings and assume that the main body of the tube is perfectly cylindrical. We also assume that the pressure difference between Ω and surrounding is $\bar{P}$. This implies that the velocity of efflux (u_{out}) remains constant. We assume the density (ρ) of fluid remains constant and we ignore all the effect of viscosity, gravity and other forces unless they are declared to be in use later on. We also ignore the pressure gradient that appears across two openings due to inflow and outflow of the fluid. Velocity by which the fluid flows inside Ω i.e. u_{in} must be increased continuously to maintain $\bar{P}$. We assume boundary wall (Γ_w) to be thin with thickness (τ) which is initially stress free and boundary material to be in-compressible. We restrict the axial extension of the boundary. So, the control volume Ω expands due to radial expansion only which has effect in thickness of boundary wall. So, we can finally describe our model of main mathematically as used in some popular literature such as [1].

$$\Gamma_w^0 = \{(r,\theta,z): r = R_0, 0 \le z \le L, 0 \le \theta \le 2\pi\}$$

$$\Gamma_w^t = \{(r,\theta,z): r = r(t), 0 \le z \le L, 0 \le \theta \le 2\pi\}$$

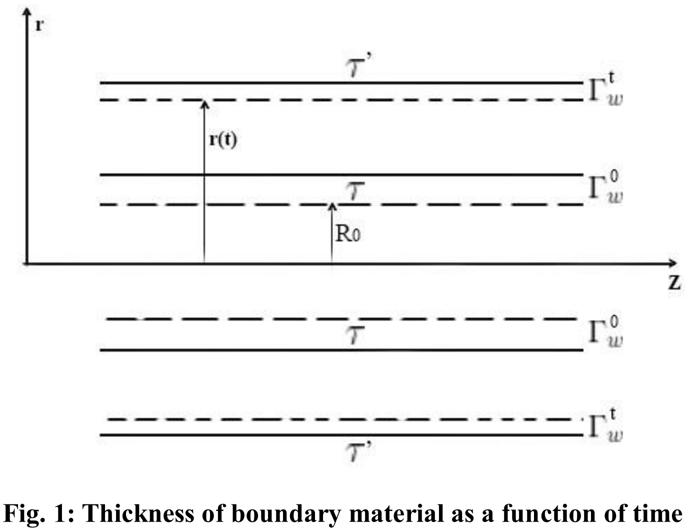

Fig. 1: Thickness of boundary material as a function of time

4. Development of Differential Equations for Describing Gross Effect

We study the gross effect, i.e. mass flow, induced force, energy exchange [4] etc. of the model we have constructed. For this, we first approach the problem by using control volume analysis, and develop differential equations from there. We use three of the most fundamental principles of physics in control volume to develop our relations. They are as follows:

4.1. Conservation of Mass

Since we have considered that the fluid in action is in-compressible, i.e. $\rho = constant$, so the volumetric flow must be conserved,

$$Q_{in} = \frac{\mathrm{d}V_\Omega}{\mathrm{d}t} + Q_{out} \tag{1}$$

$$\Rightarrow \frac{\mathrm{d}V_\Omega}{\mathrm{d}t} = A_{in}u_{in} - A_{out}\,u_{out} \tag{2}$$

where A_{in} and A_{out} are areas of Γ_{in} and Γ_{out} respectively and V_Ω is volume of Ω. A_{in} is constant but A_{out} is function of time and is given by $A_{out} = \pi r_{out}^2$. Also, $V_\Omega = \pi r^2 L$. We establish a condition that rate of expansion of the radius of Γ_{out} is equal to the rate of expansion of radius of cross section Ω., i.e. $\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{\mathrm{d}r_{out}}{\mathrm{d}t} \rightarrow 1.1$. Also, we write $\frac{\mathrm{d}r}{\mathrm{d}t} = \eta \; and \; \frac{\mathrm{d}^2r}{\mathrm{d}t^2} = \frac{\mathrm{d}\eta}{\mathrm{d}t} = \dot{\eta} \rightarrow 1.2$. Now, we can write above equation as

$$\Rightarrow 2\pi L[r\dot{\eta} + \eta^2] = A_{in}\frac{\mathrm{d}u_{in}}{\mathrm{d}t} - 2\pi u_{out}\,r_{out}\,\eta \tag{3}$$

4.2 Conservation of Linear Momentum

The mass entering (m_{in}) and (m_{out}) the mass exiting out of control volume are both functions of time. From (1), we see that we have relation for volumetric rate of flow but not volume itself. So, we have relation for mass flow ($\dot{m}_{in} = \rho A_{in}u_{in}$ and $\dot{m}_{out} = \rho A_{out}\,u_{out}$) but it is not possible to measure the mass of fluid entering or exiting at some time 't' and we can only measure the mass entering or exiting for some time interval 'Δt'. The mass entering and leaving Ω changes for every time interval Δt_i as shown in figures 2 a. and 2 b., where i = 1, 2, 3, ..., n and 'n' is number of such time intervals. If 'n' is made sufficiently large, then we get significantly good approximation for net flow of mass in Ω.

Then we have,

$$m_{in,i} = \rho A_{in}u_{in}\Delta t_i \tag{4.1}$$

$$m_{out,i} = \rho A_{out}\,u_{out}\,\Delta t_i \tag{4.2}$$

where $m_{in,i}$ and $m_{out,i}$ are mass entering and exiting Ω in time period = Δt_i.

Also, Total mass entering Ω in given period of time = $\sum_i \rho A_{in}u_{in}\Delta t_i$, And, Total mass leaving Ω in given period of time = $\sum_i \rho A_{out} u_{out} \Delta t_i$.

Now, we establish the relation of conservation of linear momentum.

$$m_{in} u_{in} = momentum\ of\ moving\ boundary\ wall + m_{out} u_{out} \tag{5}$$

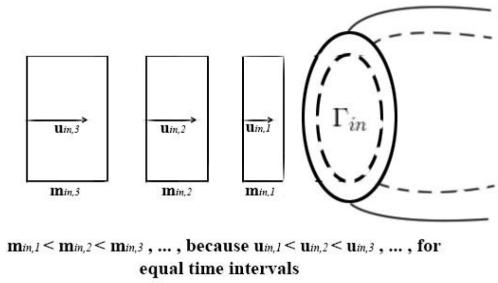

Fig. 2 a: Mass entering through Γ_{in} in certain certain time interval

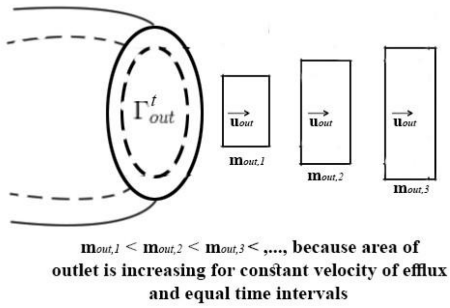

Fig. 2 b: Mass leaving through Γ_{out} in time interval

For establishing equation (6), we need to calculate momentum of moving boundary wall (P_{Γ_w}). Let us consider the wall to be of infinitesimal thickness and length, i.e. we take our elastic tube to be like a ring. Let us take a small portion of that ring of mass $d\mu$ with volume dv (where μ is the mass of the boundary wall) and the boundary is moving with velocity η[1]. Then, the momentum of that small section is given by

$$P_{\Gamma_w} = \eta \int_0^{2\pi} \int_0^{\tau} \int_0^{L} \rho_\mu r d\theta d\tau dL \ \Rightarrow P_{\Gamma_w} = 2\pi \eta \rho_\mu r L \tau$$

Here, ρ_μ is the density of the boundary material. Also, equation 6 becomes

$$m_{in} \frac{\mathrm{d}u_{in}}{\mathrm{d}t} + \dot{m}_{in} u_{in} = 2\pi \rho_\mu L r \tau [\dot{\eta} + \eta^2] + 2\pi \rho_\mu L r \eta \frac{\mathrm{d}\tau}{\mathrm{d}t} + \dot{m}_{out} u_{out} \tag{6}$$

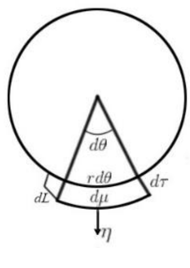

Fig. 3 a: Small volume element dv of boundary material

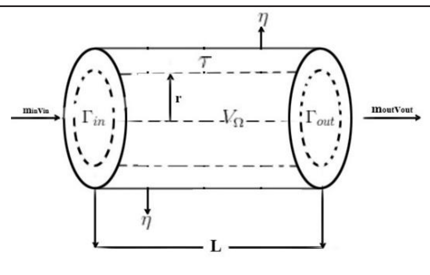

Fig. 3 b: Total momentum of boundary wall moving with momentum $\eta d\mu$

[1] the value of η should be taken for mid-point of each time interval

4.3 Conservation of Energy

We use the same concept of mass entering and mass exiting out of Ω developed during formulation of equations for conservation of momentum. We know, some of the kinetic energy of mass of fluid entering Ω will be used to move the boundary wall while some energy will be spent by mass exiting out of Ω. The total energy in the process will be conserved. So, we get

$$\frac{1}{2}m_{in}u_{in}^2 = \text{Energy used in moving the boundary wall} + \frac{1}{2}m_{out}\,u_{out}^2 \tag{7}$$

For this, we first need to find the expression for energy used in moving the boundary.

We have considered that Γ_w to be elastic. Let $\widehat{\Phi}$ be the stress exerted by the structure on the fluid. We have assumed the expansion to be axisymmetric, so we claim $\widehat{\Phi}$ is acting normally on the fluid as shown in figure 4. We know, that the expansion of Ω is due to pressure difference $\bar{P}$ and $\widehat{\Phi}$ opposes the expansion of the main body under $\bar{P}$. There is expansion only if $\bar{P} > \widehat{\Phi}$ and the expansion occurs due to the effect of net pressure $\bar{P} - \widehat{\Phi}$ experienced by Γ_w. Therefore, at some time 't', the energy spent in attaining certain shape of Ω is given by $E = (\bar{P} - \widehat{\Phi})V$, where V is the volume of Ω. This energy E is the energy spent in moving the boundary wall, thus we can write equation (7) as

$$2m_{in}u_{in}\frac{\mathrm{d}u_{in}}{\mathrm{d}t} + \dot{m}_{in}u_{in}^2 = -2\frac{\mathrm{d}\widehat{\Phi}}{\mathrm{d}t}V + 4\pi r\eta L\left(\bar{P} - \widehat{\Phi}\right) + \dot{m}_{out}\,u_{out}^2 \tag{8}$$

5. Modifications in Reynold's Transport Theorem

Some modifications in RTT were necessary for it to be compatible with our model because our model is constructed in such a way that there is two open boundaries (Γ_{in} and Γ_{out}) where mass is free to flow and there is boundary wall Γ_w from where mass can neither enter or exit. Also, in our model, the momentum of system can not be expressed in terms of MV, where M is mass in action, because V is very difficult to understand. So, we modify RTT, such that it follows all the fundamental concepts of the theorem and overcome above mentioned difficulties. Let,

$$\partial\Gamma = \Gamma_{in} \cup \Gamma_w \cup \Gamma_{out}$$

Then, from RTT, we say that

Rate of change of some factor of the system = Rate of change of some factor of the control volume + Change in factor caused due to system of boundaries i.e.

$$\frac{\mathrm{d}B_{sys}}{\mathrm{d}t} = \frac{\mathrm{d}B_{\Omega}}{\mathrm{d}t} + \int_{\partial\Gamma} d\beta^*$$

$$\Rightarrow \frac{\mathrm{d}B_{sys}}{\mathrm{d}t} = \frac{\mathrm{d}B_{\Omega}}{\mathrm{d}t} + \int_{\Gamma_{in}} d\beta^*_{\Gamma_{in}} + \int_{\Gamma_w} d\beta^*_{\Gamma_w} + \int_{\Gamma_{out}} d\beta^*_{\Gamma_{out}} \tag{9}$$

And $\beta^* = \frac{\mathrm{d}B}{\mathrm{d}t}$ is the main change from the RTT in usual form where $\beta = \frac{\mathrm{d}B}{\mathrm{d}m}$. Also, usual sign convention of RTT is followed.

5.1 Development of Relation for Mass Flow from Modified RTT

For developing relation of mass flow, we replace B by m in equation 9 i.e.

$$\frac{\mathrm{d}m_{sys}}{\mathrm{d}t} = \frac{\mathrm{d}m_{\Omega}}{\mathrm{d}t} + \int_{\dot{m}_{\Gamma_{in}}} d\dot{m}_{\Gamma_{in}} + \int_{\dot{m}_{\Gamma_{w}}} d\dot{m}_{\Gamma_{w}} + \int_{\dot{m}_{\Gamma_{out}}} d\dot{m}_{\Gamma_{out}}$$

$$\Rightarrow \frac{\mathrm{d}V_{\Omega}}{\mathrm{d}t} = A_{in}u_{in} - A_{out}u_{out} \tag{10}$$

Here, equation (10) is in agreement with equation (2). Hence, our modification of RTT is applicable for developing relation of mass flow.

5.2 Development of Momentum Relation from Modified RTT

For developing momentum relation, we we replace B by P (momentum) in equation 9 i.e.

$$\frac{\mathrm{d}P_{sys}}{\mathrm{d}t} = \frac{\mathrm{d}P_{\Omega}}{\mathrm{d}t} + \int_{\dot{P}_{\Gamma_{in}}} d\left(\frac{\mathrm{d}P}{\mathrm{d}t}\right)_{\Gamma_{in}} + \int_{\dot{P}_{\Gamma_{w}}} d\left(\frac{\mathrm{d}P}{\mathrm{d}t}\right)_{\Gamma_{w}} + \int_{\dot{P}_{\Gamma_{out}}} d\left(\frac{\mathrm{d}P}{\mathrm{d}t}\right)_{\Gamma_{out}}$$

$$\Rightarrow m_{in}\frac{\mathrm{d}u_{in}}{\mathrm{d}t} + \dot{m}_{in}u_{in} = 2\pi\rho_{\mu}L\left[\tau r\dot{\eta} + r\eta\frac{\mathrm{d}\tau}{\mathrm{d}t} + \tau\eta^2\right] + \dot{m}_{out}u_{out} \tag{11}$$

Here, equation (11) is in agreement with equation (6). Hence, our modification of RTT is applicable for developing momentum relation.

5.3 Development of Energy Relation from Modified RTT

For developing energy relation, we we replace B by E (Energy) in equation 9 i.e.

$$\frac{\mathrm{d}E_{sys}}{\mathrm{d}t} = \frac{\mathrm{d}E_{\Omega}}{\mathrm{d}t} + \int_{\dot{E}_{\Gamma_{in}}} d\left(\frac{\mathrm{d}E}{\mathrm{d}t}\right)_{\Gamma_{in}} + \int_{\dot{E}_{\Gamma_{w}}} d\left(\frac{\mathrm{d}E}{\mathrm{d}t}\right)_{\Gamma_{w}} + \int_{\dot{E}_{\Gamma_{out}}} d\left(\frac{\mathrm{d}E}{\mathrm{d}t}\right)_{\Gamma_{out}}$$

$$\Rightarrow \frac{\mathrm{d}(\frac{1}{2}m_{in}u_{in}^2)}{\mathrm{d}t} = \frac{\mathrm{d}\bar{P}V}{\mathrm{d}t} - \frac{\mathrm{d}\widehat{\Phi}V}{\mathrm{d}t} + \frac{\mathrm{d}(\frac{1}{2}m_{out}u_{out}^2)}{\mathrm{d}t}$$

$$\Rightarrow 2m_{in}u_{in}\frac{\mathrm{d}u_{in}}{\mathrm{d}t} + \dot{m}_{in}u_{in}^2 = -2\frac{\mathrm{d}\widehat{\Phi}}{\mathrm{d}t}V + 4\pi r\eta L(\bar{P} - \widehat{\Phi}) + \dot{m}_{out}u_{out}^2 \tag{12}$$

Here, equation (12) is in agreement with equation (8). Hence, our modification of RTT is applicable for developing energy relation.

6. Further Mathematical Analysis

6.1 Rate of Change in Volume

We can rearrange equation (3) from and get rate of change of rate of change in volume is given by, $\frac{\mathrm{d}^2 V_\Omega}{\mathrm{d}t^2} = A_{in}\frac{\mathrm{d}u_{in}}{\mathrm{d}t} - 2\pi u_{out} r_{out}\eta$

If $\frac{\mathrm{d}^2 V_\Omega}{\mathrm{d}t^2} < 0$, then at some point, $\frac{\mathrm{d}V_\Omega}{\mathrm{d}t}$ will be zero, i.e. $r = r_{max}$ and $\eta = 0$. Let

$$\frac{\mathrm{d}^2 V_\Omega}{\mathrm{d}t^2} \leq 0 \Rightarrow u_{in} \leq \frac{\pi u_{out}}{A_{in}} r_{out}^2 \tag{13}$$

Integrating equation (4) with respect to t, we obtain

$$\int_{\dot{V},initial}^{\dot{V},final} \frac{\mathrm{d}^2 V_\Omega}{\mathrm{d}t^2} dt = \int_{u_{in,initial}}^{u_{in,final}} A_{in}\frac{\mathrm{d}u_{in}}{\mathrm{d}t} dt - \int_{r_{out,minimum}}^{r_{out,maximum}} 2\pi u_{out} r_{out}\eta dt$$

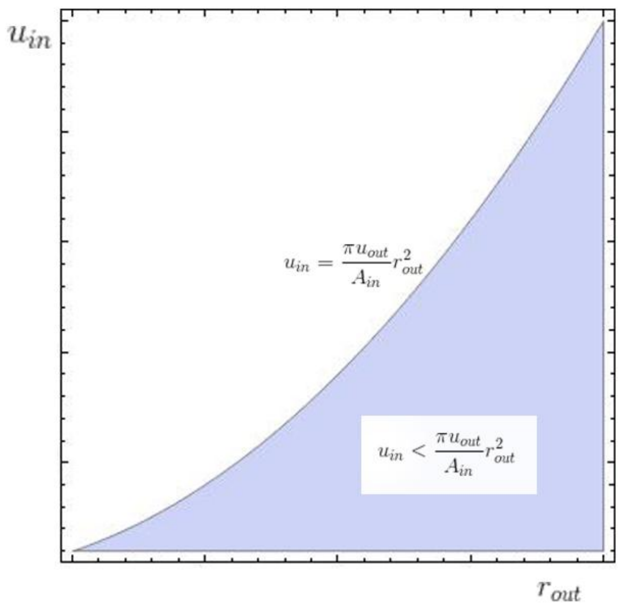

Fig. 4: Relation between fluid influx velocity and radius of outlet

Let $\dot{V}_{\Omega,final} = 0$, also from equation (2) $\dot{V}_{\Omega,initial} = A_{in} u_{in,initial} - A_{out,initial} u_{out}$,

$$\Rightarrow u_{in,final} = \frac{\pi u_{out}}{A_{in}} r_{out,final}^2 \tag{14}$$

The inequality sign of (13) changes into equality sign of (14), when $u_{in} = u_{in,final}$ and $r_{out} = r_{out,final}$. Also, Equation (14) gives relationship between radius of maximum expansion and maximum influx velocity required to establish constant pressure $\bar{P}$ in Ω.

6.2 Relationship between K and λ_τ

Let us define K as factor by which volume of Ω increases. If V be a reference volume and V' be volume after some expansion, then $K = \frac{V'}{V}$.

λ_τ, λ_r and λ_L are used in many literatures such as [2] , [5] and are defined as $\lambda_\tau = \frac{\tau'}{\tau}$, $\lambda_r = \frac{r'}{r}$ and $\lambda_L = \frac{L'}{L}$, where τ, r and L are thickness, radius and length of tube in reference configuration and τ', r' and L' are thickness, radius and length of tube after some expansion. In our case, $\lambda_L = 1$. Also, in-compressible material, $\lambda_r \lambda_\tau \lambda_L = 1$.

This is a very trivial relation but very fundamental in understanding relation between increasing volume and decreasing thickness of boundary material.

$$V' - V = \pi(r')^2 L - \pi r^2 L$$

$$\Rightarrow K = \lambda_r^2 \text{ and } K = \frac{1}{\lambda_\tau^2}$$

Also, it is important to note that $\lambda_r \geq 1$, as the system never shrinks and $0 < \lambda_\tau \leq 1$, due to in-compressibility of material.

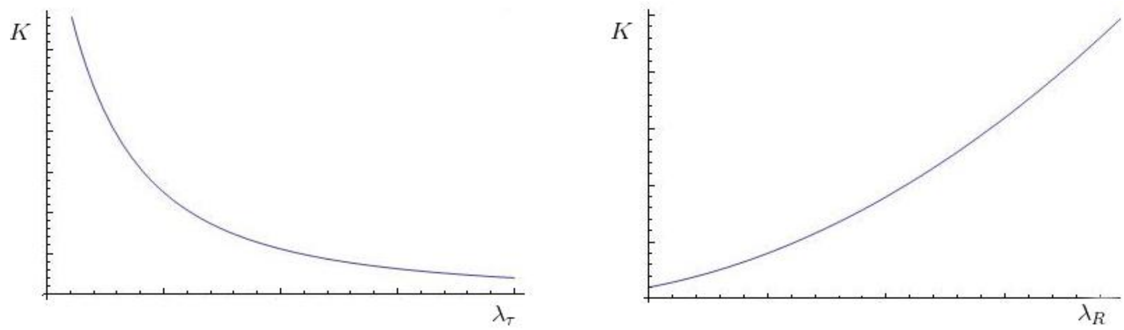

Fig. 5: Relation of K with different parameters

7. Results and Discussions

7.1 Modified Reynold's Transport Theorem

The modified form of RTT is given by

$$\frac{\mathrm{d}B_{sys}}{\mathrm{d}t} = \frac{\mathrm{d}B_\Omega}{\mathrm{d}t} + \int_{\Gamma_{in}} d\beta^*_{\Gamma_{in}} + \int_{\Gamma_w} d\beta^*_{\Gamma_w} + \int_{\Gamma_{out}} d\beta^*_{\Gamma_{out}}$$

And we see that this modified RTT describes kinematics of our model. In traditional form of RTT, it was difficult to define u such that mu describes the total momentum of the system. In our modification, we have solved that problem but we introduce a new problem of describing m.

7.2 Some other Relations

The condition for rate of change in volume change rate to be decreasing was found to be $u_{in} \leq \frac{\pi u_{out}}{A_{in}} r_{out}^2$ which is in the form of $y \leq kx^2$ and from this relation we can conclude that rate of volume expansion is decreasing for $u_{in} < \frac{\pi u_{out}}{A_{in}} r_{out}^2$ and rate of volume change is 0 when $u_{in} = \frac{\pi u_{out}}{A_{in}} r_{out}^2$.

The factor by which volume of tube changes is directly proportional to square of parameter by which radius of tube changes and inversely proportional to square of parameter by which thickness of material wall changes which is given by relations $K = \lambda_r^2$ and $K = \frac{1}{\lambda_\tau^2}$.

8. Conclusion

We develop simple model for expansion of cylindrical elastic tubes which may be significant in reducing computation time for numerical simulations. Although, it ignores many important aspects of fluid flow such as head loss, it preserves important kinematic properties of expansion. So this model is somewhat acceptable to study kinematic features of deformation of the tube such as rate of change in stress exerted by material to the fluid inside the tube and so on. We also modify Reynold's transport theorem to describe our model. Some of the graphical results obtained makes it easier to study the relationship between different parameters such as K, λ_τ and λ_R.

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