

# Mathematical Study of Temperature Distribution in Human Dermal Part during Physical Exercises

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*Received: Feb. 25, 2016*

*Revised: May 18, 2016*

*Accepted: Aug. 25, 2016*

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**Abstract:** The purpose of this paper is to model metabolic rate that governs the behavior exhibited by various exercises over the period. This model equation is used in one dimensional Pennes' bio-heat equation to study the temperature distribution in dermal part of tissue layers due to various exercises. The appropriate Dirichlet and Neumann boundary conditions are used. The solution of the bio-heat equation is then obtained using FEM technique and the simulated results are presented graphically.

**Key words:** Metabolic rate, Pennes' bio-heat equation, Finite element method

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## 1. Introduction

Human body survives in cold / hot temperature and extreme weather. In such situations body temperature is maintained by skin and core temperature regulation. Survival in hot and cold environment means human can adjust in high as well as in low temperatures conditions. Although environmental fluctuation affects skin layer temperature, body core temperature remains fixed. The body core temperature of a normal body is maintained at about 37<sup>0</sup>C. Thus, body temperature plays an important role for the proper function of various important systems of human body. The human body has remarkable capacity for regulating its core temperature constant. Activities like different types of physical exercises and aerobics cause corresponding increase in energy production to release the body temperature. Vasodilation of blood vessels increases blood circulation leading to sweating and evaporation and helping to decrease the skin temperature. The peripheral region plays an important role in temperature regulation of the human body. The human body maintains its body core temperature under the normal atmospheric conditions [9]. A decrease in either skin temperature or blood temperature stimulates the thermo-receptors to provide feedback to the thermoregulatory center within the hypothalamus (hypothalamus is a part of brain which can inhibit and release hormones) to activate the mechanisms that will conserve body heat and increase heat production. The primary mechanisms utilized by the body to increase internal temperature in cold environment are shivering, non-shivering thermo-genesis, and peripheral vasoconstriction.

During the exercise in hot environment, when the demand of body temperature regulation is highest, the cardiovascular system (cardiovascular system is a circulating system which comprises heart, blood vessel and blood which help in the circulation of gases, nutrients and waste matter throughout the body and help in metabolism process) can become burdened. The circulatory system transports heat generated by the Adenosine Triphosphate (ATP) producing metabolic pathways to the surface of the body, where heat can be transferred to the external environment. To accomplish this situation, a large portion of cardiac output must be taken up by the skin and the exercising muscles. Because the blood volume is limited to circulation, exercise poses a complex problem. An increase in blood flow to one of the areas automatically decreases blood flow to the other area. This can lead to potentially problematic situations. Exercise in hot environments increases total oxygen uptake, which causes the working muscles to utilize more glycogen and to produce more lactate compared with exercise in cold environments. Repeated bouts of exercise in the hot environment (104<sup>0</sup>F) increases heart rate and oxygen uptake significantly compared with exercise in a cool environment (48<sup>0</sup>F). Increased muscle temperature may impair skeletal muscle function and metabolism that can lead to fatigue. Moreover, increased carbohydrate utilization is directly linked to the increased secretion of epinephrine with elevated body temperature (hyperthermia).

Excessive exposure to cold environments can cause damage to both peripheral tissues and the life-supporting cardiovascular and respiratory systems. The most significant effect of hypothermia is on the heart. Cooling of tissue leads a progressive decline in heart rate. The deep breathing as in exercise in cold environments can damage the respiratory tract. However, the cold air is rapidly warmed in the trachea, even when the air inhaled is less than 13<sup>0</sup>F. Therefore, there is absolutely no shred of damage posed by inhalation of cold air. The heavy breathing through the mouth has been shown to cause a minor degree of irritation to the mouth, pharynx, trachea, and even bronchi when the air temperature is below 10<sup>0</sup>F. Metabolism is the set of physio-chemical process taking place all the time at cellular level, resulting in cell growth, production of thermal and mechanical energy, replication and elimination of waste material. The chemical activity and metabolic heat generation rate are proportional in human body whose magnitude depends on some of the physical factors. These factors are: (i) physical exercise (ii) shivering (iii) change in environment temperature and (iv) energy required for the daily activity.

Explicit mathematical treatment of heat transport in living tissue was first appeared in the pioneering work of Henry H Pennes in 1948 [12], known as bio-heat equation or Pennes bioheat equation. Afterwards lots of researches were carried out to modify the assumptions made by Pennes especially for perfusion term. Other researchers worked on the various applications of bio-heat equations. Despite appearing some modified versions of Pennes equations, researchers preferring for application sides use Pennes bio-heat equation due to its simplicity to use on various numerical techniques such as finite difference method, finite element method etc to handle the complex geometry of dermal part. To use modified version, a micro-level study of anatomical structure is needed. Saxena [13] and his group [3, 8, 14] are working on various applications of bio-heat equation. In Nepal, Gurung [7] initiated the work in this direction, and

his group [1, 2, 5, 6] are working for various applications of bio-heat equations in biology and bio-medical sciences. Kumari and Adlakha [10] investigated a numerical model to study the effect of physical exercise on temperature distribution in peripheral layers of circular shaped human limbs considering time independent metabolic rate.

## 2. Bio-heat Equation

Heat is lost and gained through the process of convection, radiation, blood perfusion and conduction, while evaporation contributes only heat loss to the environment. The total heat loss from body surface depends on the temperature difference between skin and environment. There is normal blood flow and metabolic activity when the body is at rest however, during exercises there is abnormal blood flow and metabolic activity. When a person engages in different types of physical activities, the body requires additional fuel which increases the metabolic rate and the rate of heat production. The body must use additional method to remove the additional heat produced in order to keep the internal temperature at healthy level. Different people have different thermal behavior even in the same environment. Blood perfusion is the physiological term that refers to the process of delivery of arteries blood to a capillaries bed in the biological tissue. The perfusion term was first formulated by Harry H. Pennes' in 1948. He proposed that the rate of heat transfer between blood and tissue is proportional to the product of a volumetric perfusion rate and the difference between the arterial blood temperature and the local tissue temperature.

The heat balance within the human body is a complicated process. This process involves heat diffusion, metabolic heat generation and heat convection by free fluid flow. The perfusion occurs in tissues from arteries to capillaries and veins. Cardiovascular system is the key system by which heat is distributed throughout the body from body core to limbs and head. Let  $\omega_b$  be the volumetric blood flow rate per unit volume of tissue. The energy balance per unit volume can be set as [12]:

$$\rho c \frac{\partial T}{\partial t} = \nabla \cdot (K \nabla T) + \rho_b c_b \omega_b (T_A - T) + S \quad (1)$$

Equation (1) is known as bio-heat equation. Here,

- $\rho$  = Density of tissue ( $\text{g/cm}^3$ ),
- $c$  = Specific heat capacity of tissue ( $\text{cal/g}^{\circ}\text{C}$ ),
- $K$  = Thermal conductivity of Tissue ( $\text{cal/cm min}^{\circ}\text{C}$ ),
- $\rho_b$  = Density of blood ( $\text{g/cm}^3$ ),
- $\omega_b$  = Blood perfusion rate per unit volume ( $\text{g/cm}^3 \text{ min}$ ),
- $c_b$  = Blood specific heat capacity ( $\text{cal/g}^{\circ}\text{C}$ ),
- $T$  = Environment temperature ( $^{\circ}\text{C}$ ),
- $T_A$  = Arterial blood temperature ( $^{\circ}\text{C}$ ),
- $S$  = Metabolic heat generation rate ( $\text{cal/cm}^3 \text{ min}$ ).

### 3. Metabolic Energy during Exercises

Exercise is the physical exertion of the body that makes healthy or healthier level of physical fitness - both physical and mental health. Exercise plays an important role in temperature distribution in human body, so exercises are essential in our life. Regular exercise even for half an hour benefits us mentally and physically because it purifies blood, increasing its flow and decreasing heart rate. It lowers mental stress and tends to bring blood pressure under control. The exercise increases blood flow through muscle which helps to improve muscles ability to use fats during exercise. Muscles are principal sources of metabolic heat. Exercise increases the temperature of blood and tissues due to increase the blood perfusion in muscles. The muscles contract and stretch help to increase the metabolism and increases the temperature.

There are three broad intensities of exercise.

- Light exercise: The exerciser is able to talk while exercising. Going for a walk is an example of light exercise.
- Moderate exercise: The exerciser feels slightly out of breath during the session. Examples could be walking briskly, cycling moderately, or walking up a hill.
- Vigorous exercise: The exerciser is panting during the activity. The exerciser feels body is being pushed much nearer its limit, compared to the other two intensities. This could include swimming, running, cycling fast, climbing stairs and carrying heavy weight.

In heavy exercise, the rate of intake of oxygen in the body and exhaled carbon-dioxide from the body also increase, which support on increasing the metabolism to provide the necessary energy. At the same time, heat generation also increases as by product. In low intensity exercise as sitting, typing, there is very less difference in vascular system so the metabolic rate is similar to the basal metabolic rate while in high intensity exercise as cycling, wrestling, most of the organs are come in movement so loses the energy. Thus, the metabolism of human body varies on the environment temperature and exercises. According to Gurung [8], the basal metabolic rate at the environment temperature  $15^{\circ}\text{C}$  is  $0.0357 \text{ cal/cm}^3 \text{ min}$ , while at  $23^{\circ}\text{C}$  and  $33^{\circ}\text{C}$ , it is equal to  $0.018 \text{ cal/cm}^3 \text{ min}$ . On contrary to the basal metabolic rate, Bradshaw [4] proposed the steady metabolic rate for various exercises. The proposed value for wrestling is  $0.7247 \text{ cal/cm}^3 \text{ min}$ ; for running is  $0.3165 \text{ cal/cm}^3 \text{ min}$ ; and for typing is  $0.0999 \text{ cal/cm}^3 \text{ min}$ .

### 4. Model Equation of Metabolic Energy during Exercises

The metabolism of human body has normal values at rest called basal metabolic rate (*BMR*). During physical exercise metabolic activity increases due to increase in the blood flow. The continuous increase in blood flow is controlled by the mechanism of body by rapidly producing metabolic energy at the beginning of exercise and becomes constant after certain time, so it is plausible to consider the metabolic rate,  $S(t)$ , increases exponentially in certain time and then it becomes constant over the time. We then model the metabolic rate based on exercises as

$$S(t) = BMR + \beta(1 - e^{-\alpha t}), \quad \alpha \geq 0 \quad (2)$$

where  $\beta$  is an exercise controlled parameter and  $\alpha$  is the metabolic controlled parameter, and  $\alpha = 0.05$  fits reasonably for exponential behavior of  $S(t)$  over time for extensive exercise (Wrestling).

It is observed that the minimum value of  $S(t) = BMR$ , and the maximum value of  $S(t) = BMR + \beta$ , because

$$\begin{aligned} S(t) &= BMR & \text{as } t \rightarrow 0 \\ S(t) &= BMR + \beta & \text{as } t \rightarrow \infty \end{aligned}$$

The unsteady behavior of metabolic rates during various exercises such as wrestling, running and typing due to Bradshaw [4] are as presented in the graph of Fig.1 with  $BMR = 0.0357$  (cal/cm<sup>3</sup> min) [8].

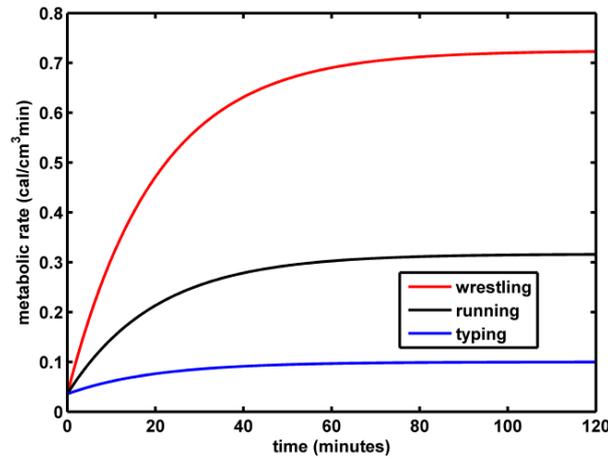
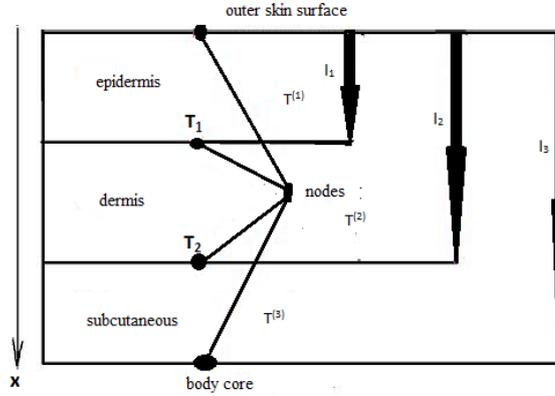


Fig. 1: Metabolic curve for different exercises.

## 5. Mathematical Formulation of Dermal Temperature

Skin is the main organ that keeps helping the temperature balance in the human body. If any illness occurs in the body, the first symptom is changing body temperature. In mathematical treatments of temperature distribution in human dermal part, the skin layers can be regarded as a physical and physiological barrier with complex structures. The three natural layers of skin are epidermis, dermis and subcutaneous tissues (SST). Let  $l_1, l_2 - l_1$  and  $l_3 - l_2$  be the thickness of the layers of epidermis, dermis and subcutaneous, respectively (Fig.2).



**Fig. 2: Schematic diagram of skin layers.**

Let  $T_0, T_1, T_2$  and  $T_3$  be the respective nodal temperatures at a distance  $x = 0, x = l_1, x = l_2$  and  $x = l_3$  measured from the outer surface of the skin and  $T^{(1)}, T^{(2)}, T^{(3)}$  be the temperature functions of epidermis, dermis and subcutaneous tissues, respectively. The governing equation that characterized the heat regulation in in-vivo tissue of human body is given by the partial differential equation (1), which we write for 1D as:

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( K \frac{\partial T}{\partial x} \right) + M (T_A - T) + S \quad (3)$$

where,  $M = \rho_b c_b \omega_b$  (cal/cm<sup>3</sup> min<sup>0</sup>C).

A normal human body has body core temperature of around 37<sup>0</sup>C. If the nude body is exposed to the environment, then the loss of heat from the body occurs due to convection, radiation and sweat evaporation. So, we consider the following two boundary conditions in the study.

- Dirichlet boundary condition:  $T_3 = \text{Body core temperature} = 37^0\text{C}$ .
- Neumann boundary condition:

$$K \frac{\partial T}{\partial x} \Big|_{x=0} = h(T - T_a) + LE \quad (4)$$

where,

$h$  = Combined heat transfer coefficient due to convection and radiation,

$T_a$  = Atmospheric temperature,

$L$  = latent heat of evaporation,

$E$  = Rate of sweat evaporation.

The bio-heat equation (3) together with the Neumann boundary condition (4) is equivalent to the minimization problem of the functional [11]:

$$I[T(x,t)] = \frac{1}{2} \int_0^l \left[ K \left( \frac{dT}{dx} \right)^2 + M (T_A - T)^2 - 2ST + \rho c \frac{dT^2}{dt} \right] dx + \frac{1}{2} h (T - T_a)^2 + LET \quad (5)$$

The nodal temperatures  $T_0$ ,  $T_1$ ,  $T_2$  and  $T_3$  are considered as global nodal values for linear finite element discretization of the dermal layers. Thus each dermal layer (epidermis, dermis and subcutaneous), tissues are the three linear elements in finite element procedure.

## 6. Solution of the Problem

The dermal layer is a complex geometry with different physical and physiological properties in the layers. So, to handle all these properties, the numerical approximation technique, the finite element method is the most appropriate method. Based on the study of the properties in the layers, the following assumptions are made:

Epidermis ( $l_0 < x < l_1$ ):

$$K = K_1 = (\text{constant}),$$

$$M = M_1 = 0,$$

$$S = S_1 = 0,$$

$$T_A = 0,$$

Dermis ( $l_1 < x < l_2$ ):

$$K = K_2 (\text{constant})$$

$$M = M_2,$$

$$S = S_2 = s_o + \beta(1 - e^{-\alpha t}), \quad s_o = BMR,$$

$$T_A = T_b,$$

$$T^{(2)} = \left[ \frac{l_2 T_1 - l_1 T_2}{l_2 - l_1} + \frac{T_2 - T_1}{l_2 - l_1} \right],$$

Subcutaneous Tissue ( $l_2 < x < l_3$ ):

$$K = K_3 (\text{constant})$$

$$M = M_3,$$

$$S = S_3 = 2[s_o + \beta(1 - e^{-\alpha t})], \quad s_o = BMR,$$

$$T^{(3)} = \left[ \frac{l_3 T_2 - l_2 T_3}{l_3 - l_2} + \frac{T_3 - T_2}{l_3 - l_2} \right],$$

$$T_A = T_3 = T_b = 37^{\circ}C,$$

Expressing the integral  $I[T(x, t)]$  in equation (5), element-wise as  $I_1$ ,  $I_2$  and  $I_3$ , for the finite element discretization of the elements epidermis, dermis and subcutaneous tissues, respectively, we have

$$I_1[T(x, t)] = \frac{1}{2} \int_0^{l_1} \left[ K \left( \frac{dT^{(1)}}{dx} \right)^2 + M_1 (T_A - T^{(1)})^2 - 2S^{(1)}T^{(1)} + \rho_1 c_1 \frac{dT^{(1)2}}{dt} \right] dx + \frac{1}{2} h (T_0 - T_a)^2 + LET_0$$

$$I_2[T(x, t)] = \frac{1}{2} \int_{l_1}^{l_2} \left[ K \left( \frac{dT^{(2)}}{dx} \right)^2 + M_2 (T_A - T^{(2)})^2 - 2S^{(2)}T^{(2)} + \rho_2 c_2 \frac{dT^{(2)2}}{dt} \right] dx$$

$$I_3[T(x, t)] = \frac{1}{2} \int_{l_2}^{l_3} \left[ K \left( \frac{dT^{(3)}}{dx} \right)^2 + M_3 (T_A - T^{(3)})^2 - 2S^{(3)}T^{(3)} + \rho_3 c_3 \frac{dT^{(3)2}}{dt} \right] dx$$

Solving the above integrals using the parameters as considered, we obtain  $I_1$ ,  $I_2$  and  $I_3$  as functions of nodal values  $T_0$ ,  $T_1$ ,  $T_2$ , and are given as below:

$$I_1 = A_1 + B_1 T_0 + D_1 T_0^2 + E_1 T_1^2 + F_1 T_0 T_1 + \alpha_1 \frac{d}{dt} (T_0^2 + T_1^2 + T_0 T_1)$$

$$I_2 = A_2 + B_2 T_1 + C_2 T_2 + D_2 T_1^2 + E_2 T_2^2 + F_2 T_1 T_2 + \alpha_2 \frac{d}{dt} (T_1^2 + T_2^2 + T_1 T_2)$$

$$I_3 = A_3 + B_3 T_2 + C_3 T_3 + D_3 T_3^2 + E_3 T_3^2 + F_3 T_2 T_3 + \alpha_3 \frac{d}{dt} (T_2^2 + T_3^2 + T_2 T_3)$$

where  $A_i$ ,  $B_i$ ,  $D_i$ ,  $E_i$ ,  $F_i$  and  $C_j$  with  $1 \leq i \leq 3$  and  $2 \leq j \leq 3$  are all constants whose values depend upon physical and physiological parameters of dermal part.

As a next step to finite element method, we differentiate  $I_1$ ,  $I_2$  and  $I_3$  with respect to  $T_0$ ,  $T_1$  and  $T_2$  and set  $\frac{dI}{dT_i} = 0$ , for  $i = 0, 1, 2$ . We then obtain the following system of ordinary differential equations:

$$PT + Q\dot{T} = R \quad (6)$$

Where,

$$P = \begin{pmatrix} 2D_1 & F_1 & 0 \\ F_1 & 2(D_2 + E_1) & F_2 \\ 0 & F_2 & 2(D_3 + E_2) \end{pmatrix}; \quad Q = \begin{pmatrix} 2\alpha_1 & \alpha_1 & 0 \\ \alpha_1 & 2(\alpha_1 + \alpha_2) & \alpha_2 \\ 0 & \alpha_2 & 2(\alpha_2 + \alpha_3) \end{pmatrix};$$

$$T = \begin{pmatrix} T_o \\ T_1 \\ T_2 \end{pmatrix}; \quad \dot{T} = \begin{pmatrix} \frac{dT_o}{dt} \\ \frac{dT_1}{dt} \\ \frac{dT_2}{dt} \end{pmatrix}; \quad R = \begin{pmatrix} -B_1 \\ -B_2 \\ -C_2 - B_3 - F_3 T_3 \end{pmatrix}$$

We now use the Crank -Nicolson method to solve the equation (6) iteratively. The Crank -Nicolson method of the above equation (6) can be written as

$$\left( Q + \frac{\Delta t}{2} P \right) T^{(i+1)} = \left( Q - \frac{\Delta t}{2} P \right) T^{(i)} + \Delta t R \quad (7)$$

The use of  $\Delta t$  is the equal time interval and  $T^{(0)}$  is the initial nodal temperature in  $3 \times 1$  matrix form. In normal condition when atmospheric temperature is below body core temperature, the tissue temperature increases from skin surface towards the body core, so at  $t = 0$ , we consider  $T_0 < T_1 < T_2 < T_3$  are in linear order towards the body core. Thus, we assume the linear equation for  $T^{(0)}$  as

$$T(x, 0) = T_0 + \kappa x \quad (8)$$

Here,  $\kappa$  in equation (8) is constant, whose numerical value is determined by taking the known value  $T_3 = T_b = 37^\circ\text{C}$  at  $x = l_3$ .

## 7. Numerical Results

The temperature distribution in the layers of dermal part is carried out numerically for sweat evaporation effect and ambient temperatures during various exercises for three cases of environment temperatures  $T_a = 15^\circ\text{C}$ ,  $23^\circ\text{C}$  and  $33^\circ\text{C}$ . The value of basal metabolic rate, blood flow mass and sweat evaporating rate at different ambient temperatures are taken differently, for same layer while the thermal conductivity of skin layers are taken various for different layers. The initial nodal temperature  $T^{(0)}$  based on the thickness of three layers of skin are calculated using the equation (8) and are presented as in Table-1.

Table 1: Thickness of skin and nodal temperature at  $t = 0$ .

	Epidermis layer	Dermis layer	Subcutaneous layer
Thickness (cm)	0.1	0.35	0.50
Nodal temperature ( $^\circ\text{C}$ )	$T_0 = 22.87$	$T_1 = 25.67$	$T_2 = 32.76$

To solve the equation (7), the values of physical and physiological constants blood mass flow ( $M$ ), basal metabolic rate ( $s_0$ ) and sweat evaporation rate ( $E$ ) at various ambient temperatures are taken as below in Table-2.

Table 2: The values of  $M$ ,  $s_0$  and  $E$  [5, 6, 11] .

$T_a(^{\circ}\text{C})$	$M = \rho_b c_b w_b$ (cal/cm <sup>3</sup> min <sup>0</sup> C)	$s_0$ (cal/cm <sup>3</sup> min)	$E$ (g/cm <sup>2</sup> min)
15 <sup>0</sup> C	0.003	0.0357	0
23 <sup>0</sup> C	0.018	0.018	0
23 <sup>0</sup> C	0.018	0.018	0.00048
33 <sup>0</sup> C	0.0315	0.018	0.00048
33 <sup>0</sup> C	0.0315	0.018	0.00096

The thermal conductivity value of three layers epidermis, dermis and subcutaneous tissue; the latent heat and the heat transfer coefficient irrespective of different atmospheric temperatures are taken as below [6].

$$\begin{aligned}
 K_1 &= 0.03\text{cal/cmmin}^{\circ}\text{C}, \\
 K_2 &= 0.045\text{cal/cm min}^{\circ}\text{C}, \\
 K_3 &= 0.06\text{cal/cm min}^{\circ}\text{C}, \\
 L &= 579\text{cal/g}, \\
 h &= 0.009\text{cal/cm}^2 \text{ min}^{\circ}\text{C}, \\
 \rho_1 &= 1.05\text{g/cm}^3, \\
 \rho_2 &= 0.996\text{g/cm}^3, \\
 \rho_3 &= 1.05\text{g/cm}^3, \\
 c_1 &= 0.83\text{cal/g}^{\circ}\text{C}, \\
 c_2 &= 0.38\text{cal/g}^{\circ}\text{C} \\
 c_3 &= 0.38\text{cal/g}^{\circ}\text{C}
 \end{aligned}$$

## 8. Discussion

The effect of sweat evaporation and ambient temperatures on the temperature distribution in dermal layers during exercises based on the above parameters values are discussed below.

### 8.1 Effect of Sweat Evaporation

The value of sweat evaporation at ambient temperature 23<sup>0</sup>C is taken as 0 and 0.00048 g/cm<sup>2</sup> min, while at 33<sup>0</sup>C is taken 0.00048 g/cm<sup>2</sup> min and 0.00096 g/cm<sup>2</sup> min for the observation of temperature of three layers of skin epidermis, dermis and subcutaneous tissue in for the exercises period of typing, running and wrestling.

Table 3: Comparison of nodal temperatures  $T_0$ ,  $T_1$  and  $T_2$  between  $E = 0.00048 \text{ g/cm}^2 \cdot \text{min}$  and  $E = 0.00096 \text{ g/cm}^2 \cdot \text{min}$  at  $T_a = 33^\circ\text{C}$ .

Exercise	Steady state nodal temperature at $E = 0.00048 \text{ g/cm}^2 \cdot \text{min}$			Steady state nodal temperature at $E = 0.00096 \text{ g/cm}^2 \cdot \text{min}$		
	$T_0(^{\circ}\text{C})$	$T_1(^{\circ}\text{C})$	$T_2(^{\circ}\text{C})$	$T_0(^{\circ}\text{C})$	$T_1(^{\circ}\text{C})$	$T_2(^{\circ}\text{C})$
Typing	33.78	34.74	36.27	31.04	32.83	35.72
Running	33.88	34.84	36.35	31.13	32.93	35.77
Wrestling	34.04	35.01	36.44	31.99	33.20	35.93

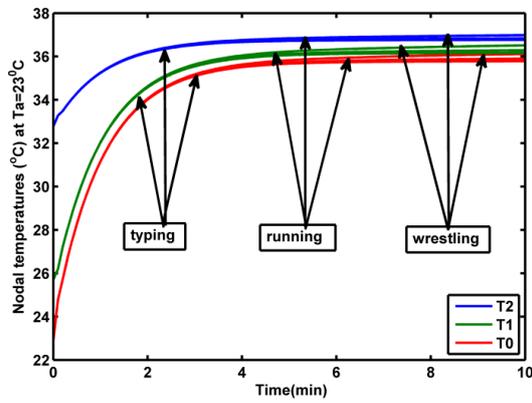


Fig. 3: Comparison of nodal temperatures  $T_0$ ,  $T_1$  and  $T_2$  when  $E = 0 \text{ g/cm}^2 \cdot \text{min}$  and ambient temperature  $23^\circ\text{C}$  during typing, running and wrestling.

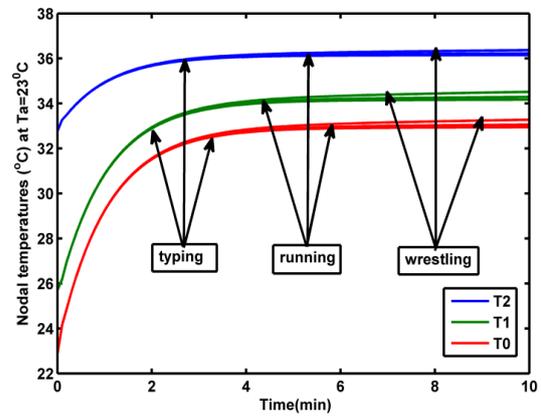


Fig. 4: Comparison of nodal temperatures  $T_0$ ,  $T_1$  and  $T_2$  when  $E = 0.00048 \text{ g/cm}^2 \cdot \text{min}$  and ambient temperature  $23^\circ\text{C}$  during typing, running and wrestling.

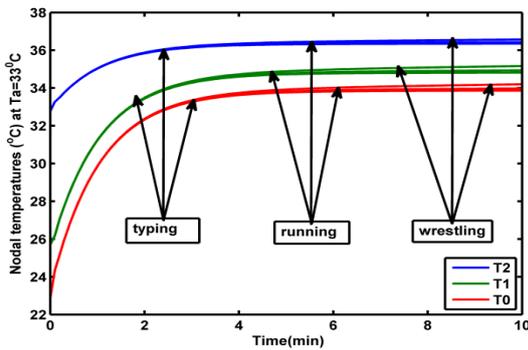


Fig. 5: Comparison of nodal temperatures  $T_0$ ,  $T_1$  and  $T_2$  when  $E = 0.00048 \text{ g/cm}^2 \cdot \text{min}$  and ambient temperature  $33^\circ\text{C}$  during typing, running and wrestling.

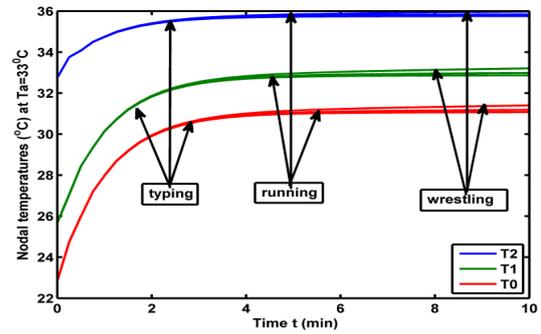
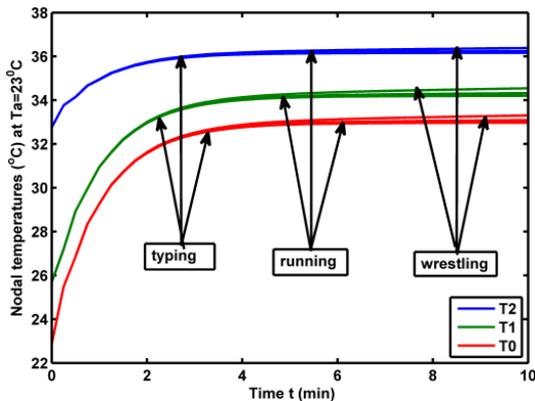


Fig. 6: Comparison of nodal temperatures  $T_0$ ,  $T_1$  and  $T_2$  when  $E = 0.00096 \text{ g/cm}^2 \cdot \text{min}$  and ambient temperature  $33^\circ\text{C}$  during typing, running and wrestling.

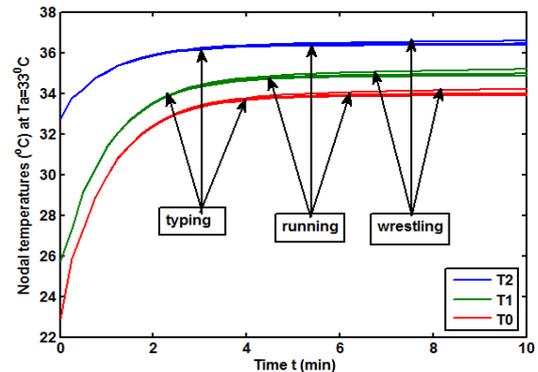
Fig. 3 to 6 represent the graphs of the time period verses interface temperature between the dermal part of tissue during typing, running and wrestling. Figures show that the temperature is slightly more in wrestling than running and running than typing. This phenomenon is due to high amount of heat energy produced in wrestling than running and running than typing during exercises due to metabolism. These figures and Table-3 represent that temperature in dermal part of body decreases while increasing the sweat evaporation rates during exercise. The same phenomena also occur without having exercises.

### 8.2 Effect of Ambient Temperature

The ambient temperature plays an important role in temperature distribution in human body. Since at high temperature, evaporation occurs in the body, the different ambient temperatures  $23^{\circ}\text{C}$  and  $33^{\circ}\text{C}$  are taken for the observation of effect of ambient temperature in the dermal part of tissue by keeping the other parameters fixed during wrestling, running and typing exercises. Figures 7 and 8 represent the graph of the time period verses nodal temperature between the dermal parts of tissue with sweat evaporation rate  $0.00048 \text{ g/cm}^2 \text{ min}$ . These figures show that the temperature value in the layers of dermal part increases with increase of atmospheric temperature irrespective of the exercise activities.



**Fig. 7:** Comparison of nodal temperatures  $T_0$ ,  $T_1$  and  $T_2$  when  $E = 0.00048 \text{ g/cm}^2 \cdot \text{min}$  and ambient temperature  $23^{\circ}\text{C}$  during typing, running and wrestling.



**Fig. 8:** comparison of nodal temperatures  $T_0$ ,  $T_1$  and  $T_2$  when  $E=0.00048 \text{ g/cm}^2 \cdot \text{min}$  and ambient temperature  $33^{\circ}\text{C}$  during typing, running and wrestling.

Table 4: Comparison of nodal temperatures  $T_0$ ,  $T_1$  and  $T_2$  between  $T_a = 23^\circ\text{C}$  and  $T_a = 33^\circ\text{C}$  at  $E = 0.00048 \text{ g/cm}^2 \text{ min}$ .

Exercise	Steady state nodal temperature at $T_a = 23^\circ\text{C}$			Steady state nodal temperature at $T_a = 33^\circ\text{C}$		
	$T_0(^{\circ}\text{C})$	$T_1(^{\circ}\text{C})$	$T_2(^{\circ}\text{C})$	$T_0(^{\circ}\text{C})$	$T_1(^{\circ}\text{C})$	$T_2(^{\circ}\text{C})$
Typing	32.94	34.16	36.13	33.78	34.74	36.27
Running	33.05	34.27	36.19	33.88	34.84	36.35
Wrestling	33.30	34.54	36.38	34.04	35.01	36.44

The graph of figures and Table-4 also reveal that skin layer temperature in case of wrestling is slightly greater than running and the same for running to typing. The slight variation in positional temperature of the skin layer during various exercises occur due to their different metabolic activities.

## 9. Conclusion

In this study, we used the mathematical model for exponential behavior of metabolic rate during various exercises, and Pennes' bio-heat equation for temperature variation in the layers of dermal part. The model for temperature variation is simulated under various physical and physiological conditions using the FEM technique. The sweat evaporation has significant effect in the living tissues temperature variation as compared to ambient temperature during various exercises. Here, we carried out the study of unsteady state of temperature distribution in the dermal layers. Our model has limit the time dependent study of metabolic rate during exercises, considering other parameters time independent. But many other control parameters like sweat evaporation rate, blood perfusion etc. are time dependent during exercises. These parameters can be considered as time dependent to handle realistic situation. But these may increase mathematical computational complexities. In our FEM model simulation, we used three linear elements epidermis, dermis and subcutaneous tissues. We can decrease the errors in FEM simulated result by subdividing the elements into sub-elements. It may increase the computation cost and time.

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