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Spectral Estimation of Electroencephalogram Signal Using Armax Model And Particle Swarm Optimization

Bijaya Gautam

Student at Master of Science Electrical Engineering, Lamar University, Beaumont, Texas, USA

Email: bijayagautam@gmail.com

Abstract

Electroencephalogram (EEG) signal is short-length time series data. Parametric and non-parametric methods are used to estimate the power spectral density (PSD) of the time series data. While the non-parametric approach of spectrum estimate of short-length data is unreliable, the parametric approach of spectrum estimate is widely suggested. In the presented work, a parametric autoregressive moving average with exogenous input (ARMAX) model was implemented to find the PSD of the EEG signal. The performance of the implemented ARMAX model was contrasted with other parametric methods like autoregressive (AR) and autoregressive moving average (ARMA) and nonparametric methods like periodogram.

Coefficients of ARMAX and (ARMA) were estimated using the particle swarm optimization (PSO) technique based on swarm intelligence. The PSO algorithm adapted with the system identification technique and statistics yielded highly satisfactory results in finding the coefficients of the ARMAX model. All the programming and visualization were performed in a MATLAB environment.

Keywords: *Electroencephalogram (EEG), ARMAX, ARMA, spectral estimation, Particle Swarm Optimization.*

1. Introduction

Electroencephalograph records the electrical activity taken from the human scalp (using sensors of a special type) over a period [1]. From the signal-processing point of view, EEG is a spatial non-stationary stochastic time-series process [2]. EEG waveforms would disclose information about certain changes due to drugs, emotions, thoughts, and diseases. Measurements of these changes available in EEG waveforms in real-time may be used to determine the status and conditions of the subject [3].

Spectral analysis is one of the techniques which aims at analyzing the spectral power in different frequency bands known as the power spectral density (PSD). Knowledge of Power Spectral Density (PSD) in the signal is useful in various situations e.g., detection of a signal masked in wideband noise [4]. Spectral analysis of stochastic signal is performed using either a parametric (model-based) or nonparametric (Fourier transform-based) approach.

This research tests the suitability of a parametric model known as the autoregressive moving average with exogenous input (ARMAX) for a spectral estimate of the EEG signal. To perform this estimation, the coefficients of the ARMAX model needed to be estimated. For this purpose, we propose employing an innovative approach, the Particle Swarm Optimization (PSO) technique. PSO is one of the stochastic optimization techniques. PSO is based on the social and personal behavior of a swarm. The proposed ARMAX model is compared with another parametric model autoregressive moving average (ARMA) and nonparametric approach periodogram.

2. Literature Review

Spectral analysis of the time series signal could be represented in the frequency (spectral) domain using finite Fourier transform (FFT) [5]. The EEG signal is short in length. Short Time Fourier Transform Method (Spectrogram Method) is a traditional approach to computing the spectrum of the short-length signal [6]. As the EEG signal is a non-stationary stochastic signal, the spectrogram "has a serious drawback, which is the implicit assumption of stationary within each segment and unsatisfactory time/frequency resolution" [7].

Instead of a direct Fourier Transform based approach, a periodogram, which is based on the square of the Fourier Transform has been suggested to estimate the PSD of a signal [8][9]. Spectral estimates obtained using estimators like periodograms are not equal to the true spectrum of a signal. However, they resemble the true spectrum. Since the research is concerned with obtaining clear information on power distribution along the frequency domain an estimator that gives a high spectral resolution is considered better than the estimator with a low spectral resolution. Periodograms are indeed the simplest type of nonparametric approach to estimate PSD. The other type is the parametric approach which relies on the model assumed for the signal generation [9]. The periodogram suffers from very high variance and is not a good estimator. Welch modified it by averaging the waveform to reduce variance [10]. The Blackman-Tukey method of periodogram smoothing [11] is another alternative Welch method. Nonparametric methods like periodogram, Welch, and Blackman are criticized as they are limited by the length of data. The consequences of having short data length are listed by Marple [12]. According to him for the signal of length N if two frequencies are separated by Δf, then we need $N \geq \frac{1}{\Delta f}$ data samples to resolve them. Correlation is also assumed to be zero beyond N. The resolution limit imposed by a short length N also causes bias.

In the Parametric methods, the output of a model (a linear system driven by white noise) is viewed as the power spectral density of the signal. E.g., Yule-Walker autoregressive (AR) structure and the Burg method [9]. Parametric methods would produce better resolution than nonparametric methods when applied to a short-length signal [13]. Maghsoudi suggests using the autoregressive moving average (ARMA) models for the EEG analysis [7]. In the method proposed by him, coefficients of the ARMA model are identified and used to depict the waveform. The benefit of the ARMA method (over the nonparametric estimation) is its ability to track the time-varying process [1]. For spectral estimation of EEG signal, Tseng evaluated the performance of parametric methods [14]. He used the Akaike information criterion (AIC) [15] for determining the orders of AR and ARMA models. The tests suggested that the AR model would require a higher model order (8.67 on average) than the ARMA model order (6.17 on average). It was also found that about 96% of the total EEG segments each 1.024 seconds long were efficiently represented by the AR model, and only about 78% could be represented by the ARMA model. He suggested parametric spectral analysis methods based on autoregressive (AR) and autoregressive moving average (ARMA) models as a better approach for the spectral estimates of the EEG signal.

There has not been much scholarly work done with ARMAX in the spectral estimation of EEG, and ARMAX may allow for a more accurate and flexible representation of the EEG time series. This research will focus on applying the "autoregressive moving average with the exogenous input" (ARMAX) method to estimate the spectrum of the EEG and compare its performance with ARMA, AR, and periodogram. Maghsoudi used Extended Least Squares (ELS) and Recursive Extended Least Squares (RELS) methods to estimate the parameters of the first order ARMA model [7]. The identified model was then used to describe the time and frequency-domain properties of the EEG waveforms. This research explores the possibility to estimate the parameters of ARMA and ARMAX models using Particle Swarm Optimization (PSO).

PSO was first introduced by Eberhart and Kennedy in 1995 They described how PSO can be applied to a nonlinear optimization problem through the simulation of a social system characterized by swarms (e.g., bees) [16]. PSO is being used in system identification problems. Huang and colleagues have demonstrated that PSO can be used for identifying an ARMAX Model for Short- term Load Forecasting [17].

3. Spectrum Estimate

Spectral density estimation of a signal would portray the distribution of the power contained in a signal over the frequency within a finite set of data [18]. The two most abundant methods of spectral estimation are as follows [9].

a. Nonparametric methods.

Nonparametric methods use signals to determine the power spectrum density. One such method that is in the scope of this research is Periodogram. If *x[n]* is a finite-length signal, a PSD estimate is the periodogram $P(e^{jw})$ [2] *that is* based on Fourier transform as shown in Eq. 1.

$$P(e^{jw}) = \frac{1}{N} X(e^{jw}) X^*(e)^{jw} = \frac{1}{N} |X(e^{jw})|^2 \quad \text{Eq.1}$$

Here, $X(e^{jw})$ is the Discrete Time Fourier Transfer (DTFT) of *x[n]* and *N* denotes the total number of samples.

b. Parametric methods.

The model represents a relation between a linear system driven by white noise input and the output [3]. The following parametric models are in the scope of this research.

i. AR Model

A simple Autoregressive (AR) model is defined by Eq. 2. The AR model is used when the current output is dependent only on the previous outputs.

$$A(q)y(n) = e(n) \quad \text{Eq. 2}$$

ii. ARMA Model

Combining Autoregressive (AR) and moving average (MA) processes, a highly flexible class of univariate processes called the ARMA [19] as described by Eq 3.

$$A(q)y(n) = e(n)C(q) \quad \text{Eq. 3}$$

iii. ARMAX Model

The autoregressive moving average model with the exogenous inputs (ARMAX model) shown in Eq. 4

$$A(q)y(n) = e(n)C(q) + B(q)u(n-k) \quad \text{Eq. 4}$$

Here, A(q), B(q) and C(q) are the polynomials of the corresponding filters' coefficients.

k is delay,

e(n) indicates white noise.

u(n) represents the deterministic signal.

y(n) indicates the output of the system.

AR model uses only A(q) filter. *e(n)* and y*(n)* indicate white noise input and system output respectively [20]. ARMA model incorporates A(q) filter (Autoregressive component) and C(q) filter (Moving Average component). ARMAX incorporates ARMA and the disturbance dynamics known as exogenous input e(n) [21].

Let's, n_a, n_b, and n_c are the orders of the A, B and C; therefore, filter polynomials A, B, and C can be written as Eq 5.

$$A(q) = 1 + a_1 q^{-1} + \cdots + a_{n_a} q^{-n_a}$$

$$B(q) = b_1 + b_2 q^{-1} + \cdots + b_{n_b} q^{-n_b+1} \quad \text{Eq. 5}$$

$$C(q) = 1 + c_1 q^{-1} + \cdots + c_{n_c} q^{-n_c}$$

In this research parameters for the AR model are obtained using the modified covariance (mcov) method described by Hayes [22]. Parameters for ARMA and ARMAX are obtained using the Particle Swarm Optimization (PSO) algorithm.

Particle Swarm Optimization

The Basic PSO algorithm [23] is described by Eq. 6.

$$v_{k+1}^{i} = w v_{k}^{i} + \alpha_1 u_1 \times \left[Pbest_{k}^{i} - R_{k}^{i}\right] + \alpha_2 u_2 \times \left[Gbest^{i} - R_{k}^{i}\right] \quad \text{Eq. 6-1}$$

$$R_{k+1}^{i} = R_{k}^{i} + v_{k+1}^{i} \quad \text{Eq. 6-2}$$

R_{k}^{i} Position of the i[th] particle during k[th] iteration.

v_{k}^{i} Velocity of the i[th] particle during k[th] iteration.

$Pbest_{k}^{i}$ Individual best fitness achieved by a i[th] particle till k[th] iteration.

$Gbest^{i}$ Global best fitness of i[th] particle till k[th] iteration.

w inertial coefficient.

α_1 and α_2 Cognitive and social parameters.

u_1 and u_2 Random numbers between 0 and 1.

PSO Algorithm Flow Diagram

The PSO algorithm flow diagram is shown in Fig. 1 [24] [25] [26].

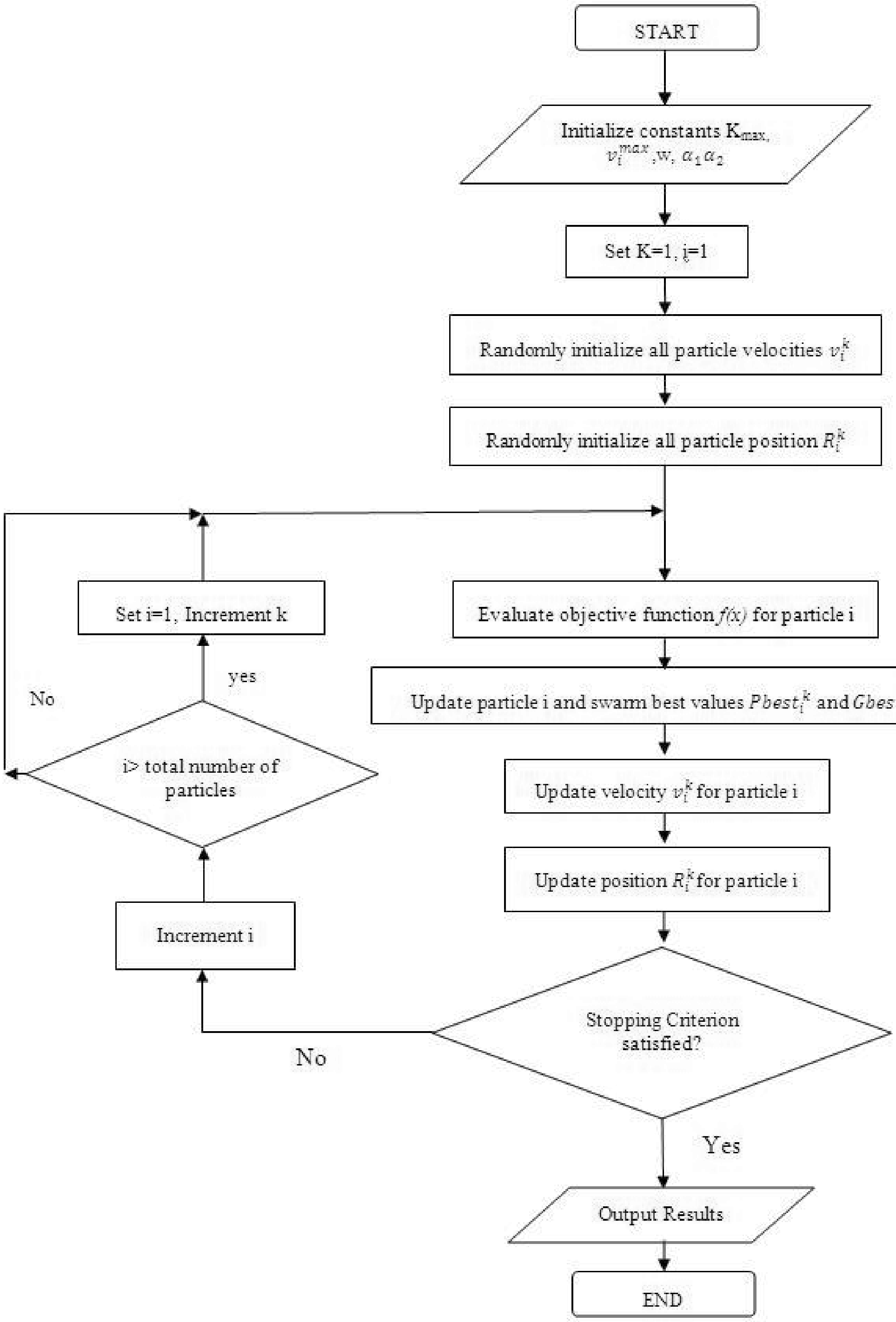

Fig 1. PSO Algorithm Flow Diagram.

Loss function

The loss function of i[th] model R_i is calculated using a prediction error $\varepsilon(n)$, which is a nX1 vector of prediction errors and a transpose of $\varepsilon(n)$ [27] as follows:

$$lf(i) = \det[\frac{1}{n}(\varepsilon(n) \times \varepsilon(n)^{\mathrm{T}})] \qquad \text{Eq. 7}$$

Prediction Error of a model at position R_i (PE) = $\varepsilon(\mathrm{n}) = \mathrm{y}(\mathrm{n}) - \hat{\mathrm{y}}(\mathrm{n})$ Eq. 8

y(n) is the target data and $\hat{y}(n)$ is the predicted output of the ARMAX model R_i. A model producing a small prediction error is considered a good model [4].

Akaike Final Prediction Error (AFPE)

Akaike's Final Prediction Error (FPE) criterion provides a measure of the model where the model is tested on a different data set [15]. After computing all the models at all positions, the most accurate model having the smallest AFPE is obtained using Eq. 9.

$$AFPE = lf(t, \theta) \times \frac{1+\frac{n}{d}}{1-\frac{n}{d}} \qquad \text{Eq. 9}$$

n is the number of values in the estimation data set.

θ represents the estimated parameters.

d is the number of estimated parameters. Order of model coefficient.

The AFPE of each R_i is the accuracy for the fit termed 'Swarm fitness' of the swarm R_i. Among all the Swarm fitness of R_i, at K^{th} iteration, the swarm fitness that is the absolute minimum (i.e., nearest to zero) is called the 'global best swarm fitness' ($Gbest_k$).

Mean Square Error

The Mean Square Error (MSE) between validation data and the model output is obtained by using Eq. 10.

$$\mathrm{MSE}(n) = \frac{1}{N}(V(n) - \hat{y}(n))^2 \qquad \text{Eq. 10}$$

When *V(n)* is the validation data, $\hat{y}(n)$ is the predicted output of the ARMAX model with $[\bar{A}\ \bar{B}\ \bar{C}]$, and *N* is the length of data. The combination of the Akaike final prediction error [15] and Cramer's rule of determinant [28] yields Eq. 11 giving us a single-valued score of the MSE.

$$MSE(det) = \mathrm{abs}[det(\frac{1}{N} MSE \times MSE^{\mathrm{T}})] \qquad \text{Eq. 11}$$

MSE(det) is calculated from the score taken from the best model ARMA/X parameters. This score signifies how well the model output may resemble the validation data.

4. Methodology

The process of estimating the parameters of the model with the real world's experimental input-output data is based on the statistical theory [7]. This research experiment follows the steps prescribed by Ljung [27].

EEG Signal

The EEG data used in this research as input was obtained from Henri Begleiter [29] from the Neurodynamics Laboratory at the State University of New York Health Center in Brooklyn. These data were recorded to examine EEG correlates of genetic predisposition to alcoholism. It contains measurements from 64 electrodes placed on the subject's scalps which were sampled at 256 Hz (3.9-msec epoch) [29][8]. For our experiment, we used 0.1172 seconds length of signal from the randomly selected data set. An example of the experimental data is shown in Fig. 2.

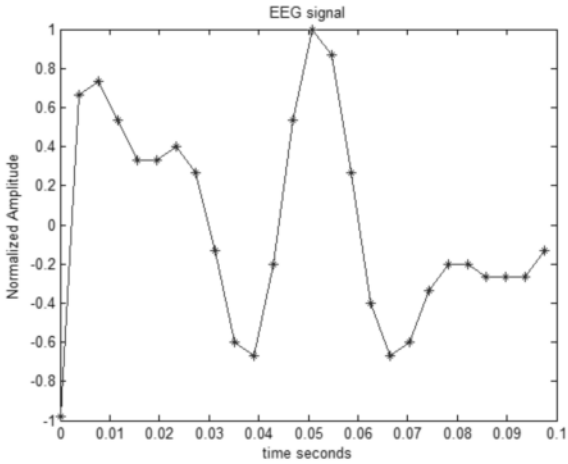

Fig 2. EEG Signal

White Noise

The discrete-time random process represented by a random vector *w* is a white noise was generated.

Exogenous Input

Unlike ARMA, an ARMAX model requires an exogenous input. This input should be a deterministic signal. In this study, the exogenous input $u(t)$ was formed by the summation of three sinusoids of 5 Hz*, 10 Hz* and *18 Hz* and sampled with sampling frequency f_s = 256 Hz (like EEG signal) for 30 epochs and total length of 0.1172 Seconds.

Models

First, we needed to verify that our algorithm would work properly. For this purpose, the ARMAX model with the following arbitrarily selected parameters was created. We call it ARMAX (A, B, C).

A= [1 -0.1400 -0.4244 0.3963 0.3921 0.3764 0.0796 0.0321]

B = [-0.2347 0.1348 -0.2084 -0.0903 0.3789 -0.0220 -0.1044 0.0021]

C = [1 0.0137 0.1123 0.1459 -0.0266 -0.1099 -0.1540 0.0419] Eq. 12

Eq. 12 represents the ARMAX [A, B, C] model of order 7. The model was simulated with exogenous input *u(n)* and white noise *e(n)*. The output of the model *y(n)* was recorded along with *u(n)* in a data set Z_e, as shown in Eq. 13. The training data set of one hundred (a pair of exogenous input *u(n)* and *y(n))* were created.

Training Data = Z_e ={ [y_1(n) u_1(n)], [y_2(n) u_2(n)], [y_3(n) u_3(n)]…..[y_i (n),u_i(n)]} Eq. 13

PSO Based Parameter Estimation

With the training data sets Z_e and noise *e(n),* ARMAX model parameters were estimated using the PSO algorithm. Parameters estimated using PSO are referred to as $\overline{A}$, $\overline{B}$ and $\overline{C}$.

i. PSO Initialization

Inertial coefficient *w* was chosen between 0.8 and 1, Cognitive coefficient α_1(step size) was chosen close to 2 and the social coefficient α_2(step size) was also chosen to 2 as suggested by Kennedy [30]. The initial trial parameter vectors R_i were randomly generated, where $R_i = uniform(a,b)^d$, $i=1,2\ldots k$. and $d = n+m+r$. Here*, k= 50* and *n, m,* and *r* were the orders of filters *A, B,* and *C* respectively. Each column

(R_i) ARMAX model is simulated with white noise and exogenous input to obtain output $\hat{y}_i$. Data sets $\hat{Z}_i$ containing pair of u_i and $\hat{y}_i$ were created and is called swarm output.

$$\hat{Z}_i = ([\hat{y}_1(n) \; u_1(n)], [\hat{y}_2(n) \; u_2(n)], [\hat{y}_3(n) \; u_2(n)] \ldots.. [\hat{y}_i(n), u_i(n)] \qquad \text{Eq. 14}$$

ii.Loss Function

The difference between the swarm output $\hat{y}$ and the output y of the training data Eq. 13 was used to compute the loss function and overall AFPE. *pbest*^{i} and *gbest*^{i} is computed and updated which results in a new swarm of matrix $\hat{R}_i$. Each column of $\hat{R}_i$ gets a new ARMAX [$\bar{A}$ $\bar{B}$ $\bar{C}$] model. All the above steps were repeated for 50 iterations.

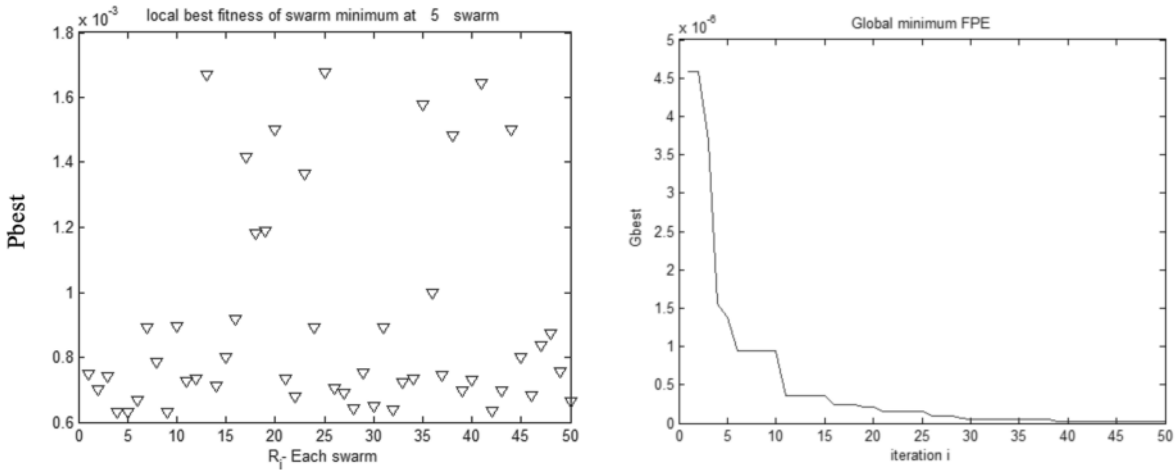

Fig 3. Pbest and Gbest

Fig 3 shows that at each step of the swarm (at each iteration), $Gbest$ is decreasing.

iii.Validation

The best solution obtained using PSO algorithm must pass the validation test. To validate the model, the validation data *V(n)* is created similarly to the training data. This data should never be used to train the model. The model is simulated, and this time the output of the model is compared with *V(n)* instead of target data *y(n)*. Fig. 4-1 demonstrates the validation data from the initial coefficients [A B C] and simulated output of the PSO-based coefficients [$\bar{A}$ $\bar{B}$ $\bar{C}$].

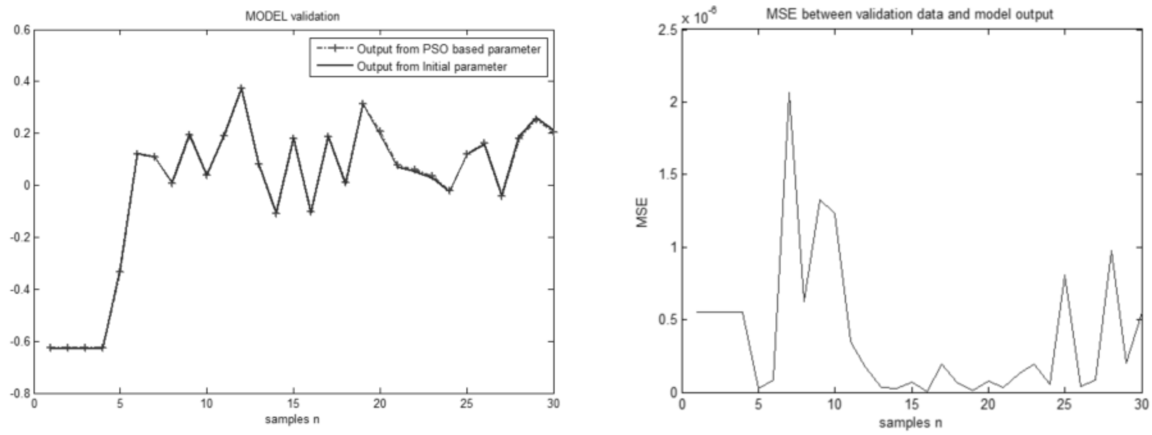

Fig 4-1. Model Validation

Fig 4-2. MSE During Model Validation

It can be observed that the model output follows the validation data very closely. Fig 4-2 shows the mean square error (MSE) defined by Eq 11. Small MSE between $V(n)$ and $\hat{y}(n)$ indicates that the performance of the estimated model is satisfactory with a good model fit.

The process of the ARMAX parameter estimation and validation was repeated for 100 independent data sets. $MSE(det)$ was computed to record the MSE as a single-valued score. The average for the *MSE(det)* was found to be as small as 2.42 E-06. This result indicates the functionality and convergence of the PSO algorithm and ARMAX model.

iv.Repeat

Once the PSO technique was validated, we used real EEG signals to estimate the ARMAX model parameters. Above steps were repeated for ARMAX of orders 3, and 5 and ARMA of orders 3, 5, and 7. Models of Orders 5 and 3 may be obtained by excluding appropriate higher-order coefficients from the filters A, B, and C.

5. Result and discussion

Spectral Analysis

Analyzing the model enables us to produce a spectrum of that model that can be used as a spectral estimate of a real EEG signal.

Frequency Response of Filters A, B, and C

The ARMAX parameters resemble three filters viz. A, B and C. The frequency response of each filter was obtained with 512 points discrete Fourier transform. The frequency responses of filters *B* and *C* are shown in Fig 5. The product of filter *B* and the exogenous input and the product of noise and filter *C* are also shown in the figure.

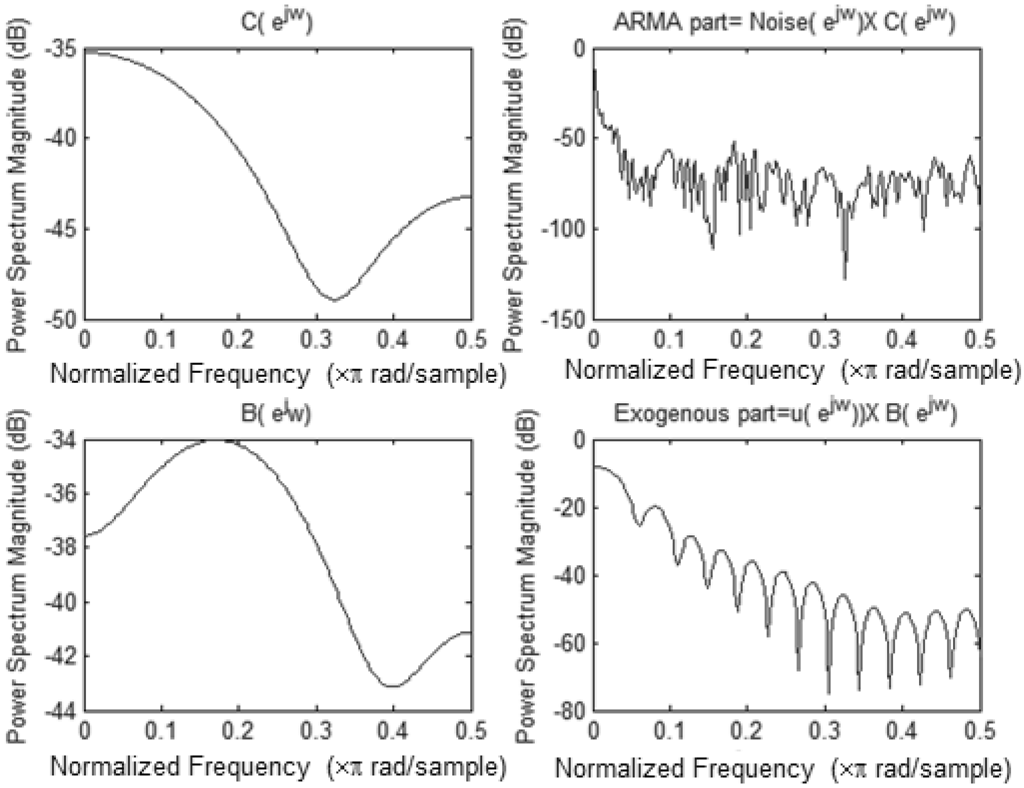

Fig 5. Frequency Response of filters C and B.

Fig 5 shows that filter C is a low-pass filter. Since the EEG may be viewed as a signal having most information within the lower frequency band, it is reasonably predictable that filter *C* is a low-pass filter. The sum of an ARMA part and an exogenous part is shown in Fig 6. This sum is filtered by filter *A*. The frequency response of filter *A* is also shown in Fig 6.

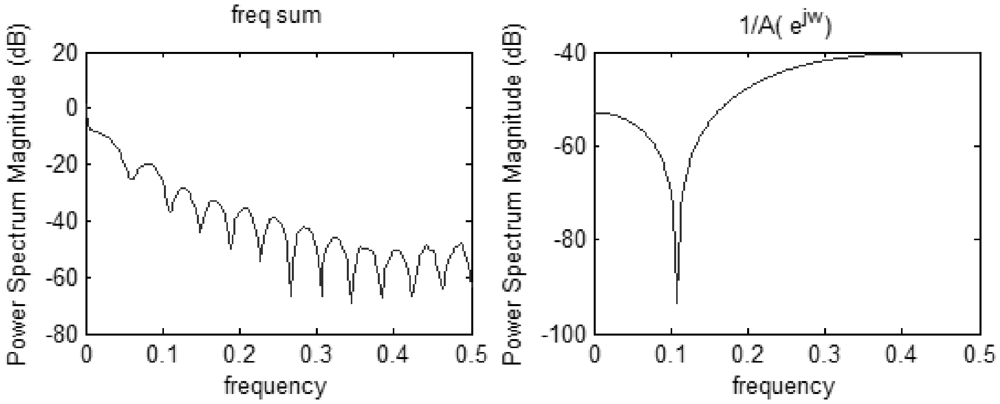

Fig 6. Frequency Response of freq sum and filter A

It could be seen in Fig 6 that filter *1/A* is a band stop filter. Filter responses of these filters are not distinct; they vary with the length of data.

ARMAX Model spectrum estimate

When the sum of the exogenous part and the ARMA part is filtered with filter *A,* the final output of the ARMAX model is obtained. This output is the spectral estimate of the EEG. The output of the third-order ARMAX model for 0.2 seconds (51 samples) of the EEG data is shown in Figure 7-1.

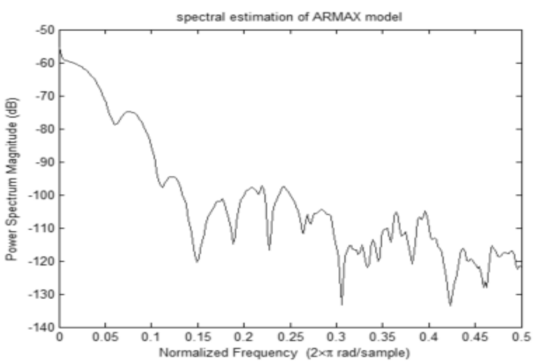

Fig 0-1. Order 3 ARMAX

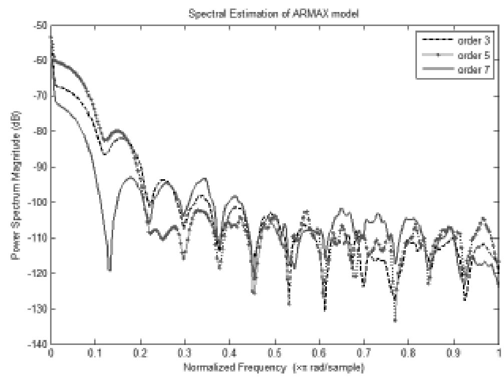

Fig 0-2. Order 3, 5, 7 ARMAX

Similar experiments were performed for the model orders 3, 5, and 7. The results of all three orders are compared in Fig 7-2 for the same EEG signal. An increase in the model order results in spectral estimates of a seemingly higher resolution.

Comparison with the ARMA

The ARMA is a special case of an ARMAX model. By keeping an exogenous input equal to zero, the same procedure used for the parameter estimation of ARMAX was implemented to estimate the parameters of an ARMA model of order 3.

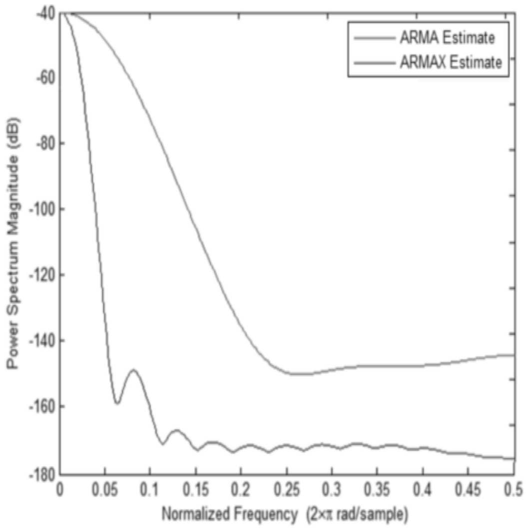

Fig 8-1. Order 3 ARMA and ARMAX

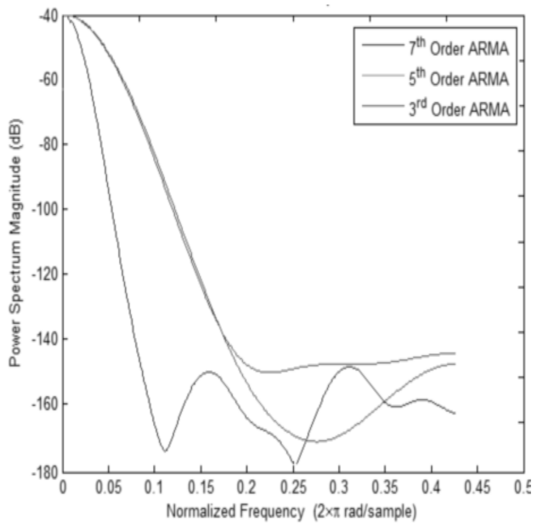

Fig 8-2 Order 3, 5, 7 ARMA and ARMAX.

Fig 8-1 shows that the ARMA model-based estimator has depicted only the general outline of the power distribution. Whereas the ARMAX model-based estimator was able to distinguish between different components of the spectrum. Fig 8-2. shows as the model order increases, spectral resolution becomes better. EEG contains most of its information in lower frequencies; therefore, a better resolution at lower frequencies is desirable for an EEG analysis. Tseng suggested that higher-order ARMA models are needed to represent an EEG signal [9][14]. Our results agree with Tseng's conclusion.

Comparison with the AR and with the Periodogram method.

Next, the comparison of an ARMAX-based spectral estimator was performed with an AR model and the periodogram method. The modified covariance technique was used to estimate the coefficients of the AR model. Comparisons were conducted for different orders of AR and ARMAX models using different fragments of EEG sequence.

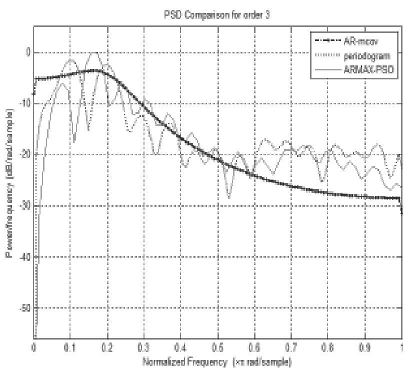

Fig 9-1 PSD Comparison for Order 3

AR model order =3
ARMAX model order =3
Data length = 0.13 Seconds
Number of samples= 34.

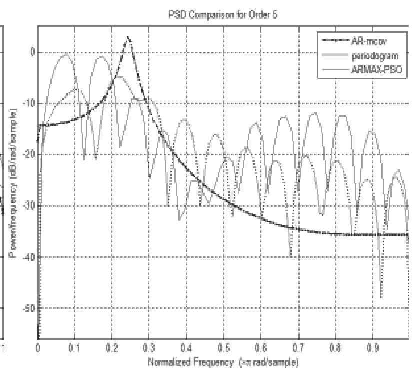

Fig 9-2 PSD Comparison for Order 5

AR model order =5
ARMAX model order =5
Data length = 0.1 Seconds
Number of samples= 26

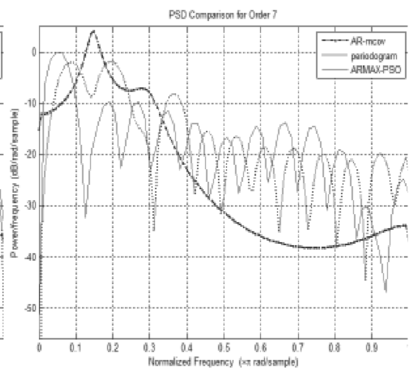

Fig 9-3 PSD Comparison for Order 7

AR model order =7
ARMAX model order =7
Data length = 0.9 Seconds.
Number of samples= 24.

Figures 9[1-3] indicate that the ARMAX-based estimates show a higher resolution. The AR method has only depicted general trends in power distribution. Like ARMA, the low-order AR models are barely suitable for an EEG spectral analysis. A DFT-based periodogram method shows a higher-resolution estimate of power distribution. However, a close inspection reveals that the periodogram at lower frequencies is unable to resolve different spectral components. The lack of distinct components at lower frequencies would make the periodogram not ideal for EEG analysis.

6. Conclusion

The EEG spectral estimate obtained by the proposed method was able to depict distinct spectral components not resolved by other methods, even compared to models of higher orders. ARMAX models of the order three were generally found suitable for EEG analysis, while the ARMA-based method generally required seventh order to produce decent spectral estimates. AR-based analyzers were unable to produce spectral estimates of sufficiently high resolution. The periodogram-based estimator, while showing decent spectral resolution for high-frequency content, failed to resolve low-frequency EEG components.

Based on these observations, we recommend the ARMAX model for spectral analysis of EEG.

7. Suggestions and recommendations

In the future, the selection of the model order of a proposed method needs to be optimized. Such an optimization would allow for avoiding over-ordering the estimator while maintaining sufficient spectral resolution. Perhaps, traditional order selection criteria (Akaike or MDL, for instance) may be modified for the ARMAX.

The study of the effects of short data fragments (while using a model parameters estimation process) is another important area of future research. Reliable spectral estimation for short data sequences is of great interest in many fields. Therefore, knowing the practical limits of the proposed method (i.e., the minimum duration of data sequence still yielding acceptable estimates) would be important for its applications.

This research may result in the development of a common tool capable to estimate the spectrum of all the channels of EEG simultaneously. The latter may be used to identify the characteristics of interest in the EEG.

Finally, the proposed spectral estimation algorithm may also be applied for the analysis of other naturally generated signals, such as speech or seismological data. Appropriate tests would be necessary before recommending the ARMAX model for those applications.

8. Acknowledgments

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