Himalayan Physics

Stern-Gerlach centennial: Parity, gradient effect, and an analogy with the Higgs field

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J.D. Bulnes[1,2], M.A.I. Travassos[1], D.A. Juraev[2,3], J.E. Ottoni[4], J. López-Bonilla[5]*

1 Department of Exact and Technological Sciences, Federal University of Amapá, Rod. J. Kubitschek, 68903-419, Macapá, AP, Brazil

2 Department of Scientific Research, Innovation and Training of Scientific and Pedagogical Staff, University of Economics and Pedagogy, Karshi, 180100, Uzbekistan

3 Department of Mathematics, Anand International College of Engineering, Near Kanota, Agra Road, Jaipur-303012, Rajasthan, India

4 Department of Physics and Mathematics, Federal University of São João del-Rei, Ouro Branco, MG, Brazil

5 ESIME−Zacatenco, Instituto Politécnico Nacional, Edif. 4, 1er. Piso, Col. Lindavista CP 07738, CDMX, México

Abstract: In this article, which celebrates 100 years of the Stern-Gerlach experiment, we identify and discuss some limitations of the mathematical field we intend to represent as the Stern-Gerlach magnetic field. We extend some recent theoretical results concerning Stern-Gerlach eigenenergies, where what we call "the gradient effect" manifests itself: the contribution of the magnetic field gradient to the self-energies. Finally, based on an analogy with the Stern-Gerlach effect, we visualized that the Higgs field must be homogeneous in its minimum energy configuration within the context of the Higgs mass generation mechanism.

Keywords: Stern-Gerlach effect • Parity transformation • Higgs field

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I. Introduction

Recently, the scientific community celebrated the centenary of the Stern-Gerlach experiment [1–3], designed by Prof. O. Stern in 1921 and carried out by experimental physicist W. Gerlach and his support team (a graduate student and a skilled laboratory technician).

Although the Stern-Gerlach experiment was designed to provide proof against the discretization of the projections of the angular momentum vector (orbital) of the electron on the atom, as Prof. N. Bohr had postulated in his atomic model[1] , and of which Prof. Stern did not agree. He discovered, to the surprise of physicists at the time, what was later interpreted, within the context of quantum mechanics,

* *Corresponding Author: joseluis.lopezbonilla@gmail.com*

[1] *The Prof. W. Heisenberg stated [4] that Bohr's theory, even around 1926, was not well regarded by many experimental physicists, which they called "atom mysticism".*

as the equivalent[2] of such a discretization[3], but for a distinct angular momentum, called intrinsic or spin, which was not known at the time of the experiment (1922).

The luck that embraced the Stern-Gerlach experiment, allowing the spin to reveal itself indirectly, was a magnetic field acting in the experiment, with which the spin coupled. Of course, the delicate and very careful nature of the experimental performance was also important, without which it would not have been possible to observe the effect [5].

Gerlach's experimental observations focused on the spatial splitting (occurring in a vertical plane, which in Figure 1 corresponds to Y = 0) of a beam of silver atoms[4] into two secondary beams, which, consistent with the quantum mechanical description, resulted from the magnetic interaction between the electron's spin magnetic moment (in the state with orbital quantum number l = 0), with an external magnetic field characterized by an intense gradient. This effect (in the vertical plane) is presented in several quantum mechanics texts [6–35], and each unfolded beam, there are only electrons in the same spin state: "up" in one beam and "down" in the other beam.

It is noted in those books that nothing is commented on concerning the unfolding of the initial beam in other planes; for example, there is no mathematical development for the spatial separation into two secondary beams in the horizontal plane (Z = 0, see Fig. 2). Nor are there any clarifications regarding the energetic separation (if it happens) by spin states in these unfolded beams. We will return to the abovementioned problems in section 2 and subsection 2.1.

Fig. 1 shows the general situation of the initial beam being separated into two secondary beams in the vertical plane. We have omitted the magnet to simplify the figure. Subsequently, approaches to various aspects of the Stern-Gerlach effect were developed [36–43].

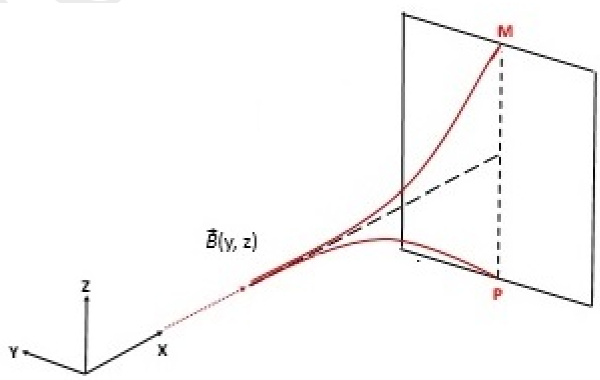

Figure 1. Separation of an incident atomic beam into two specific secondary beams, all contained in the $Y = 0$ plane, as a manifestation of the Stern-Gerlach effect. Points M and P correspond to the intersection of the secondary beams with the screen (in the figure we have omitted the source that generates the magnetic field).

[2] *In quantum mechanics they would correspond to the eigenvalues of the third component operator of the spin angular momentum vector operator.*
[3] *Relative to a reference direction defined by the externally applied magnetic field.*
[4] *Or in the variant of Phipps and Taylor [3], with hydrogen atoms with their only electron in the ground state:* $n = 1$, $l = 0$.

Furthermore, it is interesting to mention that the literature contains solutions to problems on different aspects of quantum mechanics [44–51] and developments in mathematical physics [52–63] that could find application in more formal aspects of quantum theory.

II. Inevitable deficiency in the approximate representation of the Stern-Gerlach magnetic field

For a mathematical field to be able to model the physical field within the region between the poles of the Stern-Gerlach magnet, considering the coordinate directions as shown in Fig. 1, it would have to have, according to [12], be in the following general form,

$$\mathbf{B}(y,z) = B_1(y,z)\hat{\jmath} + (B_0 + B_2(y,z))\hat{k}, \tag{1}$$

but that would not be enough. The field in (1) will correspond to a physical (magnetic) field if it meets specific requirements compatible with physics; for example, the field should verify the equation of Gauss's law of magnetostatics (1st physical requirement),

$$\nabla \cdot \mathbf{B} = 0, \tag{2}$$

or, also including $\nabla \times \mathbf{B} = \mathbf{0}$,

$$\frac{\partial B_1}{\partial y} = -\frac{\partial B_2}{\partial z} \qquad \& \qquad \frac{\partial B_2}{\partial y} = \frac{\partial B_1}{\partial z}.$$

Hence, we have that the simplest field, with magnitude gradient α, along the Y and Z coordinate directions, has the following form [31],

$$\mathbf{B}(y,z) = -\alpha y\hat{\jmath} + (B_0 + \alpha z)\hat{k}, \tag{3}$$

with B_0 being the homogeneous component and α the gradient of the magnetic field.

The field in (3), which could be adequate in a simple context, must be taken as a rough approximation for the Stern-Gerlach magnetic field, as it does not present an important characteristic of magnetic fields in general, that is (2nd requirement physics): present a pseudo-vector character in (1+3) dimensional space; that is, be invariant with respect to the parity transformation. Let's look at this.

Applying a parity transformation under the field in (3) we have,

$$\mathbf{B}(y,z) \quad \Rightarrow \quad \mathbf{B}'(-y,-z) = -\mathbf{B}(y,z) + 2B_0\hat{k}, \tag{4}$$

Wherein, it can be seen that it's not possible to manifest pseudo-vector behavior; that is, we do not have: $\mathbf{B}'(-y,-z) = \mathbf{B}(y,z)$. The mathematical field (3), however, behaves like a vector field if we assume (only mathematically) that $B_0 = 0$. Of course, making $B_0 = 0$ in (4) we have: $\mathbf{B}'(-y,-z) = -\,\mathbf{B}(y,z)$, which corresponds to a parity transformation for a vector field; thus, field $\mathbf{B}$ reduces to the expression,

$$\mathbf{B}(y,z) = -\alpha y\hat{j} + \alpha z\hat{k}. \tag{5}$$

On the other hand, in the Stern-Gerlach experiment, the energetic separation of the atomic electrons in the incident beam is determined by the homogeneous component (B_0) of the magnetic field, but the spatial separation of the initial beam into two secondary beams is determined by the gradient (α) of the magnetic field. So, since the directions of the spatial coordinates are so closely linked with a parity transformation, we have that a vector representation of the magnetic field (with $B_0 = 0$) could only be adequate in spatial problems related to the Stern-Gerlach experiment where the component B_0 is irrelevant. For example, to solve the Pauli equation considering the separation of the incident beam into two secondary beams contained in a plane other than the vertical.

It should be noted that the equation of Gauss's law of magnetostatics $\nabla \cdot \mathbf{B} = 0$, with $\mathbf{B}$ given in (5), respects parity symmetry if $\mathbf{B}(y,z)$ behaves as a vector under the transformation of parity:

$$\nabla \cdot \mathbf{B}(y,z) \quad \Rightarrow \quad \nabla' \cdot \mathbf{B}'(-y,-z) = -\nabla \cdot \Big(-\mathbf{B}(y,z)\Big) = \nabla \cdot \mathbf{B}(y,z) = 0.$$

$$\rightarrow \quad \nabla' \cdot \mathbf{B}' = 0. \tag{6}$$

As mentioned so far, we have, for example, that in the energy problem of determining the contribution of the magnetic field gradient to the Stern-Gerlach eigenenergies, we should not take $B_0 = 0$; that is, we can consider the field given in (3) as a sufficient approximation, although the fact that it has neither a pseudo-vector nor a vector character; this problem was solved in [42]. On the other hand, in the problem to determine the solutions of the Pauli equation for the states of the atomic electrons in two beams contained in the horizontal plane ($Z = 0$), the approximation with $B_0 = 0$, that is, using the field given in (5), is adequate, as shown in the appendix in [46]. These two approximations, despite their compatibility with physical requirements (partial in one case, complete in the other case), only approximately represent a Stern-Gerlach magnetic field, as details of the experimentally generated field are beyond simple mathematical representation.

We still have one issue to clarify. In the case of representing the Stern-Gerlach magnetic field by a vector field, as in (5), the invariance of Maxwell's equations in the face of a parity transformation could be broken, leading to a possible inconsistency that would not be insignificant; however, this is not the case. Let's look at this.

In the experimental context of the Stern-Gerlach effect, the following physical quantities are fixed: $\rho = 0$, $\mathbf{E} = \mathbf{0}$, $\mathbf{j} = \mathbf{0}$, so that, for example, the Ampere-Maxwell law equation reduces to the expression:

$$\nabla \times \mathbf{B} = \mathbf{0}, \tag{7}$$

the same one that does not change under parity transformation, even in the case where **B** is represented by a vector field. In the same way, the Faraday-Lenz law equation does not lose symmetry due to parity when **B** is represented by a vector field in the Stern-Gerlach context. This provides additional support for the validity of the representation given in (5).

What is the meaning of taking $B_0 = 0$?

When considering $B_0 = 0$ in expression (3) to generate (5), we are obviously not changing the physics of the magnetic field. Otherwise, we are using a mathematical resource valid in a certain context: constructing an approximation mathematics. Let's look at this. We know that every approximate expression (in the broadest sense) contains less information than the corresponding exact expression. In different approaches, different information will be missing[5].

Expression (5) is an approximation in which information aboutB_0 is missing, while the approximation given in (3) depends on information about the value of $B_0 = 0$. The difference between the situations that use approximations in which we have no control over the missing information[6] and ours is that here, we have constructed the approximation (based on certain physical requirements) so that we know what information is contained in it.

On the other hand, a situation in which it is convenient to take, precisely, $B_0 = 0$, that is, to represent the magnetic field as a vector field, is when solving the spatial problem in the horizontal plane $Z = 0$ (Fig. 2) via the Pauli equation, as shown in [46]. In this situation, we have a spatial separation without energetic separation[7] since electrons are found in both the "up" spin state and the "down" spin state in both secondary beams.

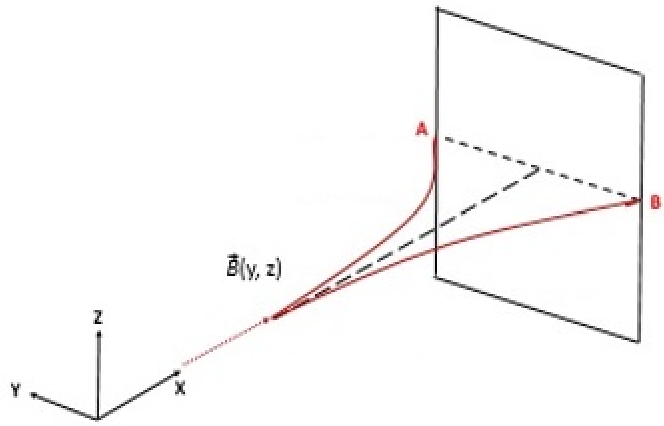

Figure 2. Separation of an incident atomic beam into two secondary beams as a manifestation of the Stern-Gerlach effect. Points A and B correspond to the intersection of the secondary beams with the bulkhead.

5 *So you wouldn't be able to find what you expected using a certain approximation!*
6 *And this loss of information cannot affect the physical system considered.*
7 *Which, if it appears, would only be compatible with $B_0 \neq 0$.*

III. The gradient effect

The contribution of the field gradient to eigenenergy

Reference [44] presents the calculations determining the eigenenergies for an electron moving in a helical magnetic field. These energies depend on the pitch of the helix, a characteristic of the field's inhomogeneity. This result served to look for a similar solution for the eigenenergies of atomic electrons in the Stern-Gerlach experiment, making it possible to show that such a solution exists [42]. It has been shown mathematically [42], in the case of electrically neutral silver atoms, with spin $S = 1/2$, entering a Stern-Gerlach magnet, that their eigenenergies carry a very small contribution from the magnetic field gradient, which produces displacements of the energy levels in quantity:

$$\xi_0 \left(\mu_B^2 \alpha^2 \hbar^2 / 2m\right)^{1/3}, \tag{8}$$

where μ_B is the Bohr magneton, α the magnitude of the field gradient, $\hbar$ the Planck constant, m the mass of the electron, and ξ_0 a constant. The Stern-Gerlach eigenenergies, incorporating the "gradient effect," are given by the expressions,

$$E_{-1/2} = \frac{P_x^2}{2m} - \mu_B B_0 - \xi_0 \left(\frac{\mu_B^2 \alpha^2 \hbar^2}{2m} \right)^{1/3}, \tag{9}$$

$$E_{+1/2} = \frac{P_x^2}{2m} + \mu_B B_0 - \xi_0 \left(\frac{\mu_B^2 \alpha^2 \hbar^2}{2m} \right)^{1/3}, \tag{10}$$

where B_0 is the homogeneous component of the magnetic field. Note in (9) and (10), just for the sake of consistency, that the gradient-dependent term has energy dimensions. The generalization for the case of atoms with any spin value S (but fixed for all atoms in the beam), with $2m_s + 1$ eigenenergies (with m_s being the magnetic quantum number), these are given by the expression,

$$E_{m_s} = \frac{P_x^2}{2m} + 2m_s \mu_B B_0 - \xi_0 \left(\frac{4m_s^2 \mu_B^2 \alpha^2 \hbar^2}{2m} \right)^{1/3}, \tag{11}$$

This result can be found in [43].

The physical Higgs[8] field and an analogy

The founders of non-relativistic quantum mechanics did not include mass (of a particle) in the list of physical observables. For clarification only, it is considered a mass observable.

[8] *Unfortunately for the scientific community, Prof. P. Higgs, who won the Nobel Prize in Physics in 2013, recently passed away.*

The state of a massive quantum particle could be expressed as a linear combination of the eigenstates of the corresponding mass operator. In general, this would mean that the particle, despite the value of its mass being included in the Schroedinger equation, could not be assigned a well-defined mass according to the Copenhagen interpretation.

On the other hand, within the scope of the most fundamental physical theories, among the initial considerations of their construction and to preserve a certain symmetry, it is assumed that the particles are not massive, despite them having a well-defined mass.

In the case of the foundations of the theory for electroweak unification, proposed by Professors S. Glashow, S. Weinberg, and A. Salam [64–67], electrons and their neutrinos, in particular, are considered massless. Due to its coupling with the so-called Higgs field, the electron acquires mass. This interaction between the electron and the Higgs field happens via a certain symmetry-breaking Higgs mechanism, but neutrinos will remain massless because they do not interact with this field.

Considering sections 3.1, and independently of the formal issues of electroweak theory, we can partially understand one aspect of it using an analogy with the Stern-Gerlach case: the Higgs field is expected to be homogeneous in the context in which electrons couple to this one and gain mass; otherwise (as we saw in the Stern-Gerlach case), an additional shift in energy (or mass) levels would be expected due to a gradient effect. In this case, the electron would have a different mass.

In fact, the Higgs field, which must be homogeneous in its minimum energy configuration in order not to violate the translational symmetry of the vacuum state, acts as if it were the homogeneous component of the Stern-Gerlach field and produces the split between the mass of the electron and its neutrino.

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