

Electromagnetic Mass and Violation of Newton's Law

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When an electron having all charge (q) distributed uniformly on its surface of sphere of radius (a), is at rest, no magnetic field is associated with it. Then the energy per unit volume of the field is given by

$$\begin{aligned} u &= \frac{11}{22} \epsilon_0 E^2 \\ &= \frac{\epsilon_0}{2} \times \left(\frac{1}{4\pi\epsilon_0} \frac{q}{a^2} \right) \frac{\epsilon_0}{2} \times \left(\frac{1}{4\pi\epsilon_0} \frac{q}{a^2} \right)^2 \\ &= \frac{1}{2} \epsilon_0 \times \frac{1}{16\pi^2\epsilon_0^2} \frac{q^2}{a^4} \frac{1}{2} \epsilon_0 \times \frac{1}{16\pi^2\epsilon_0^2} \frac{q^2}{a^4} \\ &= \frac{1}{32\pi^2\epsilon_0} \frac{q^2}{a^4} \frac{1}{32\pi^2\epsilon_0} \frac{q^2}{a^4} \end{aligned}$$

Total energy = $U_{\text{electrical}} = \int u \, 4\pi a^2 da \int u \, 4\pi a^2 da$

$$\begin{aligned} &= \int_a^\infty \frac{1}{32\pi^2\epsilon_0} \frac{q^2}{a^4} \times \int_a^\infty \frac{1}{32\pi^2\epsilon_0} \frac{q^2}{a^4} \times 4\pi a^2 da \pi a^2 da \\ &= \frac{1}{2} \frac{q^2}{4\pi\epsilon_0 a} \frac{1}{2} \frac{q^2}{4\pi\epsilon_0 a} \end{aligned}$$

Which becomes infinite if $a=0$ (point charge) indicating that there is infinite amount of energy in the field surrounding a point charge at rest, which is a matter of trouble. To analyze the problem, it is necessary to calculate the change in energy when the charged particle is moved with $v < c$. According to the theory of electrodynamics, momentum must be

same as the energy multiplied by $\frac{v}{c^2}$. The momentum density is given by

$$\begin{aligned} \vec{g} &= \epsilon_0 \vec{E} \times \vec{B} \\ \vec{g} &= \epsilon_0 \vec{E} \times \left(\frac{\vec{v}}{c^2} \times \vec{E} \right) \end{aligned}$$

This on integrating over volume gives momentum having value

$$\vec{p} = \frac{2}{3} \frac{q^2}{4\pi\epsilon_0} \frac{v}{ac^2} \vec{p} = \frac{2}{3} \frac{q^2}{4\pi\epsilon_0} \frac{v}{ac^2}$$

It shows that a charged particle carries a momentum

which is proportional to its velocity just from the electromagnetic influence. The coefficient of velocity in the expression of momentum is called mass, so the coefficient of v on right side should indicate the mass which arises only due to the effect of electrodynamics and is called electromagnetic mass i.e.

$$\begin{aligned} m_{\text{electromagnetic}} &= \frac{2}{3} \frac{q^2}{4\pi\epsilon_0} \frac{1}{ac^2} \\ m_{\text{electromagnetic}} &= \frac{2}{3} \frac{q^2}{4\pi\epsilon_0} \frac{1}{ac^2} \end{aligned}$$

This implies that the total mass of a charged particle in motion with small velocity is sum of the mechanical and the electromagnetic masses.

$$m_{\text{total}} = m_{\text{electromagnetic}} + m_{\text{mechanical}}$$

For the total mass to be electromagnetic,

$$\begin{aligned} m_{\text{total}} &= m_{\text{electromagnetic}} \\ m_{\text{total}} &= \frac{2}{3} \frac{q^2}{4\pi\epsilon_0} \frac{1}{ac^2} \\ a &= \frac{2}{3} \frac{q^2}{4\pi\epsilon_0} \frac{1}{m_{\text{total}} c^2} \\ a &= \frac{2}{3} r_0 \end{aligned}$$

Where r_0 is classical electron radius

When the electron has very high velocity, then the momentum of electron is found to be

$$\begin{aligned} p &= \frac{2}{3} \frac{q^2}{4\pi\epsilon_0} \frac{v}{ac^2} \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \\ p &= \frac{2}{3} \frac{q^2}{4\pi\epsilon_0} \frac{v}{ac^2} \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \end{aligned}$$

i.e electromagnetic mass rises with velocity, a fact discovered before the theory of relativity implying the possibility of variation of mass with velocity.

According to the theory of relativity, energy is associated with mass as U/c^2

Then the field of electron should have mass

$$m' = \frac{U_{\text{electrical}}}{c^2}$$

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Substituting value of $U_{\text{electrical}}$ we get

$$m' = \frac{1}{2} \frac{q^2}{4\pi \epsilon_0 a} \frac{1}{c^2}$$

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This is not same as $m_{\text{electromagnetic}}$. This indicates that there is discrepancy between the mass calculated from two different approaches. This difference implies that there are non electrical factors in electron which carry the additional mass holding the charges having same nature together called poincare stress. If the extra forces are taken into consideration, the results from the different approaches come out to be same. This fact indicates that it is impossible to get all the mass of a charged particle to be electromagnetic.

In other point of view, a particle is said to have mass if we have to apply force in order to accelerate it. If we have a charged particle and push on it for a while, it radiates electromagnetic wave or energy or there will be some momentum in the electromagnetic field. Therefore, to accelerate a charged particle, we must require more force than it is required to accelerate a neutral particle of the same mass, otherwise energy and momentum would not be conserved. The rate at which we work or apply force on an accelerating charge must be equal to the rate of loss of energy by radiation. So, additional force to that required by mechanical inertia due to electromagnetic interaction has to be added. This fact provides evidence for the existence of electromagnetic mass. According to Newton's third law, there must be a corresponding

force back upon us from accelerated electron .What is the source of the force which causes the pusher in the back direction?

To account for this, we can consider an electron as a charged sphere having number of pieces of charge. At rest or uniform motion, each piece repels electrically to each other piece such that forces balance causing net force zero.

When an electron is being accelerated, the force should no longer be in balance to explain the effect .This situation can be understood as the electromagnetic influences take time to go from one piece to another. When all these forces are added up, they don't cancel out. With acceleration if we look at the forces between the various parts of electron, action and reaction are not exactly equal and the electron exerts a force on itself that tends to hold back the acceleration which implies the force of an electron on itself.

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