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Topological Order in Physics

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Abstract : *In general, we know that there are four states of matter solid, liquid, gas and plasma. But there are much more states of matter. For e. g. there are ferromagnetic states of matter as revealed by the phenomenon of magnetization and superfluid states defined by the phenomenon of zero viscosity. The various phases in our colorful world are so rich that it is amazing that they can be understood systematically by the symmetry breaking theory of Landau. Topological phenomena define the topological order at macroscopic level. Topological order need new mathematical framework to describe it. More recently it is found that at microscopic level topological order is due to the long range quantum entanglement, just like the fermions fluid is due to the fermion-pair condensation. Long range quantum entanglement leads to many amazing emergent phenomena, such as fractional quantum numbers, non- Abelian statistics ad perfect conducting boundary channels. It can even provide a unified origin of light and electron i.e. gauge interactions and Fermi statistics. Light waves (gauge fields) are fluctuations of long range entanglement and electron (fermion) are defect of long range entanglements.*

Keywords: Topological order, degeneracy, Landau-symmetry, chiral spin state, string-net condensation, quantum glassiness, chern number, non-abelian statistics, fractional statistics.

1. INTRODUCTION

In physics, topological order is a kind of order in zero-temperature phase of matter also known as quantum matter. Macroscopically, topological order is defined/described by robust ground state degeneracy and quantized geometric phases of degenerate ground states. Microscopically, topological order corresponds to patterns of long-range quantum entanglement . States with different topological orders or different patterns of long range entanglements cannot change into each other without a phase transition.

Topologically ordered states have some interesting properties, such as (1) topological degeneracy and fractional statistics that can be used to realize topological quantum computer;(2) perfect conducting edge states that may have important device applications; (3) emergent gauge field and Fermi statistics that suggest a quantum information origin of elementary particles ; (4) topological entanglement entropy that reveals the entanglement origin of topological order, etc. Topological order is important in the study of several physical systems such as spin liquids, the quantum Hall effect , along with potential applications to fault-tolerant quantum computation.

We note that topological insulators and topological superconductors (beyond 1D) do not have topological order as defined above.

2. BACKGROUND

Although all matter is formed by atoms , matter can have different properties and appear in different forms, such as solid , liquid , superfluid, magnet, etc. These various forms of matter are often called states of matter or phases . According to condensed matter physics and the principle of emergence , the different properties of materials originate from the different ways in which the atoms are organized in the materials. Those different organizations of the atoms (or other particles) are formally called the orders in the materials.

Atoms can organize in many ways which lead to many different orders and many different types of materials. Landau symmetry-breaking theory provides a general understanding of these different orders. It points out that different orders really correspond to different symmetries in the organizations of the constituent atoms. As a material changes from one order to another order (i.e., as the material undergoes a phase transition), what happens is that the symmetry of the organization of the atoms changes.

For example, atoms have a random distribution in a liquid , so a liquid remains the same as we displace atoms by an arbitrary distance. We say that a liquid has a continuous translation symmetry . After a phase transition, a liquid can turn into a crystal . In a crystal, atoms organize into a regular array (a lattice). A lattice remains unchanged only when we displace it by a particular distance (integer times of lattice constant), so a crystal has only discrete translation symmetry . The phase transition between a liquid and a crystal is a transition that reduces the continuous translation symmetry of the liquid to the discrete symmetry of the crystal. Such change in symmetry is called symmetry breaking. The essence of the difference between liquids and crystals is therefore that the organizations of atoms have different symmetries in the two phases.

Landau symmetry-breaking theory is a very successful theory. For a long time, physicists believed that Landau symmetry-breaking theory describes all possible orders in materials, and all possible (continuous) phase transitions.

3 DISCOVERY AND CHARACTERIZATION

However, since late 1980s, it has become gradually apparent that Landau symmetry-breaking theory may not describe all possible orders. In an attempt to explain high temperature superconductivity the chiral spin state was introduced. At first, physicists still wanted to use Landau symmetry-breaking theory to describe the chiral spin state. They identified the chiral spin state as a state that breaks the time reversal and parity symmetries, but not the spin rotation symmetry. This should be the end of story according to Landau's symmetry breaking description of orders. However, it was quickly realized that there are many different chiral spin states that have exactly the same symmetry, so symmetry alone was not enough to characterize different chiral spin states. This means that the chiral spin states contain a new kind of order that is beyond the usual symmetry description. The proposed, new kind of order was named "topological order". The name "topological order" is motivated by the low energy effective theory of the chiral spin states which is a topological quantum field theory (TQFT). New quantum numbers, such as ground state degeneracy and the geometric phase of degenerate ground states, were introduced to characterize/define the different topological orders in chiral spin states. Recently, it was shown that topological orders can also be characterized by topological entropy.

But experiments soon indicated that chiral spin states do not describe high-temperature superconductors, and the theory of topological order became a theory with no experimental realization. However, the similarity between chiral spin states and quantum Hall states allows one to use the theory of topological order to describe different quantum Hall states. Just like chiral spin states, different quantum Hall states all have the same symmetry and are beyond the Landau symmetry-breaking description. One finds that the different orders in different quantum Hall states can indeed be described by topological orders, so the topological order does have experimental realizations.

The fractional quantum Hall (FQH) state was discovered in 1982 before the introduction of the concept of topological order in 1989. But the FQH state is not the first experimentally discovered topologically ordered state. The superconductor , discovered in 1911, is the first experimentally discovered topologically ordered state, which has topological order .

Although topologically ordered states usually appear in strongly interacting boson/fermion systems, a simple kind of topological order can also appear in free fermion systems. This kind of topological order corresponds to integral quantum Hall state, which can be characterized by the Chern number of the filled energy band if we consider integer quantum Hall state on a lattice. Theoretical calculations have proposed that such Chern numbers can be measured for a free fermion system experimentally. It is also well known that such a Chern number can be measured (may be indirectly) by edge states.

4. MECHANISM

A large class of 2+1D topological orders is realized through a mechanism called string-net condensation. This class of topological orders can have a gapped edge and are classified by unitary fusion category (or monoidal category) theory. One finds that string-net condensation can generate infinitely many different types of topological orders, which may indicate that there are many different new types of materials remaining to be discovered.

The collective motions of condensed strings give rise to excitations above the string-net condensed states. Those excitations turn out to be gauge bosons. The ends of strings are defects which correspond to another type of excitations. Those excitations are the

gauge charges and can carry Fermi or fractional statistics .

The condensations of other extended objects such as "membranes ", "brane-nets", and fractals also lead to topologically ordered phases and "quantum glassiness".

Examples of topologically ordered states

3D s-wave superconductors (Many textbooks ignore the dynamical U(1) gauge field and treat 3D superconductors as symmetry breaking states.)

Integer quantum Hall states (Those topological orders have no fractionalized quasiparticles excitations and are called invertible topological orders.)

Fractional quantum Hall states (which have fractionalized quasiparticles which fractional charges and fractional statistics or even non-abelian statistics. Chern-Simons gauge theories are their low energy effective theory)

Chiral spin state (which can be viewed as fractional-quantum-Hall analogue in spin liquids, with Chern-Simons gauge theory as low energy effective theory)

Z2-topological order or Z2 spin liquid (with Z2 gauge theory as low energy effective theory.

Herbertsmithite may realize such Z2 spin liquid.)

5. MATHEMATICAL FOUNDATION

We know that group theory is the mathematical foundation of symmetry breaking orders. What is the mathematical foundation of topological order? It was found that a subclass of 2+1D topological orders—Abelian topological orders—can be classified by a K-matrix approach. The string-net condensation suggests that tensor category (such as fusion category or monoidal category) is part of the mathematical foundation of topological order in 2+1D. The more recent researches suggest that (up to invertible topological orders that have no fractionalized excitations):

2+1D bosonic topological orders are classified by unitary modular tensor categories.

2+1D bosonic topological orders with symmetry G are classified by G-crossed tensor categories.

2+1D bosonic/fermionic topological orders with symmetry G are classified by unitary braided fusion categories over symmetric fusion category, that has modular extension. Topological order in higher dimensions may be related to n-Category theory. Quantum operator algebra is a very important mathematical tool in studying topological orders. Some also suggest that topological order is mathematically described by extended quantum symmetry. .

Applications

The materials described by Landau symmetry-breaking theory have had a substantial impact on technology. For example, ferromagnetic materials that break spin rotation symmetry can be used as the media of digital information storage. A hard drive made of ferromagnetic materials can store gigabytes of information. Liquid crystals that break the rotational symmetry of molecules find wide application in display technology; nowadays one can hardly find a household without a liquid crystal display somewhere in it. Crystals that break translation symmetry lead to well defined electronic bands which in turn allow us to make semiconducting devices such as transistors . Different types of topological orders are even richer than different types of symmetry-breaking orders. This suggests their potential for exciting, novel applications.

One theorized application would be to use topologically ordered states as media for quantum computing in a technique known as topological quantum computing . A topologically ordered state is a state with complicated non-local quantum entanglement . The non-locality means that the quantum entanglement in a topologically ordered state is distributed among many different particles. As a result, the pattern of quantum entanglements cannot be destroyed by local perturbations. This significantly reduces the effect of decoherence . This suggests that if we use different quantum entanglements in a topologically ordered state to encode quantum information, the information may last much longer. The quantum information encoded by the topological quantum entanglements can also be manipulated by dragging the topological defects around each other. This process may provide a physical apparatus for performing quantum computations Therefore, topologically ordered states may provide natural media for both quantum memory and quantum computation. Such realizations of quantum memory and quantum computation may potentially be made fault tolerant .

Topologically ordered states in general have a special property that they contain non-trivial boundary states. In many cases, those boundary states become perfect conducting channel that can conduct electricity without generating heat. This can be another potential application of topological order in electronic devices.

Similar to topological order, topological insulators also have gapless boundary states. The boundary states of topological insulators play a key role in the detection and the application of topological insulators. This observation naturally leads to a question: are topological insulators examples of topologically ordered states? In fact topological insulators are different from topologically ordered states defined in this article. Topological insulators only have short-ranged entanglements and have no topological order, while the topological order defined in this article is a pattern of long-range entanglement. Topological order is robust against any perturbations. It has emergent gauge theory, emergent fractional charge and fractional statistics. In contrast, topological insulators are robust only against perturbations that respect time-reversal and U(1) symmetries. Their quasi-particle excitations have no fractional charge and fractional statistics. Strictly speaking, topological insulator is an example of SPT order , where the first example of SPT order is the Haldane phase of spin-1 chain.

6. CONCLUSIONS

Landau symmetry-breaking theory is a cornerstone of condensed matter physics . It is used to define the territory of condensed matter research. The existence of topological order appears to indicate that nature is much richer than Landau symmetry-breaking theory has so far indicated. So topological order opens up a new direction in condensed matter physics—a new direction of highly entangled quantum matter. We realize that quantum phases of matter (i.e. the zero-temperature phases of matter) can be divided into two classes: long range entangled states and short range entangled states. Topological order is the notion that describes the long range entangled states: topological order = pattern of long range entanglements. Short range entangled states are trivial in the sense that they all belong to one phase. However, in the presence of symmetry, even short range entangled states are nontrivial and can belong to different phases. Those phases are said to contain SPT order . SPT order generalizes the notion of topological insulator to interacting systems. Some suggest that topological order (or more precisely, string-net condensation) in local bosonic (spin) models have the potential to provide a unified origin for photons , electrons and other elementary particles in our universe.

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