Role of Reliability Analysis in Structural Design

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Abstract: Modern structures require more critical and complex designs; the need for accurate and efficient approaches to assess uncertainties in loads, geometry, material properties, manufacturing processes involved and also the operational environment, has increased significantly. Reliability assessment techniques help to develop the initial guidance for robust designs. In this context , the classical methods such as theory of probability, statistical methods and reliability analysis methods are often used by structural engineers. Some of the methods which have been developed in the later stages include Monte Carlo Sampling, Latin Hyper Cube Sampling, First and Second Order Reliability Methods, Stochastic Finite Element Method and Stochastic Optimization. In addition, in those structural problems where randomness is relatively small, a deterministic model is usually used rather than a Stochastic Model. However, when the level of uncertainty is high, Stochastic approaches are necessary for system analysis and design. Number of probabilistic analysis tools have been developed to qualify uncertainties, but the most complex systems are still designed with simplified rules and schemes, such as factor of safety based designs. However, these traditional design processes do not directly account for the random nature of the most input parameters. Factor of safety is used to maintain some degree of safety in the structural design. Generally, the factor of safety is understood to be the ratio of the expected strength of response to the expected load. In practice, both the strength and load are variables, the values of which are scattered about their respective mean values. When the scatter of the variables is considered, the factor of safety could potentially be less than unity and the traditional factor of safety based design would fail. More likely is that the factor of safety is too conservative, which leads to an over expensive design.

Keywords: Reliability analysis, Structural design, Probability Density Function

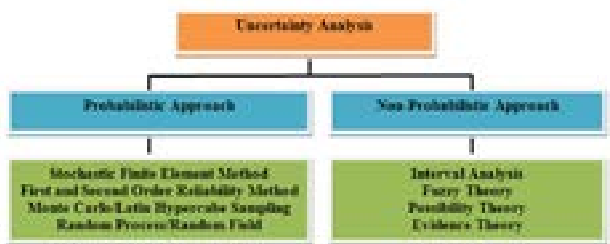
Uncertainty and its Analysis

Identification/prediction of uncertainties is directly linked to Reliability Analysis. In the modern competitive world, engineering communities' motto should be, "***If it works, make it better***". Compared to the deterministic approach based on safety factors, the stochastic approach improves design reliability. The stochastic approach provides number of advantages to engineers. The various statistical results, which include mean value, variance and confidence interval, can provide a broader perspective and more complete description of the given structural system; one that takes more factors and uncertainties into account.

Probability theory treats the likelihood of a given event's occurrence and quantifies certain measures of random events. The new methodologies, which can consider the randomness or uncertainty in the data or model, are known as Uncertainty Analysis of Stochastic Analysis. These methods facilitate robust designs and provide the designer with guarantee of satisfaction in the presence of a given amount of uncertainty.

The competence and limitations of the representations have already been delimitated by classifying uncertainties into two categories i.e. Aleatory and Epistemic. Aleatory (random or objective) uncertainty is also called Irreducible or Inherent uncertainty. Epistemic (subjective) uncertainty is reducible uncertainty that stems from lack of knowledge and data.

The uncertainty analysis is broadly carried out, usually in two distinct categories, as described below;



Probabilistic approach is based on theoretical foundation of the Probability Density Function (PDF) information and introduces the use of random variables, processes and fields to represent uncertainty. The Non-probabilistic approach manages imprecise knowledge about the true value of the parameters.

Reliability and its importance

An engineering structure's response depends on many uncertain factors such as loads, boundary conditions, stiffness and mass properties. The response (critical location stresses, resonant frequencies etc.) is considered satisfactory when the design requirements imposed on the structural behavior are met within an acceptable degree of certainty. Each of these requirements is termed as Limit State or Constraint.

The study of structural reliability is concerned with the calculation and prediction of the probability of the limit state violations at any stage during the structure's life. The probability of occurrence of an event such as limit state violation is a numerical measure of the chance of its occurring. Once the probability is determined, the next goal is to choose design alternatives that improve structural reliability and minimize the risk of failure.

Methods of reliability analysis are rapidly finding application in the multi-disciplinary design environment because of the engineering system's stringent performance requirements, narrow margins of safety, liability and market competition. In structural design problems involving uncertainties, a structure designed using deterministic approach may have a greater probability of failure than a structure, of the same cost, designed using the probabilistic approach that accounts for uncertainties. This is because the design requirements are precisely satisfied in the deterministic approach, and any violation of the parameters could potentially violate the system constraints.

Mathematical equations commonly used in probabilistic representations

For a continuous random variable, Fx(x) is calculated by integrating the PDF of all values of X≤x

$$Fx(x) = \int_{-\infty}^{x} f_x(S)\, ds \qquad \textit{Equation 3.1}$$

The expected value or average is used to describe the central tendency of a random variable. If fx(x)is the probability density function of X, the mean is given by the

$$\mu_x = E(X) = \int_{-\infty}^{\infty} f_x(x)\, dx \qquad \textit{Equation 3.2}$$

The Gaussian (or normal) distribution is generally used in many engineering and science fields due to its simplicity and convenience, especially a theoretical basis of the central limit theorem. The central limit theorem states that the sum of many arbitrarily distributed random variables asymptotically follows a normal distribution when the sample size becomes large. This distribution is often used for small coefficient of variation cases, such as young's modulus, Poisson's ratio and other material properties. The Gaussian distribution is given as below;

$$f_x(x) = \frac{1}{\sigma_x\sqrt{2\pi}} exp\left[-\frac{1}{2}\left(\frac{x-\mu_x}{\sigma_x}\right)^2\right], -\infty < x < \infty \qquad \textit{Equation 3.3}$$

In addition to above, the lognormal distribution also plays an important role in probabilistic design because negative values of engineering phenomena are sometimes physically impossible. Therefore, lognormal distribution are found in description of fatigue failure, failure rates and other phenomena involving a large range of data.

The PDF of "y" is given as below;

$$f_y(y) = \frac{1}{\sqrt{2\pi}\sigma_y} exp\left[-\frac{1}{2}\left(\frac{y-\mu_y}{\sigma_y}\right)^2\right] -\infty < y < \infty \qquad \textit{Equation 3.4}$$

Gamma distribution consists of gamma function. This distribution is important because it allows us to define two families of random variables, the exponential and chi-square, which are used extensively in applied engineering and statistics.

The density function associated with the gamma distribution is defined by;

$$f_x(x) = \frac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}, 0 \le x < \infty \qquad \textit{Equation 3.5}$$

Similar to above, Weibull distribution is well suited for describing the weakest link phenomena or a situation where there are competing flaws to failure. The probability density function under Weibull distribution is defined as below;

$$f_x(x) = \frac{\alpha x^{\alpha-1}}{\beta^{\alpha}} exp\left[-\left(\frac{x}{\beta}\right)^{\alpha}\right], x \ge 0, \alpha > 0, \beta > 0 \qquad \textit{Equation 3.6}$$

Solution Techniques for Structural Reliability

If a structure or part of the structure exceeds the specific limit, the structure or part of the structure is unable to perform as required, then the specific limit is called Limit State. The structure will be considered unreliable if the failure probability of the structure limit state exceeds the required value. For most of the structures, the limit state can be divided into two categories;

Ultimate limits states are related to the structural collapse i.e. part of the structure or the entire collapse of the structure. Examples of the most common ultimate limit states are corrosion, fatigue, deterioration, fire, plastic mechanism, progressive collapse, fracture etc. such a limit state should have a very low probability of occurrence.

Serviceability limit states are related to disruption of the normal use of the structures. Examples of serviceability limit states are excessive deflection, excessive vibration, drainage, leakage, local damage etc. Since there is a less danger than in the case of the ultimate limit states, a higher probability of occurrence may be tolerated in such limit states. However, people may not use structures that yield too much deflections, vibrations etc.

Generally, the limit state indicates the margin of safety between the resistance and the load of structures. The limit state i.e. g(.) function and the probability of failure "Pf" could be defined as follows;

$$g(X) = R(X) – S(X) \qquad \textit{Equation 4.1}$$

$$P_f = P\,[g(.) < 0] \qquad \textit{Equation 4.2}$$

Where "R" is the resistance and "S" is the loading of the system. Both "R(.)" and "S(.)" are functions of random variables "X". The notation "g(.)<0" denotes the failure region. Likewise, "g(.)=0" and "g(.)>0" indicate the failure surface and safe region respectively.

The mean and the standard deviation of the limit states, "g(.)" can be determined from the elementary definition of the mean and the variance. The mean of "g(.)" is as below;

Equation 4.3

$$\mu_g = \mu_R - \mu_S$$

Where "µR" and "µs" are the means of "R & S" respectively and the standard deviation of "g(.)" is as below;

Equation 4.4

$$\sigma_g = \sqrt{\sigma_R^2 + \sigma_S^2 - 2\rho_{RS}\sigma_R\sigma_S}$$

Where "ρRS" is the correlation coefficient between "R & S" and "σR&σS" are the standard deviations of "R & S" respectively.

The safety index or reliability index "β" is defined as below;

Equation 4.5

$$\beta = \frac{\mu_g}{\sigma_g} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2 - 2\rho_{RS}\sigma_R\sigma_S}}$$

If the resistance and the load are uncorrelated (ρRS = 0) the safety index becomes the following;

$$\beta = \frac{\mu_g}{\sigma_g} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}}$$

Equation4.6

The safety index indicates the distance of the mean margin of safety from "g(.)". The idea behind the safety index is that the distance from the location measure "µg" to the limit state surface provides a good measure of reliability.

Another well-known definition of the reliability analysis is the safety factor i.e. "F" and same is defined as below;

$$F = \frac{R}{S}$$

Equation 4.7

Failure occurs when "F=1" and if the safety factors are assumed to be normally distributed, the safety index is given by the following;

$$\beta = \frac{\mu_F - 1}{\sigma_F}$$

Equation 4.8

First and Second Order Reliability Method

The Taylor series expansion is often used to linearize the limit state "g(X)=0". In this approach, the first or the second order Taylor series expansion is used to estimate reliability. These methods are referred to as the First Order Second Moment (FOSM) and Second Order Second Moment (SOSM) methods respectively.

FOSM is also referred to as Mean Value First Order Second Moment Method (MVFOSM), since it is a point expansion method at the mean point and the second moment is the highest order statistical result used in this analysis. Although, implementation of FOSM is simple, it has been shown that the accuracy is not acceptable for low probability of failure i.e. "Pf< 10-5" or for highly non-linear responses. In SOSM, the addition of the second order term increases computational efforts significantly, yet the improvement in accuracy is often minimal.

Conclusion

Structural elements in a framed structure are usually analyzed in a conventional manner. Failure of such structure may have several modes that should be considered when analyzing the overall structural reliability. However, even in the case of one structural member (say a beam or a column), several failure modes (limit states functions) have to be taken into account. For example: a continuous beam may fail due to positive or negative moments or due to shear. In general, the reliability of the structural system depends on the reliability of its elements and the relevant failure modes. As a rule, there is a high co-relation between the properties of elements in different parts of the structure. In some cases, loads may also be mutually dependent. In addition, limit states for the whole structure such as the overall deflection due to foundation settlement may be significant.

Therefore, the reliability analysis shall be playing a major role in assessing the overall reliability of the structure. Reliability under variable loads with intermittencies is also an important area where the reliability analysis shall be playing an important role.

In a nutshell, the reliability analysis propagation in theory and also in the structural analysis shall be the new area for research in developing economies.

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