Mathematical Modeling of Omkareshwar Hydro-electric Plant at Narmada River

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Abstract: Madhya Pradesh state is forced to optimize the utilization of its water resources due to continuous depletion of the fossil fuels and its present power generation scenario. The state has two cascade schemes of hydro-electric plants one at Chambal river and the other at Narmada river. Narmada river at present has four main hydro plants in cascading mode starting from Bargi, Indira Sagar and Omkareshwar in Madhya Pradesh to Sardar Sarovar in the neighboring state of Gujrat. The installed capacities of 2810 MW of these hydro plants feed to the major portion of state's electricity demand.

The proper scheduling of the hydroelectric plants mean, water resources must be used in such a way that, water discharge at an upstream plant is converted into electric energy at downstream plants without spillage. Hydroelectric scheduling of the plants requires a judicious modeling of each of the hydro electric plant for an improved efficiency and arrest possible losses. The paper presents the mathematical model of the Omkareshwar hydroelectric plant as part of cascade scheme at Narmada River in Madhya Pradesh.

Key words: Hydro-electric system, hydro turbine model, reservoir model, tailrace model, water flow equation.

List of Symbols

P_j^t = Electrical power generated in jth unit of the plant at time t (MW)

X_j^t = Reservoir storage of the plant at time t (cubic metre)

H_j^t = Head for the plant at time t (metre)

HF_j^t = Reservoir elevation of the plant at time t (metre)

HT_j^t = Tailrace elevation of the plant at time t considering the total discharge (metre)

HT_{jn}^t = Tailrace elevation at time t considering the discharge in Narmada river (metre)

Y_j^t = Natural inflow in the reservoir at time t (cubic metre)

Z_k^t = Inflows from the upstream plant in the reservoir at time t (cubic metre)

U_j^t = Discharge through j the turbine unit of the plant at time t (cubic metre)

V_j^t = Spill from the reservoir at time t (cubic metre)

Q_j^t = Total Discharge through the plant at time t including spill and turbines discharge (cubic metre)

Q_{jn}^t = Part of Discharge through the plant at time t in Narmada river (cubic metre)

t = Time index

= Time delay between Water flow from upstream plant to Omkareshwar (seconds)

= Hydro turbine model constant (where i=1,2---6)

B_i = Reservoir storage model constant (where i=1,2---5)

C_i = Tail race elevation model (Considering total discharge) constants (where i=1,2—5)

C_i' = Tail race elevation model (Considering discharge in Narmada river) constants (where I = 1,2—5)

QP = Quadratic Polynomial

CP = Cubic Polynomial

FP = Fourth order polynomial

FIP = Fifth order polynomial

SP = Sixth order polynomial

SEP = Seventh order polynomial

DS = Data set

$RMSE$ = Root mean square value

SSE = Sum of square due to error

Introduction

Modeling of the Hydro electric plant is a very complex task and as such there is no uniform modeling as each one is unique to its location and requirement. The diversity of these designs makes its necessary to model each one individually. The parameters of modeling are nonlinear and highly dependent on the control variables. In each case of the power station study, the coefficients has to be determined by collating actual plant data.

To develop the model of the Omkareshwar Hydro-electric plant, the following data is used:

1. Hill Chart of the turbine unit of the plant
2. Area capacity curve of the reservoir of the Omkareshwar hydro-electric plant
3. Tail race water rating curve of the Omkareshwar hydro-electric plant
   a. Total Discharge at Dam Axis
   b. Discharge in Narmada river
4. Water inflow into each reservoir as a function of upstream plant discharge, water transport delay and inflows between reservoirs.
5. Water to energy conversion factor as a function

of net head and discharge through each turbine.

Hydro-electric system description

Narmada river is an inter-state river originating from Amarkantak in the state of Madhya Pradesh and passing through Rajasthan, Maharashtra and Gujrat states with a total flow length of 1312 km. Narmada Valley Development Authority is trying to generate more power using cascade hydro plant scheme through Narmada water resources (as shown in Fig 1). In order to utilize the potential of available water resources for power generation, even the irrigation canal of the project is used for power generation by installing a canal head power house. However, modeling of these plants are not included in this paper.

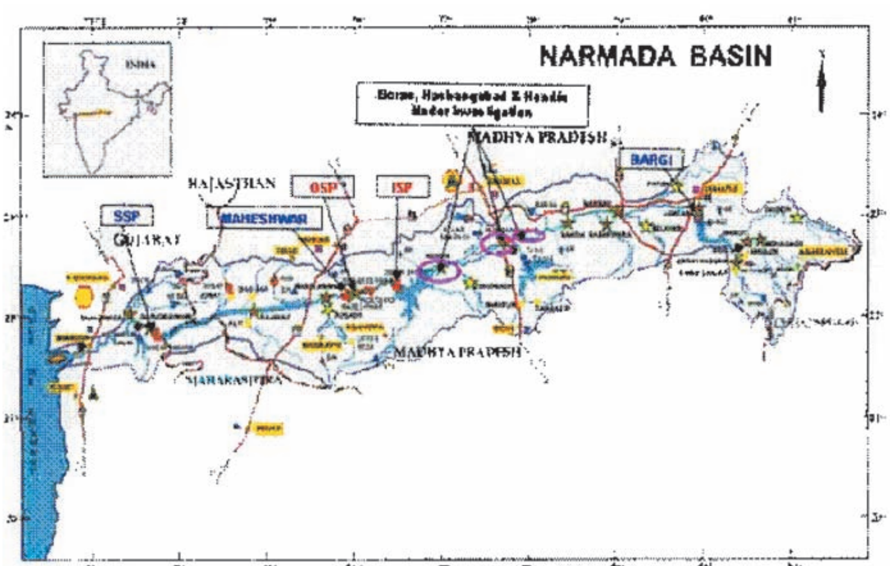

Fig 1: Location map of hydro plants at Narmada basin (source: Narmada Hydro-electric Development Company)

The cascade hydro-electric plants (as shown in fig. 2) on Narmada river consists of four major hydro plants in Madhya Pradesh namely '*Rani Avantibai Sagar*', '*Indira Sagar*', '*Omkareshwar*', and '*Maheshwar*'. One is in neighbouring state of Gujarat i.e.'*Saradar Sarovar*' project. These plants are under different management structures. Start from upstream plant Raniavantibai sagar is owned by MPGENCO (Madhya Pradesh Power Generation Company), Indira Sagar and Omkareshwar are under NHDC (Narmada Hydro-electric Development Company a joint venture of National Hydro-electric Development company and Govt. of MP) Maheshwar is with S. Kumars and Sardar Sarovar is under Maharastra Government with 57% share of Madhya Pradesh in electricity production.

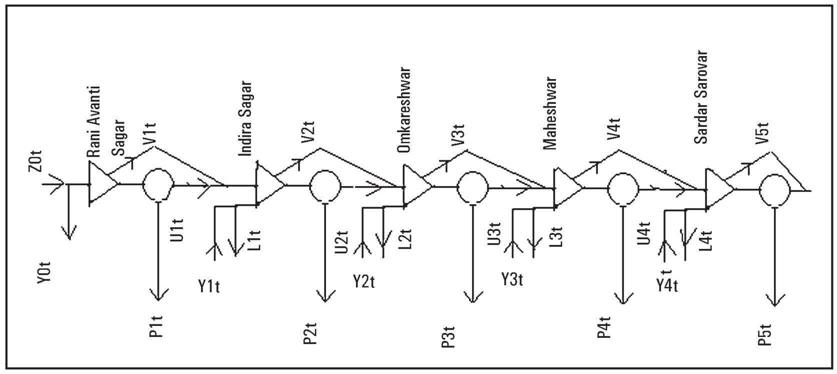

Fig 2: Cascade Hydroelectric system at Narmada river in Madhya Pradesh

In a scenario where management structures are different and in conflict with each other in their approach to maximize power generation, discharge and spillage become issues of contention. This is where the mathematical model required to explain the proper scheduling to help maximize the power generation.

In present case of series of cascade schemes the maximum TRC (Tail race level) of the upstream reservoir is an FRL (Full reservoir level) for its immediate downstream plant. Hence the increase in the level of reservoir at downstream level can affect the head of its immediate upstream plant [6]. Hence a proper scheduling is required for cascade scheme to effectively utilize the water resources of the Narmada river

Omkareshwar hydro-electric plant model

Omkareshwar hydro-electric plant with a capacity of 520 MW, with 8 units each of 65 MW is a multipurpose project. It fulfills the partial requirement of the irrigation and power generation for the state of Madhya Pradesh. It is a run-of-the river type with a small reservoir with a live storage capacity of 0.299 Million cubic metres (MCM) and gross capacity of the 0.987 MCM. This plant has Francis turbines and the rated head and discharge are 32 metre and 237.25 Cubic metres/sec respectively. It is at 40 Km river distance from its upstream plant Indira Sagar project.

Generation at Omkareshwar plant is a nonlinear in its function of reservoir content and discharges. There is substantial head variation resulting from both reservoir and tailrace water elevation variation in the case of low and medium head plants. Reservoir elevation will vary with the inflows in the reservoir and the existing reservoir contents. Tail water elevation depends upon the discharge through turbine.

Maximum Reservoir level of this plant is 196.60 m and its maximum TRC level is 162.76m. There is an high hydraulic coupling between the Indira Sagar, Omkareshwar and Maheswar and Sardar Sarovar projects. The modeling of the Omkareshwar hydro plant is covering the each area like reservoir, hydro turbine, tail race, water flow modeling.

Hydro turbine model

It is very important to model the hydro turbine, as it is an objective function which has to maximize in the hydroelectric scheduling problem. The power generation in the hydro plant is a function of the discharge through turbine and head. In case of big reservoir the effect of head variation can be negligible, but in small reservoir plant like Omkareshwar hydro plant the effect of variation of head should be considered in the modeling of the plant. Head is a function of the reservoir storage. Hence it can be represented in form of storage in the hydro turbine model equation.

To model this plant a hill chart [7] is used. Various data set from the typical hill chart has been obtained at various head and discharge. The relation of the power generation has been formed by using the nonlinear curve fitting techniques by least square method in MATLAB 7.0 Version software. The second order quadratic equation [5] is very well satisfying the relation of generated power in term of storage and discharge as per data available from hill chart.

$$P_j^{\,t} = A_1(H_j^{\,t})^2 + A_2(U_j^{\,t})^2 + A_3(H_j^{\,t})(U_j^{\,t}) + A_4(H_j^{\,t}) + A_5(U_j^{\,t}) + A_6$$

Where the constants of the hydro turbine models are calculated as

A_1= -0.009331818
A_2= -0.00035529
A_3= 0.0140485215
A_4= 0.413454476
A_5= 0.140660308
A_6= -27.22095302

Reservoir modeling

Water stored in reservoir is the main source of the hydro-electric plant, as it decides the energy content and the gross head of that plant. It is affected by the inflows to the reservoir as well. Outflows from reservoir is either through turbines or spillway. The model relate the level of reservoir in terms of Reservoir storage or volume of stored water in reservoir. This model has been developed with the data of area capacity curve [7] by applying suitable curve fitting techniques for various type of polynomial the graphical results are obtained as shown in fig 3.

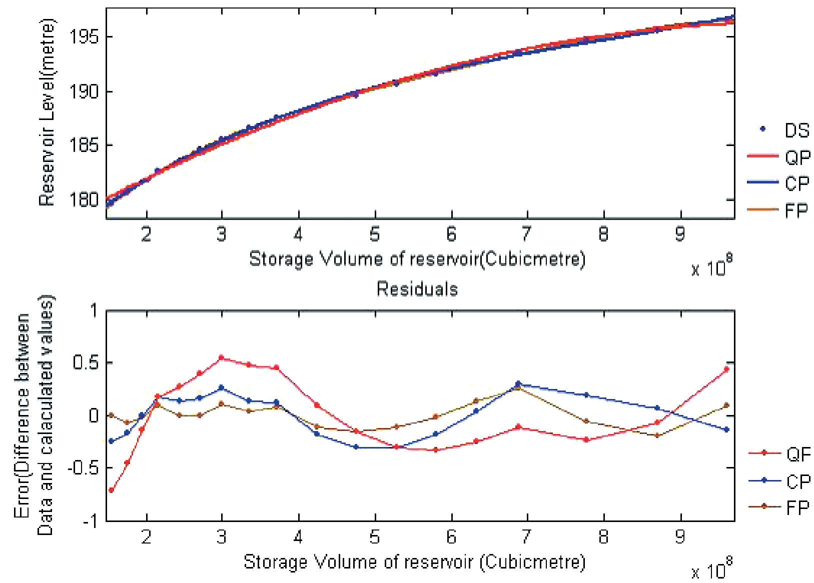

Fig 3: Simulation results for Reservoir modeling

After analyzing the graphical results and calculated errors the value of RMSE is 0.13013 and SSE is 0.22016 which are the lowest in case of fourth degree polynomial, hence a fourth order polynomial equation satisfying the characteristic of the area capacity curve and the constants of the equations are:

$$HF_J^{\,t} = B_1(X_j^{\,t})^4 + B_2(X_j^{\,t})^3 + B_3(X_j^{\,t})^2 + B_4(X_j^{\,t}) + B$$

B_1 = -6.601*10^-35
B_2 = 1.711*10^-25
B_3 = -1.718*10^-16
B_4 = 9.487*10^-8
B_5 = 168.5

The net head will be calculated by the given equation

$$H_j^{\,t} = \left(HF_j^{\,t}\right) - \left(HT_j^{\,t}\right) - \text{Head Loss}$$

Due to the non availability of data regarding head loss it can be assumed negligible.

Tailrace elevation model

As per design of the Hydroelectric plant, the station discharges through turbine and in spillway can raise the tailrace elevation and hence decrease the effective head. The tail race water after dam axis is divided into two rivers, part of it goes in Kaveri river and maximum goes to Narmada river. The water going in Narmada river can only be used as an inflow to its immediate downstream hydro plant that is Maheshwar (under construction) or Sardar Sarovar hydro plant. Tail water rating curve [7] has been used in two cases, one is considering total discharge and second one with discharge in Narmada river as second case act as an inflows to its downstream plant.

Case 1: Considering total discharge at dam axis

Using tail race water level curve [7] at dam axis graphical results are obtained, although after analyzing graphical results and the calculated error it can be seen that the higher order polynomial provides better fit but to reduce the complexity in the optimization problem of hydro scheduling a fourth order polynomial is considered with RMSE Value is 0 .50591 and SSE value 6.91076.

Where

C_1= -1.117*10 $^{-18}$

C_2= 2.212*10 $^{-13}$

C_3= -1.556*10^{-8}

C_4= .0007019

C_5= 162.9|

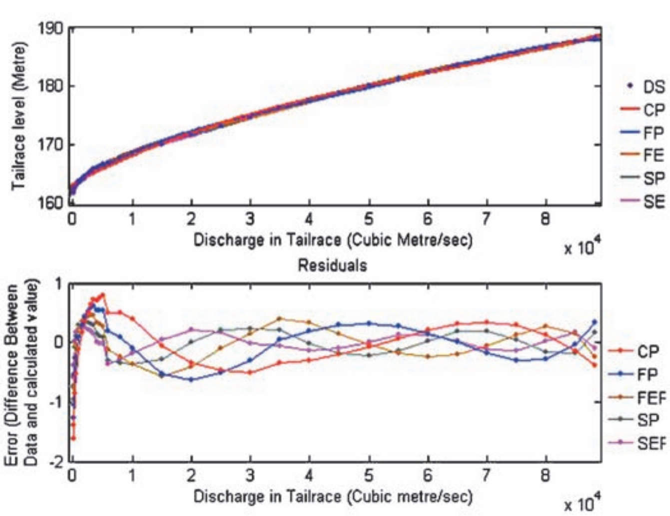

Fig 4 : Simulation results for Tailrace elevation model

Case 2: Discharges in Narmada river

Similar to earlier case although higher order polynomial can provide better fit but here a fourth order polynomial with RMSE 0.38850 and SSE 4.07523 is considered and calculated constants are as

$$HT_{Jn}^{t} = C_1^{'}(Q_{jn}^{t})^4 + C_2^{'}(Q_{jn}^{t})^3 + C_3^{'}(Q_{jn}^{t})^2 + C_4^{'}(Q_{jn}^{t}) + C_5^{'}$$

Where

$C_1^{'}$ = -3.568*10 $^{-17}$

$C_2^{'}$ = 2.859*10 $^{-12}$

$C_3^{'}$ = -7.518*10 $^{-8}$

$C_4^{'}$ = .001392

$C_5^{'}$ = 161

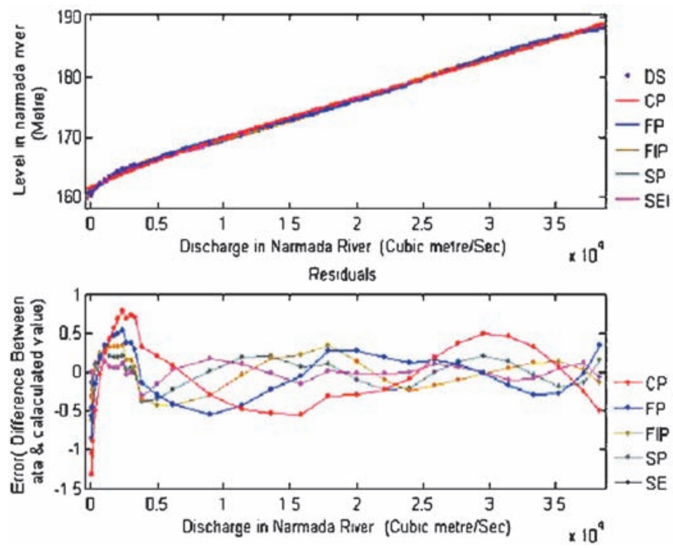

Fig 5 : Simulation results for Narmada river elevation at downstream of Omkareshwar plant

Water flow equation

Hydro-electric system at Narmada river is a type of series cascade system and the discharge from the immediate upstream reservoir is one of the factor in deciding the operating head of the plant as well the reservoir volume. Water flow equation relates the content of the reservoir at any instant with due consideration of time delay to reach water from upstream plant to the Omkareshwar plant [3].

Total Inflow into station

$$I_j^t = Y_j^t + Z_k^{t-\xi} \text{ where } (Z_k^{t-\xi} = U_k^{t-\xi} + V_k^{t-\xi})$$

Storage in reservoir at time t

$$X_j^t = X_j^{t-1} + Y_j^t - U_j^t - V_j^t + (U_k^{t-\xi} + V_k^{t-\xi})$$

Water travel time is approximately 5 hrs between the immediate upstream project of Omkareshwar to the Omkareshwar hydro-electric project.

Conclusion

The development of the suitable model for the Omkareshwar hydroelectric plant is done from the typical curves received from competent authority of the project. The model coefficients and a graphical comparison of the actual data provide satisfactory results. This model can be used for the Optimal Generation, scheduling of the cascade hydro system at Narmada river at real time, short term, mediun term, long term basis.

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