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Some Explanations of the Shapes of Yield Curves with a Focus on the Flattening Long End.

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INTRODUCTION

Knowledge of the Term Structure of Interest Rates and Yield Curves is of fundamental importance to financial decision making and can be used for various purposes, such as hedging, arbitrage, and speculation. The information content in the term structure of interest rates has been explored by many authors, for example by Fama and E.F. Schiller et al. and the interest in the shape of yield curves is persistent as is evidenced by some very recent articles, for example by Siegel, Nelson, Livingston, and Jain.

This paper reports traditional explanations of the general shapes of yield curves and some recent explanations of the flattening of yield curves for long maturities, with a purpose of providing a basis for further research in this area.

This section contains discussion of the important empirical tendencies concerning yield curves. In Section 2 we review the popular theories explaining the shapes of yield curves. Section 3 will focus on the explanation of the flat long end of yield curves and finally Section 4 contains concluding remarks.

Definition and Measurement

The relation between promised security yields and security maturities (on securities that are otherwise identical except in their terms to maturity) is referred to as the Term Structure of Interest Rates. When expressed graphically, this relation is known as a yield curve, although Bierwag and Grove argue that yield curves are not always good surrogates of the term structure of interest rates. Yield curves are always plotted for securities with similar degrees of default risk.

Despite the fact that term to maturity structure of interest rates is so important to financial decision making, its measurement is not straightforward. Some of the reasons as pointed out by Siegel and Nelson are: lack of default free securites at all maturities, the existence of various sources of heterogeniety in actual securities, taxation, transaction costs, and the fact that we do not record simultaneous trades on all securities. Although most of the literature on the term structure deals with default free bonds, it is not as restrictive as it appears. This is so because, by using a martingale pricing process, the problem of valuing (default) risk assets can be reduced to one of discounting at the riskless rates.

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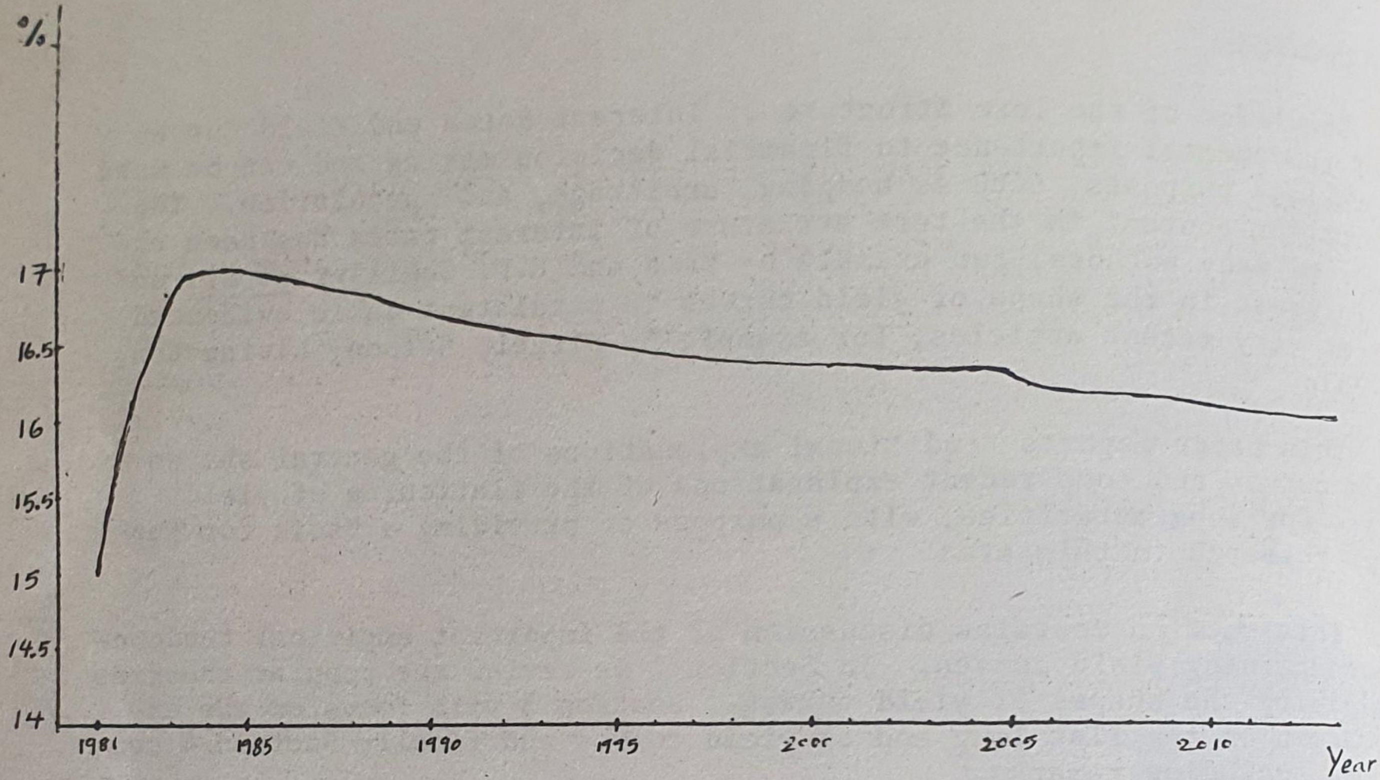
Empirical Shapes of Yield Curves

As mentioned above, often yield curves are reported for Treasury Securities and look as the following. As Wood and Wood point out, it

Figure 1: Yields of US Treasury Securities, September 30, 1981, based on closing bid quotes.



Note: This diagram is adopted from Wood and Wood: the curve filled by eye and based only on the most active issues.

should be remembered that such yield curves are only approximations of a true yield curve because even Treasury Securities, although free of default risk, differ in many respects in addition to term to maturity, for example in special tax features, and call features. However, the principal empirical tendencies can be summarised as follows:

(a) Yield curves tend to have positive slopes when yields are low and to have somewhat negative slopes when yields are high.

(b) Yield curves tend to be sharply upward sloping in the very early maturities producing bumps as in the above figure.

(c) Yield curves tend to become level as maturity increases regardless of their slopes in early maturities - most yield curves are relatively flat at the long end of maturities.

We can add another empirical tendency to the above list, as described by Radcliffe.

(d) The level and shape of yield curves vary considerably over time in the manner shown in Figure 2, reproduced from Radcliffe.

Figure 2: Hypothetical Yield Curves at Various Stages of Economic Activity.

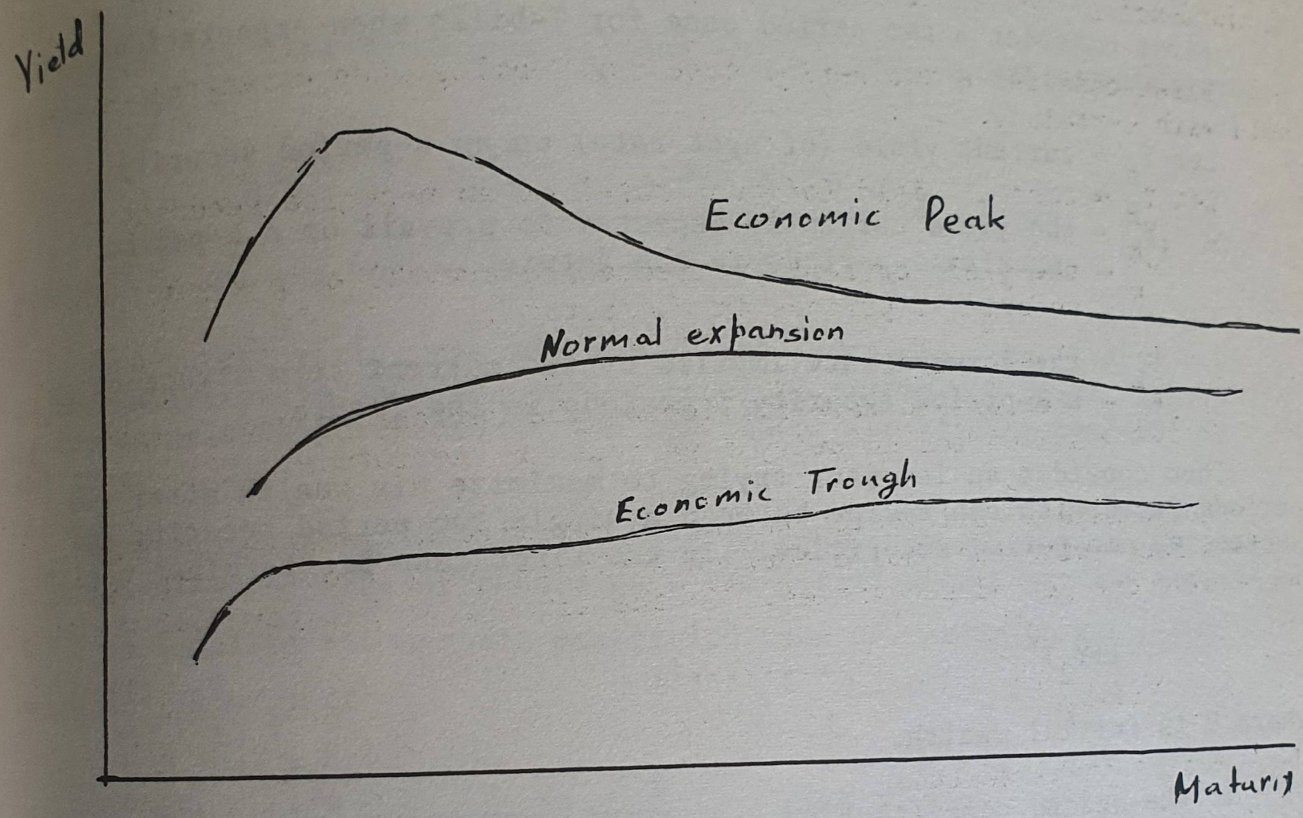


Figure 2 illustrates the empirical fact that yield curves are low and upward sloping during periods of slack economic activity. During rapid business growth and full employment they tend to be high and downward sloping at intermediate range of maturities.

There are three popular theories of the term structure of interest rates: (1) (Unbiased) Expectations Theory (ET), (2) Liquidity (Term) Preference Theory (LPT), and (3) Market Segmentation Theory (MST) - In the next section we will briefly discuss each of these theories and see how they try to explain the above mentioned empirical tendencies.

Traditional Explanations of Yield Curves

The Expectations Theory (ET)

This theory originated from Fisher's work and has since played a central role to all other theories. In addition as Ingersoll points out, in continuous time models, the expectations hypothesis plays the same pivotal role that risk neutrality does for option pricing. At least four versions of the ET are common in the literature all of which are derived from arbitrage relations among the various spot and forward rates, but not all of which are compatible with each other.

Basically, ET asserts that long term yields are averages of current and expected short term yields. Following Wood and Wood a simple presentation of the theory is as:

Traditional Expectations Theory

First consider a two period case for T-bills when expectations are held with certainty.

Let Y_n = current yield (of spot rate) on an n-period security

${}_iY^e_k$ = the yield currently expected to prevail on a k-period security i periods in the future.

${}_iF_k$ = the forward rate implied by the current term structure on a k-period security i periods in the future.

Then consider an investor trying to maximize his wealth after two periods faced with the choice between a single two period security and successive one period securities. In the first case his terminal wealth is

$$V(1+Y_2)^2 \qquad \ldots\ldots\ldots \qquad (1)$$

where V is initial wealth.

In the second case (that is, a one period security now and reinvestment of proceeds in another one period security next period), the terminal wealth is

$$V(1+Y_1)\ (1+{}_1Y^e_1) \qquad \ldots\ldots\ldots \qquad (2)$$

According to ET the investor is indifferent between the two alternatives if and only if

$$(1+Y_2)^2 = (1+Y_1)(1+{}_1Y^e_1), \text{ or, } 1+Y_2 = [(1+Y_1)(1+{}_1Y^e_1)]^{\frac{1}{2}} \quad \ldots \quad (3)$$

This equation suggests an important implication of ET, that long term yields are geometric averages of current and expected short term yields. This implication can easily be extended to n periods as,

$$1+Y_n = [(1+Y_1)(1+{}_1Y^e_1)\ldots.(1+{}_{n-1}Y^e_1)]^{1/n} \quad \ldots.. \quad (4)$$

Thus, both a rising and a falling yield curve can be explained by a series of rising and falling expected short term yields, respectively. It can also be easily shown that the expected one period rates on all these securities (e.g., those corresponding to Y_1, Y_2...) are the same regardless of the length of the investment horizon.

Applying arbitrage relations another important implication of ET is that the implied forward rates given by the following equation are in fact the market's expectations of future rates.

$$(1+Y_n) = (1+Y_1)(1+{}_1F_1)\ldots(1+{}_{n-1}F_1)^{1/n} \quad \ldots\ldots \quad (5)$$

ET asserts that equations (4) and (5) are equivalent for all maturities, that is,

$${}_1Y_1^e = {}_1F_1,\ {}_2Y_1^e = {}_2F_2, \text{ and so on.}$$

Uncertainty and the Modern Expectations Theory (MET)

When future yields are not known with certainty, the assertion of ET that all expected holding-period rates of return are equal, regardless of the securities purchased or the lengths of the holding periods, is not true.

We will prove the above statement in case of two periods. The expected one-period rate of return on a two-period single payment security $({}_1R_2^e)$ is equal to the certain one-period rate of return on a one-period security $({}_1R_1^e{=}Y_1)$ if

$$(1+{}_1R_2^e) = E\frac{{}_1P_1}{P_2} = E\left(\frac{V}{1+{}_1\tilde{Y}_1}\right) \Big/ \frac{V}{(1+Y_2)^2} = (1+Y_2)^2\, E\left(\frac{1}{1+{}_1Y_1}\right) = 1+Y_1 \ldots (6)$$

where P_2 and Y_2 are the current price and yield to maturity on the two-period security and $\tilde{P}_1$ and ${}_1\tilde{Y}_1$ are the price and yield (both uncertain) on a one-period security one period in the future, and V is the amount invested.

On the other hand, the two-period rate of return on successive one-period securities equals the certain two-period return on a two-period security if

$$(1+Y_1)\, E(1+{}_1\tilde{Y}_1) = (1+Y_2)^2 \quad \ldots\ldots \quad (7)$$

But equations (6) and (7) cannot in general both be true - that is, expected one-period and two period rates of return cannot in general both be equal for one and two-period securities - because

$$E\left(\frac{1}{1+{}_1\tilde{Y}_1}\right) \geq \frac{1}{E(1+{}_1\tilde{Y}_1)} \quad \ldots\ldots \quad (8)$$

by Jensen's inequality since $\frac{1}{1+{}_1\tilde{Y}_1}$ is a convex function.

The equality in (8) would hold only under certainty.

This inherent contradiction in the ET under uncertainty can be avoided following the suggestion of Cox, Ingersoll and Ross (CIR) if it is postulated that expected holding period returns are equal only for one specific holding period, the "shortest" interval. Furthermore, CIR have shown that only this choice of holding period is consistent with equilibrium when trading is continuous and future yields are uncertain.

The main conclusions of the MET are:

(a) Risk-neutral investors are indifferent between one and two period securities under uncertainty, only if Y_2 is less than the average of current and expected short term yields.

(b) This implies a downward bias in the yield curve becoming more pronounced as uncertainty increases.

(c) However, this downward bias may be offset or reversed if risk averse investors with short holdings periods require expected return premium on long term securities.

(d) Thus, the upward pressure of risk aversion and the downward bias of yield curve implied by MET may be the cause for the flattening long ends of yield curves. On the other hand the former effect dominates the latter to produce the usually found upward slopes of the yield curves over the first 6 or 12 months to maturity.

Liquidity or Term Preference Theory (LPT)

This theory originated in Hicks' classic work on Value and Capital and directly introduces uncertainty and risk aversion into the expectations theory. It is also a kind of refined version of the market segmentation theory (discussed in next subsection), because it asserts that investors have preferred maturity habitats but can be induced to trade in other maturity segments if offered an inducement to do so in the form of a higher rate of return. This yield inducement is referred to as liquidity premium. However, as Wood and Wood point out, the term, "Term Premium" would be more appropriate than the popular name liquidity premium because the theory is concerned with risk due to price volatility rather than liquidity. This point is even more relevant in the case of T-bills (in which most empirical works are done), because the differences in the liquidity of T-bills of different maturities are negligible or nonexistent. See also Nelson on this point.

Equation (5) is still valid in LPT except that implied forward rates are now composed of the expected spot rates plus a liquidity (or term) premium. This can be expressed as,

$$ {}_{i}F_{1} = {}_{i}Y^{e}_{1} + L_{i} \qquad \ldots\ldots \qquad (9)$$

Therefore, equation (5) reduces to

$$1+Y_n = (1+Y_1)(1+{}_1Y_1^e+L_1)\ldots\ldots(1+{}_{n-1}Y_1^e+L_{n-1})^{1/n} \qquad \ldots\ldots (10)$$

It is usually believed that $L_1, \ldots L_{n-1}$ are positive and increasing with maturity because of the assumption that risk averting lenders prefer to invest short term and risk averting borrowers prefer to borrow long term. The reason for the difference in behaviours of lenders and borrowers is related to the matching principle. Radcliffe expresses this principle as, "If borrowers use the proceeds from a sale of securities to invest in long term real assets but must finance with short term borrowings, they incur an income risk.... . Conversely, if lenders (such as households) have short term investment horizons, they incur a principal risk when lending long term. Thus, lenders will demand a premium on long term issues .... a premium which borrowers will be willing to pay ...".

But Modigliani and Stuch argue that liquidity premiums may sometimes be decreasing (implying a downward bias in yield curves) with maturity when lenders prefer to longer term than borrowers would wish to borrow. One example of such an investor who may prefer long term securities is a life insurance company dominated by young policy holders. Empirical evidence often shows that liquidity premiums are positive although reaching a peak at short maturities between 6 to 9 months as in Mc Culloch's estimates. This can also explain the long end of yield curves

Market Segmentation Theory (MST)

This theory states that economic units which demand or supply financial resources have maturity preferences (preferred habitats) which effectively create a number of largely independent market segments.

Whereas the ET asserts that default free securities of different maturities are perfect substitutes and the LPT asserts that they are imperfect substitutes, the MST asserts that the market for securities of different maturities are so tightly compartmentalized that maturity groups are nonsubstitutable and the yield curve is broken up into distinct independent segments.

Therefore, the slope of the yield curve is determined completely by the relative supplies and demands within each market segment.

This theory explains the different shapes of yield curves during different stages of business activity as below.

Trough: During the trough of business cycles the yield curve is almost flat except for the very early maturity instruments which are what firms rely upon to provide liquidity needs. The low level of economic activity will cause businesses to accumulate large liquidity reserves instead of reinvesting funds in unneeded inventory, receivable, and plant. This tendency is reinforced by firms' precaution against short term insolvency problems made more likely by the depressed level of economic activity. The net result will be a large supply of funds to the money market, causing very low short term rates (see Figure 2 above)

Recovery: As business activity begin to pick up, excess liquidity is spent on working capital and plant additions. Moreover, total demand for credit (of all maturities) also rises causing short term rates to rise relatively more (from very depressed level) than proportionately to long term rates.

Peak: Finally, during the peak of the cycle, rates are the highest in the middle maturity range due to large demands for credit needed to support the procyclical expansion in receivables and inventory balances.

Thus, the varying shapes of yield curves are the result of the changing investment needs of the business firms over the course of the business cycles.

One implication of MST is that there is arbitrage opportunity for those who can accurately (but differently from the market as a whole) forecast the demand and supply situations in the different market segments. For example, a simple rule of thumb strategy would be to buy bonds (of long maturity) at the peak of economic activity and to sell at the trough of a recession. However, the important question is whether the market as a whole does see and seek such opportunities or not. According to the unbiased expectations theory a fully arbitraged market would eliminate all such profits and bring about the relationship between long and short rates as predicted in equations (4) and (5) above.

In the next section we focus on the flattening of the long end of yield curves.

THE FLATTENING OF YIELD CURVES AT LONG MATURITIES

Some Traditional Explanations

As pointed out in Section 1, regardless of the early maturity slopes of yield curves, it is consistently observed that yield curves become flat at the intermediate to long maturity end.

Because of this consistent tendency of yield curves, it is clear that any theory of term structure must be able to explain why this should happen. Malkiel has emphasized this point by saying that a term structure "theory must account for the pervasive tendency of the yields curves to level out as term to maturity increases and to develop what is called 'shoulder' irrespective of the general shape in the early maturities." Milkiel stipulates that the shape of yield curves depends on the sensitivity of bond price to changes in interest rates and that yield curves would tend to be flat for long maturities because long term bonds of different maturities have essentially the same price sensitivity to interest rate changes. But this approach depends on one empirical tendency to explain another empirical tendency without much theoretical basis.

Similarly, Lutz has tried to explain the flattening of yield curves by constant long term forward rates for long maturities. But, Livingston and Jain point out that this view would be justified only if investors would use the same forcasted rate as the best predictor of all distant rates because of the uncertainties attached to expectations of future rates. Moreover, we will see in the next subsection that the behavioural explanations and assumptions about forward rates, zero coupon discount rates, or bond price volatilities as those of Milkiel and Lutz are unnecessary for explaining the flat end of yield curves.

In Section 2 we hinted that the traditional term structure theories can also provide some explanations for this trend. For example the upward pressure of risk aversion and the implied downward bias in yield curves of MET can offset each other for long maturities to produce a flat end. Similarly, the LPT can explain this trend by the fact that term premiums reach a peak at early maturity levels.

However, such behavioural explanations appear to be ad hoc because the yields are not explicitly expressed in terms of term to maturity so as to enable us to categorically determine the slope of the yield curve as term to maturity increases. Therefore, in the following subsections we will review some recent and formal explanations of the flattening of the yield curves at long maturity end.

Flattening of Par Bonds

Livingston and Jain present a theoretical proof that flattening of yield curves for par bonds is inevitable for long maturities, without any assumptions about forward rates, zero coupon discount rates, bond price volatility, or whether or not investors can differentiate expectations of distant future rates from each other. The authors formalize the statement of Schaefer that constant coupon yield curves "are ... asymptotically horizontal no matter what shape the spot rate (zero coupon rate) curve adopts."

Livingston and Jain prove this statement in the following way:

Assuming perfect market without taxes, the price (P_n) of an n period default free noncallable bond with coupon C is

$$P_n = \sum_{j=1}^{n} \frac{C}{(1+R_j)^j} + \frac{1}{(1+R_n)^n} \quad \ldots\ldots \quad (1)$$

where R_j's are discount rates (assumed nonnegative) of zero coupon bonds (spot rates) of maturity j and represent the true term structure of interest rates.

Bond price expressed as a function of yield to maturity Y_n is,

$$P_n = \sum_{j=1}^{n} \frac{C}{(1+Y_n)^j} + \frac{1}{(1+Y_n)^n} \quad \ldots\ldots \quad (2)$$

Then a flat yield curve is defined by the following relation.

The difference $Y_{n+k}-Y_n$, is close to zero for $k \geq 1$.....(3)

For a par bond set $P_n=1$, and note that C equals Y_n. Therefore, solving for Y_n from (1) and (2) gives

$$Y_n = \frac{1-(1+R_n)^{-n}}{\sum_{j=1}^{n} 1/(1+R_j)^j} = \frac{1-D_n}{\sum_{j=1}^{n} D_j} = \frac{1-D_n}{A_n}$$

where $D_j = \frac{1}{(1+R_j)^j}$ and $A_n = \sum_{j=1}^{n} D_j$.

Then the following theorems are proved (see Livingston and Jain, Appendix).

<u>Theorem 1.</u> If $Y_{n+k} > Y_n$ for any $k \geq 1$, then $Y_{n+k}-Y_n \leq \frac{1}{n}$.

<u>Theorem 2.</u> If $Y_{n+k} < Y_n$ for any $k \geq 1$, then $Y_n-Y_{n+k} \leq Y_n \left[\frac{k}{n+k}\right]$

Thus, the two theorems show that the rise or decline in a par bond yield curve must be close to zero for long maturities. In other words, par bond yield curves must be flat for long maturities. The authors als suggest an intuitive explanation for this result, "The yield to maturity for a coupon bearing bond is a complicated. Weighted average of the zer coupon discount rates. As maturity increases, the weights for the longe term zero coupon rates tend to become relatively small, because the present value factors for the distant time periods tend to become of lesser relative importance, thereby implying flattening of the yield curve...."

Finally, Livingston and Jain also argue that these results for par bonds can be extended to non-par bonds, since it can be shown that, for a Taylor expansion evaluated at par, yield to maturity for non-par bonds is, within a reasonable interval around par, approximately equal to yield to maturity for par bonds for long maturities.

Flattening for Intermediate Maturities

As discussed above Livingson and Jain have shown that flattening yield curves are inevitable for very long maturities. However, Livingston suggests that the widely observed flattening of yield curves for intermediate maturities has not been explained by the above mentioned work. Therefore, Livingson tries to explain the flattening of yield curves even for mid-range maturities under some 'weak' bounds for forward interest rates.

First the case of zero coupon bonds is examined, and then tnat of coupon-bearing par bonds.

Case of Zero Coupon Securities

In a perfect market the price of a default free, non-callable, zero coupon bond with maturity n and U.S. $ 1 par value is given as

$$D_n = (1+R_n)^n \quad \ldots\ldots \quad (1)$$

Denoting forward rates by $f_1, \ldots, f_n$ we have

$$(n+R_n)^n = (1+R_1)\ (1+f_2)\ \ldots\ (1+f_n) \quad \ldots\ldots \quad (2)$$

This gives,

$$R_{n+1} - R_n = (1+R_n)^{\frac{n}{n+1}}\ (1+f_{n+1})^{\frac{1}{n+1}} - (1+R_n) \quad \ldots\ldots \quad (3)$$

Livingston proves that the following equation holds approximately in case of discrete compounding, and exactly in case of continuous compounding

$$R_{n+1} - R_n = \frac{f_{n+1} - R_n}{n+1} \quad \ldots\ldots \quad (4)$$

Equation (4) clearly shows that a unit change in the forward rate (f_{n+1}) relative to the n period spot rate (R_n) results in a change in the spot rate equal to $\frac{1}{n+1}$, which gets smaller and smaller a n gets larger - that is, the yield curve flattens.

Now suppose $f_{n+1} = w\ R_n$, so that equation 4 becomes,

$$R_{n+1} - R_n = \frac{(w-1)R_n}{n+1} - \quad \ldots\ldots \quad (5)$$

As illustrated numerically by Livingston, plugging some reasonable values of R_n and w, we find that the yield curve starts flattening as early as 10 to 15 years to maturity.

However, it should be noted that this result is based on the assumption made about forward rates before deriving equation (5).

Similarly, Livingson examines the case of coupon bearing par bonds. He proceeds as in flattening of par bonds and derives the result that $Y_{n+k} - Y_n$ is a function of reciprocal of term to maturity n. In addition he shows that there is much more marked flattening for par bonds relative to zero coupon bonds. For example assuming that f_{n+j} is less than or equal to wY_n, he shows that for a rising yield curve,

$$Y_{n+k} - Y_n \leq \frac{w-1}{wn+1} \qquad \ldots\ldots \quad (6)$$

For reasonable values of the parameters, Livingston argues that this relation gives tighter bounds for yields than Livingston and Jain.

Asymptotic Flattening and Stability of Forward Rates

Most estimation methods of term structure depend heavily on polynomial splines or exponential splines. But Shea finds that exponential splines are subject to the same shortcomings that polynomial splines are. The main problem with polynomials or exponentials is that they imply that implicit forward rates increase or decrease explosively as maturity is extended.

Therefore, Nelson and Siegel avoid the use of polynomials in deriving models of yield curves which can explain the observed shapes. The authors apply the expectations theory to the assumption that spot rates are generated by a differential equation. Then the forward rate will be solution to the equations.

For example, if the instantaneous forward rate at maturity m is r(m), it may be given by the solution to a second order differential equation

$$f(m) = b_0+b_1\exp(-m/t_1)+b_2\exp(-m/t_2) \qquad \ldots\ldots \quad (1)$$

where t_1 and t_2 are time constants, and b_0, b_1, and b_2 are determined by initial conditions.

According to the expectations theory, the yield to maturity, Y(m) is the average of the forward rates and is given by

$$Y(m) = \frac{1}{m} \int_0^m f(s)ds \qquad \ldots\ldots \quad (2)$$

Since the forward rate curves can take different shapes (monotonic, humped or S type) depending on the parameters of equation 1, the yield to maturity will also take similar shapes. However, Nelson and Siegel find that equation 1 is overparameterized. Therefore, following the advice of Friedman, they stipulate a more parsimonious model by restricting the equation to equal roots (and $t_1 = t_2 = t$).

$$f(m)=b_0+b_1\exp(-m/t)+b_2(m/t)\exp(-m/t) \qquad \ldots\ldots \quad (3)$$

Now integrating equation (2) we obtain

$$Y(m) = b_0 + (b_1+b_2)\,[1-\exp(-m/t)]/\tfrac{m}{t} - b_2\exp(-m/t) \qquad \ldots\ldots (4)$$

Thus, the limiting value of Y(m) as $m \to \infty$ is b_0, and as m gets small is (b_0+b_1), same as for f(m). In conclusion, the yield curve may take different shapes according to the values of the parameters in equation (4), but all curves are flat asymptotically.

Note that this result is based on the stability of the forward rates function. In other words, the long maturity behaviours of both forward rates and yield curves are dominated by decay that is proportional to the reciprocal of maturity. However, this model considers only pure discount bonds.

Similarly, Nelson and Siegel prove flattening of yield curves at long term maturities (for pure discount bonds) to be approximately proportional to the reciprocal of the time to maturity under fairly general conditions, and suggest the use of a "reciprocal maturity yield curve" for easier interpretation.

They start with equation (2) given above and add two assumptions:

(a) Forward rate has a finite limiting value,

$$f(\infty) = \lim_{m\to\infty} f(m) \text{ exists and is finite} \qquad \ldots\ldots (5)$$

(b) Forward rates approach this asymptotic value quickly enough,

$$\lim_{m\to\infty} \int_0^m [f(s)-f(\infty)]\,ds = c \text{ exists and is finite} \qquad \ldots\ldots (6)$$

This is the area between the forward rate curve and its asymptotic value.

Nelson and Siegel argue that this is not a very restrictive assumption, since exponential decay $f(m)-f(\infty) \sim \exp(-m/t)$, for positive t, easily satisfies this assumption, as does the much slower algebraic decay $f(m)-f(\infty) \sim m^{-\alpha}$ for large m with any $\alpha > 1$.

From equation (2) and the assumptions, it follows that,

$$Y(\infty) = \lim_{m\to\infty} Y(m) = f(\infty) \qquad \ldots\ldots (7)$$

The authors then prove that

$$Y(m) = Y(\infty) + \frac{c}{m} + f(m) \qquad \ldots\ldots (8)$$

where c is defined in (6).

From equation (8) it is clear that the yield curve approaches a finite asymptotic value as maturity increases, whether the yield curve is initially rising or not.

CONCLUDING REMARKS

This paper reviewed the traditional and recent explanations of the shapes of yield curves, with emphasis on the long maturity behaviour. The emphasis was due to the frequently and consistently observed flattening of yield curves toward the long end regardless of the initial slopes. This fact suggests that models of yield curves should consider such behaviour as a constraint on the modelling. The man purpose of this paper is to provide a basis and motivation for further research in the area of term structure and yield curves. A future research may be directed toward developing a model based on economic reasoning (rather than ad hoc assumptions about the behaviours of forward rates and the expectations of investors) - a model which can adequately explain the different shapes of yield curves and at the same time lends itself to reasonable estimation and testing.

SELECTED REFERENCES

Bierwag, G. and M. Grove (1967), "A Model of the Term Structure of Interest Rates", Review of Economics and Statistics 49.

____ (1971), "A Model of the Structure of Prices of Marketable U.S. Treasury Securities", Journal of Money, Credit and Banking.

Chambers, D.R., W.T. Carleton, and D.W. Waldman (1984), "A New Approach to the Estimation of the Term Structure of Interest Rates", Journal of Financial and Quantitative Analysis, 19.

Cox, J.C., J.E. Ingersoll, and Stephen A. Ross (1981), "A Reexamination of Traditional Hypotheses about the Term Structure of Interest Rates", Journal of Finance.

Dothan, Uri (1978), "On the Term Structure of Interest Rates", Journal of Financial Economics.

Durand, D. (1942), "Basic Yields of Corporate Bonds, 1900-1942", Technical Paper 3, New York, NBER.

____ (1958), "A Quarterly Series of Corporate Basic Bond Yields, 1952-57, and Some Attendance Reservations", Journal of Financial 13.

____, and Willis J. Winn (1947), "Basic Yields of Bonds, 1926-47: Their Measurement and Pattern", Technical Paper 6, New York, NBER.

Fama, E.F. (1976), "Inflation Uncertainty and Expected Returns on Treasury Bills", Journal of Political Economy.

____ (1976), "Forward Rates as Predictors of Future Spot Rates", Journal of Financial Economics.

Fama, E.F. (1984), "The Information in the Term Structure", Journal of Financial Economics 13.

Fisher, I. (1896), "Appreciation and Interest", Publications of the American Economic Association, 11.

___ (1930), Theory of Interest, Macmillan, NY.

Hamburger, M.J., and E.N. Platt (1975), "The Expectations Hypothesis and the Efficiency of the Treasury Bill Market", Review of Economics and Statistics.

Hicks, J.R. (1946), Value and Capital, 2nd ed., Clarendon Press, Oxford.

Ingersoll, J.E. (1987), Theory of Financial Decision Making, Rowman and Littlefield.

Livingston, M. (1987), "Flattening of Bond Yield Curves", Journal of Financial Research, 10, 1.

Livingston, M. and S. Jain (1982), "Flattening of Bond Yield Curves for Long Maturities", Journal of Finance.

Malkiel, B. (1966), The Term Structure of Interest Rates, Englewood Cliffs, N.J., Prentice Hall.

McCulloch, J.H. (1971), "Measuring the Term Structure of Interest Rates", Journal of Business, 44.

___ (1975), "An Estimate of the Liquidity Premium", Journal of Political Economy, 83.

Modigliani, F., and R.C. Sutch (1966), "Innovation in Interest Rate Policy", American Economic Review.

___ and ___ (1967), "Debt Management and the Term Structure of Interest Rates: An Empirical Analysis of Recent Experience", Journal of Political Economy, 75.

Nelson, C. (1972), The Term Structure of Interest Rates, NY: Basic Bks.

Radcliffe, R.C. (1987), Investment, 2nd ed., Scott, Foresman and Co.

Schaefer (1977), "The Problem with Redemption Yields", Financial Analysts Journal, 33.

Shea, G.S. (1984), "Pitfalls in Smoothing Interest Rate Term Structure Data: Equilibrium Models and Spline Approximations", Journal of Financial and Quantitative Analysis, 19.

Siegel, A.F., and C.R. Nelson (1987), "Parsimonious Modelling of Yield Curves", Journal of Business, 60.

Siegel, A.F., and C.R. Nelson (1988), "Long Term Behaviour of Yield Curves", Journal of Fiancial and Quantitative Analysis, 23.

Wood, J.R. (1983), "Are Yield Curves Normally Upward Sloping? The Term Structure of Interest Rates, 1962-1982", Economic Perspective, Federal Reserve Bank of Chicago.

Wood, J.R. and Norma L. Wood (1985), "Financial Markets", Harcourt Brace Jovanovich, Inc.