

# Sensitivity of Tax Yields and Their Forecasting

G. S. Monga\*

In this paper, some methods of forecasting tax revenue have been examined. The earlier part deals with various methods of estimating elasticity and buoyancy. The role of discretionary changes is also considered. Apart from a brief survey of various techniques employed in forecasting tax revenue some new techniques and suggestions have been given.

## Elasticity and Buoyancy

The concepts of elasticity and buoyancy of taxes are often used to examine the responsiveness of tax collections to variations in national income. The measurement of elasticity requires constancy of the tax rate and base over the period of time under examination. To meet this artificial and static assumption of constancy, elasticity has to be estimated after adjusting the data for any discretionary changes that may have been made during the period without any change in the tax structure or base of the tax. Buoyancy is estimated without allowing for discretionary changes. It uses the data as they are and compared to the elasticity measure, may reflect the excess burden of the tax system.

The frequency of discretionary changes does not necessarily follow any framed rule and the magnitude of these changes can differ significantly from one year to another.

---

\* Dr. Monga is professor of Economics at the university of Bombay.

There may or may not be any relation of these changes with the existing revenues. Even in a single year, alterations in different taxes may vary widely and new taxes may be introduced and some older ones abolished. For forecasting purposes, the discretionary changes cannot be assumed to have the same effect throughout.

A historical tax revenue series cannot be fully adjusted for discretionary changes but depending on their nature and frequency in the given series, some methods of adjustments are available. These are proportional adjustment method, constant rate structure method, dummy variable approach and Divisia index, the last two not requiring any prior purging of the series.

### Adjustments for Discretionary Changes

The proportional adjustments method of Frest (1962) is straightforward and has been applied to the data of several countries: by Mansfield (1972) for Peru, Baas and Dixon (1974) for UK, and Byrne (1979) for Zambia.

The method requires the budget estimates of tax yield resulting from discretionary measures. Only simple adjustments are involved through a constant proportionality factor. The proportional adjustment method has the drawback that the budget estimates of the tax yield which are required for adjustment are often difficult to obtain. However, once the adjusted tax data are obtained, obviously assuming constancy of elasticity of tax with respect to income, the procedure is straightforward.

There is a reference year which provides the basic structure of rate and each year's actual yield is adjusted to allow for the impact of discretionary changes by using a series of multiplicative factors. A simplified version of Prest's formula is given by:

$$T_{ij} = T_j \lambda_j \lambda_{j-1} \dots \lambda_2$$

Where,  $T_j$  is the actual yield in year  $j$

$T_{ij}$  is the adjusted yield with reference to year  $i$

and  $\lambda_j = \frac{T_j - d_j}{T_j}$  Where  $d_j$  is the effect of the discretionary change in year  $j$

With year 1 taken as the reference year we will have:

$$T_{11} = T_1, T_{12} = T_2 \lambda_2 = T_2 - d_2$$

$$T_{13} = T_3 \lambda_3 \lambda_2 = (T_3 - d_3) (T_2 - d_2) / T_2$$

$$T_{14} = T_4 \lambda_4 \lambda_3 \lambda_2 = (T_4 - d_4) (T_3 - d_3) (T_2 - d_2) / T_2 T_3$$

In general,

$$T_{1k} = T_k \lambda_k \lambda_{k-1} \dots \lambda_2$$

The observations before the reference year can be similarly adjusted. The choice of the reference year does not make any difference to the elasticity measure.

In Sahota's (1961) version of adjustment for discretionary change, the new rate and base are allowed to influence tax elasticity. Although the method adjusts for discretionary changes in the year in which they take place, a recasting of the model as done by Bahl (1972) leads to the same result as that of prest.

The Sahota expression to determine the actual tax receipts excluding discretionary effects in the year  $i$  may be written as :

$$I_i = \frac{T'_i}{T_i} I_{i-1}$$

Where  $T'_i$  stands for  $i$ th year tax collections adjusted to rates in year  $i-1$ .

The empirical results obtained from this method should be the same as those from the Prest method. Other proportional adjustment methods do not differ much from the Prest method. In the constant structure method, a series showing the appropriate yield of a constant rate tax is constructed. The choice of a proper reference year is important here because the elasticity of the tax structure in the reference year influences any forecast made from the data. It is usual to take the current year as the reference period to base

the forecast on the elasticity of the existing structure. The method which amounts to the "reconstruction of a simulated time series based on the current tax structure, probably results in the most appropriate historical tax income relationship for forecasting purposes. (It) requires separate consideration of each major tax whereas the data adjustment method is a generalised procedure applicable to all taxes" (Bahl : 1972)

### Dummy Variables

If T and Y are free from extraneous influences, a loglinear relation between the two gives an estimate of income elasticity. But since administrative and discretionary changes are usually present, dummy variables may be used as additional explanatory factors. It can be a simple linear relation like:

$$\ln T = \alpha + (\beta + \gamma D) \ln Y$$

where the dummy variable D is associated with the discretionary change. D takes the value 1 or 0 depending on whether there is or is not a discretionary change. To avoid the loss of degrees of freedom, not many dummy variables can be introduced even if there are many discretionary changes. The method does not require purging of the series but that may not be a great advantage. Singer gives an alternative equation using dummy variables  $D_i$  :

$$T = \alpha + \beta Y + \sum_{i=1}^n C_i D_i$$

Where T and Y may be in aggregate or per capita terms. Also the equation may be used with  $\ln T$  and  $\ln Y$  in place of T and Y. Obviously  $C_i$  turns out to be representing change in elasticity occurring because of discretionary changes.

### Divisia Index

In the Divisia index approach of Choudhary (1979) to measure the revenue

effects of discretionary changes, the latter are considered as analogous to technical change, assuming that the underlying tax function is homogeneous. The procedure is elaborate and the tax revenue elasticity is estimated by adjusting the buoyancy by a simple transformation of a Divisia index of discretionary changes.

The method does not require the prior elimination of discretionary effects in tax elasticity estimation. This is done by first estimating the effects of discretionary measures on revenue by an index which isolates automatic from total growth of revenue. The buoyancy measure obtained by any standard technique is then adjusted by a suitable transformation of the index of discretionary revenue to find tax yield elasticity. The method which uses historical data does not require past discretionary changes. It may not always give good results and the proportional adjustment method is suggested in case full information on discretionary changes is available. The method is described briefly here.

Assuming a stable aggregate tax function relating tax yield and the base, the upward movement along the tax revenue can be represented by tax elasticity relative to growth in base. The shape of the tax function remains unaltered in the absence of discretionary change which when introduced would bring about a movement along the tax yield curve and may also induce a shift in the same.

The Divisia index of the revenue effects of discretionary tax changes equals the percentage increase in total tax yield divided by percentage automatic increase. The index must possess the invariance property for which the tax function must be continuously differentiable. It is possible to do away with the linear homogeneity assumption which is rather restrictive particularly because of progressive tax rate structures. Choudhary (1979) begins with a continuously differentiable aggregate tax yield function given by:

$$T(t) = f(x_1(t), \dots, x_k(t); t)$$

Where  $x_i(t)$  is the proxy tax base with  $k$  categories of taxes ( $i = 1, \dots, k$ ) and the time variable  $t$  is a proxy for discretionary measures.

If  $D(t)$  is the Divisia index of discretionary change and  $\dot{D} = \frac{dD/dt}{D}$  then

$\dot{D}(t) = \dot{f}_t / f$  and  $\beta_i(t) = \frac{f_i x_i}{f}$ . The index of growth of tax revenue owing to discretionary measures over the period  $(0, n)$  is:

$$D(n) = \frac{T(n)}{T(0)} \exp \left\{ -\sum_1^k \int_0^n \beta_i(t) \dot{x}_i(t) dt \right\} \text{ where } \dot{x} = \frac{dx/dt}{x}$$

and  $D(0)$  is set equal to unity. Assuming a constant  $\beta_i = \beta$  we have:

$$\ln D(n) = \ln \frac{T(n)}{T(0)} - \sum_1^k \beta \ln x_i(n) / x_i(0)$$

The index  $D(n)$  may provide a reasonable measure of the effects of discretionary changes. If the degree of homogeneity of the tax function  $T(t) = f [x_1(t), \dots, x_k(t), t]$  is assumed to be  $r > 0$  and if the growth rates of all the bases are equal to that of GDP or any income variable  $x(t)$  then the buoyancy  $\mu$  of tax yield can be found from the tax function  $T(t) = a x^r D^*(t) = a x^\mu$ .

Where  $D^*$ , as a special case of  $D$  denotes an index of revenue growth due to discretionary changes. For the time interval  $(0, n)$  we have

$$D^*(n) = [x(n) / x(0)]^{\mu-r}$$

Which equals unity when elasticity is the same as buoyancy in the case of no discretionary change. If the buoyancy is found from the tax function  $T = ax^\mu$  and the index  $D$  from the underlying tax function, we get  $\hat{\gamma} = \mu - \frac{\ln D(n)}{\ln x(n)/x(0)}$ . Depending on the index being disaggregative or aggregative there will be two values of  $\gamma$  which will be equal if the bases grew at rates proportional to GDP growth rate. Since historical data are usually annual we may use the discrete value of  $\beta_i$  given by

$$\beta_i(t) = \frac{T_i(t) - T_i(t-1)}{X_i(t) - X_i(t-1)} \cdot \frac{X_i(t)}{T(t)}$$

using this we get the weighted average of  $\beta_i$ 's given by

$$\beta = \frac{1}{n} \sum_1^n \beta_i(t) \dot{X}_i \bar{X}_i \text{ where } n \bar{X}_i = \int_0^n \dot{X}_i dt = \ln \dot{X}_i(n) / X_i(0)$$

### Tax Revenue Forecasting

The linear regression model is commonly used to forecast tax revenue and we will consider here several variations and other possibilities. Before that we examine the tax elasticity coefficient which is a useful parameter in revenue forecasting. It measures the progress of the tax structure and administrative improvement. A low measure of elasticity points out the need for additional efforts to mobilise resources and for adoption of a proper strategy to make the tax system revenue buoyant. It means that the buoyancy measure alone may not be a good substitute for elasticity or built-in flexibility. If the tax revenue series has been purged of discretionary changes, its relation with incomes may be easily established. Writing  $\Delta T_i = T_i - T_{i-1}$ ,  $T = T_{i-1}$ ,  $Y = Y_{i-1}$  in the definition

$$\eta = \frac{\Delta T \cdot Y}{\Delta Y \cdot T}, \text{ we have } T_i = T_{i-1} (1 + \eta \Delta Y / Y_{i-1})$$

which may be alternatively written, using  $c = \Delta T / \Delta Y$  for the flexibility coefficient,

$$T_i = T_{i-1} + c \Delta Y$$

for  $k$  lags, this may be written

$$T_i = T_{i-k} + c \Delta Y$$

where now  $\Delta Y = Y_i - Y_{i-k}$ .

Depending on the item to be forecast, use may be made in these equations of the elasticity of total tax or its various components. This may also be done for two or three major tax groups separately.

### Tax Functions

To estimate a simple tax income relation it may be useful to fit a linear regression of tax yield  $T$  on income  $Y$ :

$$\ln T = \ln \alpha + \beta \ln Y \text{ or } T = \alpha Y^\beta$$

The regression coefficient can be easily seen to be the constant income elasticity coefficient provided there are no discretionary tax changes over the period under consideration or, if there are such changes, they have been removed. If the historical series has not been purged of discretionary changes, the regression

coefficient estimated from the same equation is termed buoyancy. Depending on the availability of data and the number of observations, additional explanatory variables may be introduced in the tax function and their significance tested.

Confining ourselves to a single explanatory variable  $Y$  and assuming absence of discretionary changes, the constancy of elasticity deserves further examination. Taxes may be related to their base  $B$  and the base related to income  $Y$ , in the equations:

$$\ln T = \alpha_1 + \beta_1 \ln B \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (1)$$

$$\ln B = \alpha_2 + \beta_2 \ln Y \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (2)$$

By substitution,  $\ln T = (\alpha_1 + \beta_1 \alpha_2) + \beta_1 \beta_2 \ln Y$

Obviously now the constancy of elasticity requires the constancy of the product  $\beta_1 \beta_2$ . This means that the constancy of either  $\beta_1$  or  $\beta_2$  implies that of the other or, the variations in the two compensate each other to keep  $\beta_1 \beta_2$  constant.

Returning to equations (2) and (3) we find that if the elasticity coefficient is partitioned

$$\eta = \frac{\Delta T}{\Delta Y} \frac{Y}{T} = \left( \frac{\Delta T}{\Delta B} \frac{B}{T} \right) \left( \frac{\Delta B}{\Delta Y} \frac{Y}{B} \right)$$

the two brackets on the right stand for  $\beta_1$  and  $\beta_2$  which are, respectively, the responsiveness of taxes to changes in the base and the responsiveness of base to income changes.

Equations (1) and (2) may be visualised as constituting a simple recursive model. Equation (2) can be used to obtain the estimate of  $\ln B$ . If the estimated

values  $\hat{\ln B}$  are substituted in (1) we have

$$\ln T = \alpha + \beta_1 \hat{\ln B}$$

Instead of using  $\beta_1$  obtained directly, it can be obtained thus and can be used to examine the constancy of  $\beta_1 \beta_2$ . Lags may also be introduced in the equations.

It is useful to employ single equation models to represent tax functions, changes in whose parameters represent changes in the tax structure. Linear models

with two or more variables are usually not employed because the number of observations available is not large enough and also because reliable forecasting of many explanatory variables may not be easy. Moreover, in the case of developing countries, owing to the presence of an unorganised sector and historical and institutional circumstances, it is not easy to suggest exact models. However, multiple regression with some additional explanatory factors may be tried.

On the left side of the tax equation, the tax revenue  $T$  need not be the only explained factor. It could as well be replaced by the ratio  $\gamma_T$  (taxable income to total income) or the ratio  $\gamma_g$  (taxes to GNP) or the ratio  $\gamma_a$  (actual tax revenue to taxable income). Depending on the left side factor, the explanatory variables may be suitably chosen from per capita income  $Y_p$ , population  $N$ , exemption range  $G$ , development and nondevelopmental expenditures  $S$  and even dummy variables  $D$  representing discretionary changes, policy factors or other exogeneous effects. Let  $Z$  stand for one of  $T_a$ ,  $\gamma_T$ ,  $\gamma_a$ , etc. and  $X_i$ 's stand for some or more of  $Y$ ,  $Y/N$ ,  $N$ ,  $G$ ,  $S$ ,  $D$ , etc. Dummy variables can be used in a variety of well known ways. A typical equation then would be:  $Z = \alpha + \beta_1 X_1 + \beta_2 X_2 + \dots + \gamma D$ .

The above methodology can be applied to total tax revenue as well as to individual taxes with suitable modifications. Each case has to be considered on its merit in an empirical study provided biases resulting from multicollinearity, autocorrelation etc. are taken care of.

Further modification is possible in the case of total tax revenue which can be explained by additional factors like the various components of total revenue. But of bias may be introduced in this case.

It is usual to think of discretionary changes as simple administrative matter having nothing to do with other variables. But in practice, decisions about discretionary changes may be affected by the existing tax revenues and incomes, as well as by their lagged values and past discretionary changes. Thus discretionary cha-

nges  $D_t$  may be considered to be a function of  $D_{t-1}$ ,  $T_{t-1}$ ,  $Y_{t-1}$  etc. In particular, one may experiment with a relation like

$$\ln D_t = a + b_1 \ln D_{t-1} + b_2 \ln T_{t-1} + b_3 \ln Y_t$$

On the right side may be introduced variables like  $Z = T/Y$  and on the left an alternative variable like  $D/Y$  may be tried.

**Other Tax Functions**

It is possible to construct tax functions by making suitable assumptions about the elasticity coefficient. Thus elasticity may be made a function of tax income ratios or, as Singer (1970) does it, elasticity is made a linear function of the rate structure:

$$\eta_i = a + b (Z^* - Z_i)$$

where  $Z$  is the tax rate  $T/Y$ ,  $Z^*$  is the maximum tax rate and  $Z_i$  is the average tax rate in year  $i$ . If absence of cyclical fluctuations is assumed during the period of study and  $\eta_i$  can be found for successive years, this relation or any similar relation can be estimated directly. It is a differential equation which may also be solved by substituting for  $\eta$  and  $Z$  and assuming  $Z^*$  to be a constant. Thus

$$\frac{dY}{dT} \frac{Y}{T} = a + b \frac{T}{Y}$$

so that  $\ln T = \alpha \ln Y + \ln (\alpha - 1 - \beta T/Y)$

where the constants  $\alpha$  and  $\beta$  can be written in terms of  $a$ ,  $b$ ,  $Z^*$ . The result may also be expressed as  $(T/Y)^\alpha = (\alpha - 1) - \beta (T/Y)$

The relation cannot be estimated easily.

A variety of other relations may be similarly derived by assuming elasticity to depend on some power of  $T/Y$ ,  $\Delta Y/Y$  etc.

If we use the adaptive expectation scheme we may write

$$T_t^* - T_{t-1}^* = \beta (T_{t-1} - T_{t-1}^*) \dots \dots \dots (3)$$

where  $T_t^*$  is the expected  $T$  for time  $t$ , and  $\beta$  is the adaptive expectation coe-

efficient ( $0 < \beta < 1$ ). We have then from (3) the expected value of  $T_t$  as a geometrically weighted average of all previous actual values of  $T$ :

$$T_t^* = \sum_{i=1}^{\infty} \beta(1-\beta)^{i-1} T_{t-i}$$

The adaptive expectations model may be used as a first approximation. It takes into account the authorities expectational behaviour in making their decisions though it does not allow for any constraints.

### Endogeneous Dummies

To incorporate the effect of discretionary changes on tax revenues ( $T$ ) it is usual to fit a regression of  $T$  on income  $Y$  and an extra explanatory variable  $D$  which is a dummy variable. However there is another side of the problem in which  $D$  may be looked upon as an endogenous rather than exogenous variable. This is because  $D$  itself may be determined by administrative choices on the basis of expected tax revenues and other factors. This argument should carry weight in the case of some taxes.

The usual regression methods will not work here. Use can be made of a variety of discrete regression models. The choice will depend on whether  $D$  is binary (taking only two values 0, 1) or whether it can take more values as for instance when discretionary changes fall into three or more intervals. For the binary case we have the linear probability model of Ladd (1966) which is related to the linear discriminant function. The probit model as suggested by Goldberger (1964) may also be used. All these models have to be employed with approximate statistical precautions. Use can be also made of simultaneous recursive models as has been done by Heckman (1978). This requires a separate treatment.

### Selected References

1. **Bahl, R. W. (1972)** "Alternative Methods for Tax Revenue Forecasting in Developing Countries: A Conceptual Analysis" *IMF Staff Papers*, Fiscal Affairs Department Working Paper no. DM/72/83, October 16.
2. **Bass, Hassel J and Dixon, A. (1974)**. "The Elasticity of the British Tax system" *IMF, FAD, Working Paper No. DM/74/96, Sep. 23.*
3. **Byrne, William J (1979)** "The Elasticity of the Tax system of Zambia, 1966-77" *IMF, FAD, Working Paper No. DM/79/87 Nov. 27.*
4. **Choudhari, N. N. (1979)** "Measuring the Elasticity of Tax Revenue: A Divisia Index Approach", *IMF Staff Papers*, Vol 26.
5. **Goldberger, A. S. (1964)** *Econometric Theory* (John wiley.)
6. **Heckman, J. (1978)** "Dummy Exogenous Variables in Simultaneous Equation Systems" *Econometrica*, Vol. 46.
7. **Ladd, G. (1966)** "Linear Regression Functions and Discrete Functions" *Econometrica*, Vol. 34.
8. **Mansfield, Charles Y. (1972)** "Elasticity and Buoyancy of a Tax system: A Method Applied to Paraguay", *IMF Staff Papers*", Vol. 19, No. 2, July.
9. **Prest, A. R. (1962)** "The Sensitivity of the Yield of Personal Income Tax in the U.K." *Economic Journal*, Vol. 72.
10. **Sahota, G. S. (1961)** *Indian Tax Structure and Economic Development* (Bombay: Asia Publishing House)
11. **Singer, Neil M. (1968)** "The Use of Dummy Variables in Estimating the Income Elasticity of State Income Tax Revenues" *National Tax Journal*, Vol. 21.
12. —————(1990) "Estimating State Income Tax Revenues: A New Approach" *The Review of Economics and statistics*, Vol. 52