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The Economics of Sanchaya Kosh: Mathematical Analysis@

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The main purpose of this article is to explore a method of finding out the amount withdrawable from the Sanchaya Kosh after an employee gets retired or resigns, as relevant. The Sanchaya Kosh can use the formula developed in this article instead of using the traditional methods and save valuable time of its employees. It, of course, implies that the interest is charged the way it is assumed in this article. I have tried to make assumptions and set problems which would conform to the reality. The article, however, does not touch upon the subject of how the Sanchaya Kosh runs, how the profit is available to Sanchaya Kosh and so on. It is concerned with the deduction of a general formula for calculating the amount due to the employees at the end of a referred period of years. Furthermore, the different factors that affect the amount are analysed and a method of solving a complex problem involvong more than one aspect is highlighted. Since the percent to be added by the Government, the rate of interest and other factors may vary from time to time, efforts have been made to generalise and find solutions that may be applied under different situations.

For the convenience of comparing the traditional methcd of calculation with the method that is being suggested in this article, the traditional method is shown in bird's eye view

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@ Sanchaya Kosh is the prevalent Nepali translation of the Provident Fund, in the first part of other article.[1] In the second part, a formula is obtained for calculating the amount payable to an employee without taking into account the increase in the salary, increase in grade, limitation of the grade and the dividend that is added to the employees' fund after a certain period of years. The third part has taken into account the increase in salary, increase in grade and limitation of the grade and the dividend. The possibility of the beginning and termination of employment not conforming to the conditions laid down in the second part and the case in which the grade starts not from the second year is analysed in the fourth part. The fifth part provides a few practical examples. The methodology to be applied for the solution of a problem involving more than one of the above cases is explained in the concluding section of the fifth part. Finally, an inference is drawn at the last part of the article.

Part I

In this part, I have tried to give a general description of the traditional method of calculation. As a basis for highlighting the method of calculation that is being used, I have gone through my own record filled by Sanchaya Kosh of Nepal. I hope that this is similar in all other cases.

At the end of every month, the monthly total contribution which is the sum of contribution from employees' side and an equal amount from HMG's side (which constitutes 20% of the salary) is deposited in Sanchaya Kosh. The interest to the sum deposited upto the end of the month Srawan) is calculated at the end of the month Bhadra.[2] The average interest coefficient on the money which is being deposited during period of any number of days and months within a year is arbitr arily taken as half of the maximum interest coefficient. Since the rate of interest per annum, R, is equal to 10%, the maximum interest coefficient is $10/100=0.1$ and the average interest coefficient is, therefore, 0.05. Suppose a person started job as an Engineer with salary of Rs. 700 per month at the beginning of 11th month of the financial year, i,e the beginning of Jestha of, say, 2038.[3] The total contribution of the months Jestha, Asadh and Srawan of 2038 will be Rs. 420. Since an employee gets his first grade starting from the beginning of the financial

1. The traditional method refers to the method that is being used by the Employees' Provident Fund of Nepal

2. The twelve months according to Nepali Calender are: Baisakh, Jestha, Asadh, Srawan, Bhadra, Asoj Kartik, Marga, Poush, Magha, Phagun and Chaitra. Baisakh is the 1st month of the calender.

3. In Nepal the financial year starts from the beginning of Srawan of the 1st year and terminates by the end of Asadh of the second year.

year and provided that he has worked a complete financial year the Engineer in question will start to get grade starting from the Srawan of 2039 only. As the interest is calculated at the end of Bhadra and as the above-mentioned sum of Rs. 420 will be liable to interest at the rate of average interest coefficient, i,e, 0.05, the interest will be Rs. 21. So the amount of money deposited at the end of Bhadra of 2038 will be Rs. 441. This amount does not take into account the monthly total contribution of Bhadra of 2038. The monthly total contributions of Bhadra, Asoj, Kartik, Marga, Poush, Magha, Phagun and Chaitra of 2038 and Baisakh, Jestha and Asadh of 2039 are all equal to Rs. 140 each and supposing the grade of the Engineer as Rs. 15, the monthly total contribution of Srawan of 2039 is Rs. 143. So, the sum of the yearly total contribution which is liable to interest at the rate of average interest coefficient will be Rs. 1683 ($11 \times 140 + 143$) and the interest on this money is Rs. 84. 15. The interest on Rs. 441 already in deposit at the end of Bhadra of 2038 gets an interest at the rate of maximum interest coefficient i,e, 0.1. So, the interest on this money is Rs. 44.10 and the total interest on the total deposit of the Engineer will be Rs. 128.25 (84. 15+44.10) Hence the amount deposited at the end of Bhadra of 2039 without the account of monthly total contribution of Bhadra of that year is Rs. 2252.25 (441+1683+128.25). Suppose, a dividend of 5% of the money deposited at the end of Bhadra of 2039 is added to the employees, fund, then the total money deposit of the employee at the end of Bhadra of 2039 is Rs. 2252.25+Rs.2252.25 $\times$ 5/100 or Rs 2364.86. This will be liable to interest at the rate of maximum interest coefficient 0.1 while the money that is being deposited between Bhadra of 2039 and Srawan of 2040 will be liable to interest at the rate of average interest coefficient 0.05.

Two things should be noted here, Firstly, there is no consistency in whether the yearly interest provided at the end of Bhadra includes the interest on monthly total contribution of Bhadra itself. It depends, I suppose, on whether the monthly total contribution of Bhadra is deposited on the account of Sanchaya Kosh before the calculation begins. It is desirable that the above-mentioned and other inconsistencies be avoided. Even the monthly total contribution of the employees who work outside Kathmandu Valley and whose salary arrives late, can be deposited on the account of Sanchaya Kosh by the Ministries and Departments concerned which are located in Kathmandu on the basis of their record in a time uniform to all. Secondly, the government fiscal year starts from Srawan and accordingly the grade changes from that month onwards. Since the interest on money deposit provided by Sanchaya Kosh at the end of Bhadra includes interest on the monthly total contribution of Srawan which is usually different from those of other months. This causes separate consideration of the monthly total contribution of Srawan to be made. This is, of course, not desirable.

Part II

General Formula for the calculation of amount at the end of n yrs

Generalised formula are developed with the following assumptions:-

(a) The instalments of money are deposited in Sanchaya Kosh at the end of a month.

(b) The interest on the money which is deposited within the first year is calculated at the end of the first month of the second year.[4]

(c) The employment begins from the first day of the first month.

(d) The grade is liable from the first month of the second year.

(e) The employment terminates after a whole number of years.

(f) Employment automatically implies permanent job.

Let X be the money which is already in deposit in Sanchaya Kosh when the employee started to contribute P percent from his monthly salary, Y.[5] G is the increment of salary per year. The ratio of the monthly salary which is equal to P/100 shall be denoted by small p.

Let total contribution be the sum of contribution to the Sanchaya Kosh from employees' side and the percent from HMG's side and k be the ratio of the total contribution to the contribution from the employees. If percent from HMG's side is equal to contribution then $k=2$.

R is the interest rate per annum of the sum deposited. r indicates the corresponding ratio i,e $R=100r$.

The monthly total contribution to Sanchaya Kosh is equal to Ykp. Therefore, the total contribution in a year is equal to 12Ykp.

As was mentioned earlier the interest is calculated at the end of first month of the second year and comprises the interest on money deposited in the first year.

4. This assumption is made for the convenience of deduction and does not, in any way, narrow the field of application of the method suggested.

5. The introduction of X may seem obscure because when the employment begins, no money will be deposited in Sanchaya Kosh. But in other cases which are analysed in part four of the article such money becomes reality. Introduction of X does not affect the validity of the derivations presented in the first three parts of the article.

The further calculations shall be eased with the introduction of a term "interest coefficient". The term "interest coeficient" shows the ratio of the interest on money for number of days of deposit of money in Sanchaya Kosh and the money itself.

The money which is deposited at the end of the first month will have a interest coefficient denoted by r.

The instalment which is deposited at the end of second month gets interest of 11 months only and so the interest coefficient is 11r/12.

The instalment which is deposited at the end of third month is liable to interest of 10 months only and the interest coefficient is 10r/12 and so on.

Since equal instalments are deposited at the end of each month and twelve instalments are deposited in a year, the average interestcoefficient for the period of one year is equal to

$$r_{av} = \frac{r + 11r/12 + 10r/12 + \ldots\ldots + r/12}{12}$$

$$\text{or,} \quad r_{av} = \frac{13}{24} r \quad \ldots \ldots \quad [1.0]$$

Average interest coefficient for a period of one year shows the rate of interest for the money deposited within a year.

The amount of money deposited within the first year i.e. X when calculated at the end of the first month of the second year will consist of X, the interest on it at the rate of r, the money 12Ykp which is deposited in instalments, and the interest on it at the rate of r_{av}.

In the second year, the money which is already deposited at the end of first year is liable to the interest rate of r, but the money which is doposited every month due to the salary Y is liable to the interest rate of r_{av}. For the convenience of calculation, the terms "constant sum" and "variable sum" shall be used. The sum which is liable to the interest rate of r will be denoted by the term "constant sum" and the sum which is liable to the interest rate of r_{av} will be denoted by the term "variable sum".

Thus the amount at the end of nth year, A_n will be composed of the constant sum for the nth year C_n, the variable sum for the same year V_n, the interest on C_n at the rate of r and the interest on V_n at the rate of r_{av}.

That is, $A_n = C_n + V_n + C_n r + V_n r_{av} = C_n (1+r) + V_n (1+r_{av})$

$$= C_n r_1 + V_n r_{av1} \quad \ldots \quad [1.1]$$

$$\text{where } r_1 = 1 + r, \; r_{av1} = 1 + r_{av} \quad \ldots \quad [1.1.1]$$

For the first year, C_1 will be X and V_1 will be 12Ykp.

So, from [1.1], $A_1 = X r_1 + 12Ykp\, r_{av1}$

This sum A_1 will be liable to interest rate of r in the second year and hence it will be constant sum for the second year and since the employee will get one grade, so V_2 will be equal to 12Ykp + 12Gkp

$$C_2 = A_1 = X r_1 + 12Ykp\, r_{av1}$$

$$V_2 = 12Ykp + 12Gkp$$

Putting n=2 in [1.1] above, $A_2 = C_2 Y_1 + V_2 r_{av1}$

or, $A_2 = (Xr_1 + 12Ykp\, r_{av1})\, r_1 + (12Ykp + 12Gkp)\, r_{av1}$

$$= Xr_1^2 + 12Ykp\, r_{av1} (1 + r_1) + 12\, Gkpr_{av1}$$

For the third year, A_2 will be constant sum and since the employee will get two grades, so V_3 will be equal to $12Ykp + 12 \times 2$ Gkp

i,e $C_3 = A_2 = Xr_1^2 + 12\, Ykpr_{av1} (1 + r_1) + 12\, Gkpr_{av1}$

$$V_3 = 12\, Ykp + 12 \times 2\, Gkp$$

Putting n=3 in [1.1], the value of A_3 will be equal to

$$A_3 = C_3 r_1 + V_3 r_{av1}$$

$$= [Xr_1^2 + 12 Ykpr_{av1} (1 + r_1) + 12 Gkpr_{av1}] r_1 + (12 Ykp + 12 \times 2 Gkp) r_{av1}$$

$$= Xr_1^3 + 12 Ykpr_{av1} (1 + r_1 + r_1^2) + 12 Gkpr_{av1} (2 + r_1)$$

Putting n=4 in [1.1], the value of A_4 will be equal to

$$A_4 = C_4 r_1 + V_4 r_{av1}$$

where, $C_4 = A_3 = Xr_1^3 + 12 Ykpr_{av1} (1 + r_1 + r_1^2) + 12 Gkpr_{av1} (2 + r_1)$

$$V_4 = 12 Ykp + 12 \times 3 Gkp$$

Since, the employee will get three grades in the fourth year, substitution of C_4 and V_4 in the expression for A_4 gives

$$A_4 = [Xr_1^3 + 12 Ykpr_{av1} (1 + r_1 + r_1^2) + 12 Gkpr_{av1} (2 + r_1)] r_1 + (12 Ykp + 12 \times 2 Gkp) r_{av1}$$

$$= Xr_1^4 + 12 Ykpr_{av1} (1 + r_1 + r_1^2 + r_1^3) + 12 Gkpr_{av1} (3 + 2 r_1 + r_1^2)$$

It will not be difficult to prove that A_n will be equal to

$$A_n = Xr_1^n + 12 Ykpr_{av1} (1 + r_1 + r_1^2 + \dots + r_1^{n-1}) + 12 Gkpr_{av1} [(n-1) + (n-2) r_1 + \dots + r_1^{(n-2)}] \quad \dots\dots [1.2]$$

The sum of the series in the third component of expression [1.2] shall be calculated now.

Suppose, $S_{n-1} = (n-1) + (n-2) r_1 + (n-3) r_1^2 + \dots\dots + r_1^{(n-2)}$ .... [1.2.1]

Multiplying both sides by r_1, $r_1 S_{n-1}$ will give

$$r_1 S_{n-1} = (n-1) r_1 + (n-2) r_1^2 + (n-3) r_1^3 + \ldots\ldots + 2r_1^{(n-2)} + r_1^{(n-1)} \quad \ldots\ldots\ldots \quad [1.2.2]$$

Substracting [1.2.1] from [1.2.2], we get

$$S_{n-1} = \frac{[r_1 + r_1^2 + \ldots\ldots + r_1^{(n-1)} - (n-1)]}{(r_1 - 1)}$$

The sum of the geometric series $r_1 + r_1^2 + \ldots + r_1^{(n-1)}$ is equal to $\frac{[r_1^n - r_1]}{(r_1 - 1)}$ and from [1.1.1] where it was assumed $r_1 = 1 + r$, $r_1 - 1$ will be equal to r.

Taking this all into account and simplifying, we get,

$$S_{n-1} = \frac{r_1^n - n r - 1}{r^2} \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad [1.2.3]$$

and since the sum of the geometric series

$$1 + r_1 + r_1^2 + \ldots\ldots\ldots + r_1^{(n-1)} = \frac{(r_1^n - 1)}{r_{1-1}} = \frac{(r_1^n - 1)}{r}$$

the expression in [1.2] will take the following form

$$A_n = Xr_1^n + 12\, Ykpr_{av1} \frac{(r_1^n - 1)}{r} + 12\, Gkpr_{av1} \frac{[r_1^n - n r - 1]}{r^2} \quad \ldots\ldots\ldots \quad [1.3]$$

Part III

This part consists of three sections: the first section will take into account the increase in salary by ΔY and increase in grade by ΔG after a referred period of years. Let us assume that increase takes place from the beginning of $(n_1 + 1)$ years. So, from that year onwards the increase in amount shall be attributed to ΔY and ΔG as well. The amount attributed to $\triangle Y$ and $\triangle G$ will be called additional amount and denoted by A_{An} . The amount obtained by addition of A_n and A_{An} shall be termed as the whole amount and denoted by A_{wn} .

The constant sum and variable sum components in the nth year of additional amount will be denoted by C_{An} and V_{An} respectively.

Apparently, the formula in [1.1] will hold true for the additional amount as well.

So, $A_{An} = C_{An}\, r_1 + V_{An}\, r_{av1}$ ... ... [2.1.1]

It is obvious that in the $(n_1 + 1)$ th year, the constant sum component of the additional amount is zero, that is

$C_{A(n_1+1)} = 0$ and $A_{A(n_1+1)} = V_{A(n_1+1)}\, r_{av1}$ ... ... [2.1.1.1]

$V_{A(n_1+1)} = 12\,\triangle Ypk + 12\, n_1\, \triangle Gpk$ ... ... [2.1.1.2]

One thing should be clarified at this point. If the grade increases by $\triangle G$ in the $(n_1 + 1)$ th year, the total grade for that year will not be $n_1\,(G + \triangle G)$. This is the reason why the expression [2.1.1.2] contains coefficient (n_1) in its second term.

In view of [2.1.1.2.] the expression [2.1.1.1] can be expressed as

$$A_{A(n_1+1)} = [12\,\triangle Ypk + 12\, n_1\, \triangle Gpk]\, r_{av1}$$

$$= 12\,\triangle Ypk\, r_{av1} + 12\, n_1\, \triangle Gpk\, r_{av1}$$

The constant sum component of the additional amount in the $(n_1 + 2)$ th year will be $A_{A(n_1+1)}$, that is,

$$C_{A(n_1+2)} = A_{A(n_1+1)} = [12\,\triangle Ypk + 12\, n_1\, \triangle Gpk]\, r_{av1}$$

$$V_{A(n_1+2)} = 12\,\triangle Ypk + 12\,(n_1 + 1)\,\triangle Gpk$$

From [2.1.1.] above, $A_{A(n_1+2)} = C_{A(n_1+2)} r_1 + V_{A(n_1+2)} r_{avl}$

$$= [12 \triangle Ypk + 12 (n_1 \triangle Gpk] r_{avl} r_1 + (12 \triangle Ypk + 12 (n_1 + 1) \triangle Gpk) r_{avl}$$

$$= 12 \triangle Ypk\, r_{avl} (1 + r_1) + 12 \triangle Gpk\, r_{avl} [(n_1 + 1) + n_1 r_1]$$

Proceeding likewise, it will not be difficult to find that

$$A_{An} = 12 \triangle Ypkr_{avl} [1 + r_1 + r_1^2 + \ldots\ldots + r_1^{(n-n_1-1)}]$$

$$+ 12 \triangle Gpkr_{avl} [(n-1) + (n-2) r_1 + (n-3) r_1^2 + \ldots\ldots + n_1 r_1^{(n-n_1-1)}$$

$$\ldots\ldots\ldots\ldots [2.1.2]$$

The sum of the geometric series in the first term of the expression [2.1.2] is given by

$$1 + r_1 + r_1^2 + r_1^3 + \ldots\ldots + r_1^{(n-n_1-1)} = \frac{[r_1^{(n-n_1)} - 1]}{(r_1 - 1)}$$

and since $r_1 - 1 = r$, so the sum $= \dfrac{[r_1^{(n-n_1)} - 1]}{r}$

The sum of the series entering into the second component of expression [2.1.2] shall be calculated as follows:-

Let the sum be denoted by S_{n1}, then

$$S_{n1} = (n-1) + (n-2) r_1 + (n-3) r_1^2 + \ldots\ldots + (n_1) r_1^{(n-n_1-1)} \ldots\ldots\ldots [2.1.2{\cdot}1]$$

$$r_1 S_{n1} = (n-1) r_1 + (n-2) r_1^2 + (n-3) r_1^3 + \ldots\ldots + (n_1 + 1 r_1^{(n-n_1-1)}$$

$$+ n_1 r_1^{(n-n_1)} \ldots\ldots\ldots [2.1.2.2$$

Subtracting [2.1.2.1] from [2.1.2,2],

$$S_{n1} (r_1 - 1) = r_1 + r_1^2 + r_1^3 + \ldots\ldots + r_1^{(n-n_1-1)} + n_1 r_1^{(n-n_1)} - (n-1)$$

$$= \frac{[r_1^{(n-n_1)} - r_1]}{(r_1 - 1)} + n_1 \; r_1^{(n-n_1)} - (n-1)$$

With the account of $(r_1 - 1) = r$ and with simplification

$$S_{n1} = \frac{[n_1 \; r_1^{(n-n_1+1)} - (n_1 - 1)\; r_1^{(n-n_1)} - nr - 1]}{r^2}$$

From [2.1.2],

$$A_{An} = 12 \; \Delta Ypkr_{av1} \frac{[r_1^{(n-n_1)} - 1]}{r}$$

$$+ 12 \; \Delta Gpkr_{av1} \frac{[n_1 \; r_1^{(n-n_1+1)} - (n_1 - 1)\; r_1^{(n-n_1)} - nr - 1]}{r^2} \quad \ldots\ldots\ldots\ldots [2.1.3]$$

Since $A_{wn} = A_n + A_{An}$, where A_n is given by the formula in [1.3] and A_{An} by [2.1.3], the value of the whole amount after n years with the account of increase in the salary by ΔY and increase in the grade by ΔG after n_1 years will be given by

$$A_{Wn} = Xr_1^n + 12 \; Ykpr_{av1} \frac{(r_1^n - 1)}{r} + 12 \; Gkpr_{av1} \frac{r[_1^n - nr - 1]}{r^2} + 12 \; \Delta \; Ypkr_{av1}$$

$$\frac{[r_1^{(n-n_1)} - 1]}{r} + 12 \; \Delta Gpkr_{av1} \frac{[n_1 \; r^{(n-n_1+1)} - (n_1 - 1)\; r_1^{(n-n_1)} - nr - 1]}{r^2} \quad \ldots\ldots [2.1.3]$$

In the second section, the effect of grade limitation after a period of n_2 years will be explored. The grade in n_2 th year is equal to $(n_2 - 1)$ G and limitation in grade means $G_{n2+1} = G_{n2+2} = \dots\dots = G_n = G_{n2} = (n_2 - 1)$ G.

If the general formula [1.3] is perused, it will be found that the limitation of grade affects only the grade component of A_n .

From [1.2], $G_{An} = 12\, Gkpr_{avl} \; [(n-1) + (n-2)\, r_1 + (n-3)\, r_1^2 + \dots\dots + r_1^{(n-2)}]$

where G_{An} is the grade component of the amount obtainable from the beginning of the second year.

Till the limitation of grade and grade component may be calculated applying the above formula , i,e $G_{An_2} = 12\, Gkpr_{avl} \; [(n_1 - 1) + (n_1 - 2)\, r_1 + \dots\dots + r_1^{(n_1 - 2)}]$

$$\text{or, } G_{An_2} = 12\, Gkpr_{avl} \frac{[r_1^{\,n_2} - n_2\, r - 1]}{r^2} \quad \dots \dots \dots \dots \quad [2.2.0.1]$$

For $(n_2 + 1)$th year, G_{An_2} will be constant sum and the variable sum will be $12\,(n_2 - 1)$Gkp.

$$\text{So, } G_{A(n_2 + 1)} = G_{An_2}\; r_1 + 12\,(n_2 - 1)\; Gkpr_{avl} \quad \dots \dots \quad [2.2.0.2]$$

This will be the constant sum for $(n_2 + 2)$th year and the variable sum will be $12\,(n_2 - 1)$ Gkp.

So, $G_{A(n_2 + 2)} = G_{A(n_2 + 1)}\, r_1 + 12\,(n_2 - 1)\, Gkpr_{avl}$, By substitution from [2.2.0.2], $G_{A(n_2 + 2)} = G_{An_2}\; r_1^2 + 12\,(n_2 - 1)\, Gkpr_{avl}\; (1 + r_1)$

Proceeding similarly we can derive,

$$G_{An} = G_{An_2}\; r_1^{(n-n_2)} + 12\,(n_2 - 1)\, Gkpr_{avl}\; [1 + r_1 + \dots\dots + r_1^{(n-n_2 - 1)}]$$

$$= 12\,Gkpr_{avl}\,\frac{[r_1^{n_2} - n_2\,r - 1]}{r^2} \times r_1^{(n-n_2)} + 12\,(n_2 - 1)\,Gkpr_{avl}\,\frac{[r_1^{(n-n_2)} - 1]}{r}$$

The whole amount is given by

$$A_{nGL} = Xr_1^n + 12\,Ykpr_{avl}\,\frac{(r_1^n - 1)}{r} + 12\,Gkpr_{avl}\,\frac{[r_1^{n_2} - n_2\,r - 1]}{r^2}\,r_1^{(n-n_2)} + 12\,(n_2 - 1)\,Gkpr_{avl}\,\frac{[r_1^{(n-n_2)} - 1]}{r} \quad \ldots \ldots \ldots [2.2.1]$$

Formula [2.2.1] gives the amount after a period of n years when the grade is limited after n_2 years.

In the third section, the dividend provided by the Sanchaya Kosh to the employees fund shall be taken into account. Let D be a coefficient indicating the ratio between the dividend and the amount which is deposited in the Sanchaya Kosh at the end of n_3 years. So the sum which is liable to interest in the $(n_3 + 1)$th year will be constant sum for that year $C_{(n_3 + 1)}$, the variable sum $V_{(n_3 + 1)}$ and the dividend DA_{n3}. Two versions shall have to be considered depending on whether the sum DA_{n3} is constant sum or variable sum in the $(n_3 + 1)$th year. Since the dividend constitutes D part of A_{n3}, for the $(n_3 + 1)$th year it will be constant sum if DA_{n3} is added to the employees' fund in the beginning of $(n_3 + 1)$th year. If $A_{(n_3 + 1)D}$ means amount at the end of $(n_3 + 1)$ years with the account of dividend

Then, $A_{(n_3 + 1)D} = A_{(n_3 + 1)} + DA_{n3}\,r_1$

$$A_{(n_3 + 2)D} = A_{(n_3 + 2)} + DA_{n_3}\,r_1^2$$

and so on.

The amount at the end of n years with the account of dividend provided at the end of n_3 years will be equal to

$$A_{nD} = A_n + DA_{n_3} \; r_1^{(n-n_3)} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad [2.3.1]$$

The dividend will be treated as variable sum for the $(n_3 + 1)$th year if the dividend will be added to the employees fund not at beginning of $(n_3 + 1)$th year but any time within that year.

Suppose the dividend is added to the employees' fund d_d days before the end of $(n_3 + 1)$th year. Since there are 365 days in a year, the dividend DA_{n_3} will be liable to interest of only d_d days and the interest coefficient for DA_{n_3} will be equal to $r_d = \frac{d_d}{365} r$. With the account of this coefficient, if $A_{(n_3 + 1)D}$ indicates the total amount at the end of $(n_3 + 1)$th year when the dividend is variable sum, then it is given by

$$A_{(n_3 + 1)D} = A_{(n_3 + 1)} + DA_{n_3} \; r_{d_1}$$

$$\text{Where } r_{d_1} = 1 + r_d$$

The sum $DA_{n_3} \; r_{d_1}$ shall be a constant sum for $(n_3 + 2)$th year.

So, $A_{(n_3 + 2)D} = A_{(n_3 + 2)} + DA_{n_3} \; r_{d_1} \; r_1$.

Similarly,

$$A_{(n_3 + 3)D} = A_{(n_3 + 3)} + DA_{n_3} \; r_{d_1} \; r_1^2 \quad \text{and so on.}$$

A'_{nD} will be equal to

$$A_{nD} = A_n + DA_{n_3} \; r_{d_1} \; r_1^{(n-n_3-1)} \quad \dots \quad [2.3.2]$$

The formulas [2.3.2] should be applied depending on whether the dividend is added to the employees' fund in the beginning of $(n_3 + 1)$th year or any time in the $(n_3 + 1)$th year.

Part IV

This part is also divided into three sections. The first section deals with the non-conformity of the start of employment keeping intact the assumptions specified in part one of the article. Let m' be the number of full months and d' the number of days in the incomplete month for which the employee contributes money to the Sanchaya Kosh in the first year. It is required to find the average interest coefficient for the first year. Let it be denoted by r'_{av}.

Obviously, $$r'_{av} = \frac{30.\frac{r}{12} + 30.\frac{2r}{12} + 30.\frac{3r}{12} + \dots + \frac{30m'r}{12} + \frac{d'(m'+1)r}{12}}{30\,m' + d'}$$

Where 30 indicates that there are thirty days in a month which is not quite precise but it eases the calculation.

Simplification will give the following value of r'_{av}

$$r'_{av} = \frac{(m'+1)(15\,m'+d')}{12\,(30\,m'+d')}\, r \quad \dots \quad [3.1.1]$$

For the complete year, $m' = 12$, $d' = 0$, and $r'_{av} = \frac{13}{24} r = r_{av}$.

It is not difficult to find that the amount at the end of n years in which the first incomplete year consists of m' months and d' days with the account of the average interest coefficient for the first year as calculated above will be equal to $A_{ni1} = Xr'_{av1}\, r_1^{(n-1)} + (m' + \frac{d'}{30})$

$$Ykpr'_{av1} \cdot r_1^{(n-1)} + 12\, Ykpr_{av1} \frac{[r_1^{(n-1)} - 1]}{r} + 12\, Gkpr_{av1} \frac{[r_1^{(n-1)} - (n-1)\, r - 1]}{r^2} \quad \ldots\ldots \quad [3.1.2]$$

Where A_{ni1} is the amount at the end of n years with the first incomplete year of m' months and d' days.

$$r'_{av1} = 1 + r'_{av}$$

It is to be noted that the component of the grade changes and the formula assumes that the grade is provided from the beginning of the third year. So, n is replaced by)n–1) in the grade component .

In case of limitation of grade after n_2 years, the amount at the end of n years will be equal to $A_{nGi1} = Xr'_{av1}\, r_1^{(n-1)} + (m' + \frac{d'}{30})\, Ykpr_{av1}\, r_1^{(n-1)} + 12\, Ykpr_{av1}$

$$\frac{[r_1^{(n-1)} - 1]}{r} + 12\, Gkpr_{av1} \frac{[r_1^{(n_2 - 1)} - (n_2 - 1)\, r - 1]}{r^2}\, r_1^{(n-n_2)} + 12\,(n_2 - 2)\, Gkpr_{av1}$$

$$\frac{[r_1^{(n-n_2)} - 1]}{r} \quad \ldots \quad \ldots \quad \ldots \quad [3.1.3]$$

Where A_{nGi1} is the amount at the end of n years in which the first incomplete year contains m' months and d' days and the grade is limited after n years. It is important to note that the last component contains the coefficient $(n_2 - 2)$ instead of $(n_2 - 1)$ which shows that the grade is obtained from the beginning of note the second year but third year.

The second section considers the case in which the date of deposit of the final instal-ment and the date of withdrawal of money from the Sanchaya Kosh do not conform with the assumptions laid down in the first part of the article.

Let m'' and d'' be the respective number of complete months and days of the incomplete month in the n th year in which the money is deposited and m'' and d'' the respective number of complete months and days of incomplete month in the same year after which the money is extracted from the Sanchaya Kosh.

Clearly, $m''' \geqslant m''$.

The interest coefficient for the amount $A_{(n-1)}$ when it is deposited in the Sanchaya Kosh for m''' months and d''' days shall be given by

$$r''' = \frac{(30\,m''' + d''')}{360}\, r \quad \ldots \quad [3.2.1]$$

here r''' is the interest coefficient for $A_{(n-1)}$ in the nth year.

Some clarification is necessary for the interpretation of the formula in [3.2.1].

The coefficients 30 and 360 show the number of days in a month and a year respectively. These figuree are assumptive and are chosen so as not to contradict the fact that a year contains 12 months. In practice the coefficients should be replaced be real ones. If, from the other side, m''' months and d''' days comprise d_1 days the formula for r''' will be

$$r''' = \frac{d_1}{365}\, r \quad \ldots \quad [3.2.1.1]$$

The interest eocfficient of the money which is deposited in the Sanchaya Kosh at the end of first month of n th year is

$$\frac{[\,30\,(m''' - 1) + d'''\,]}{30\,m''' + d'''}\, r''' = r''' \left(1 - \frac{30}{30\,m''' + d'''}\right)$$

The interest coefficient of the money which is deposited in the Sanchaya Kosh at the end of second month of the n th year is

$$\frac{[\,30\,(m''' - 2) + d'''\,]}{30\,m''' + d'''}\, r''' = r''' \left(1 - \frac{30 \times 2}{30\,m''' + d'''}\right)$$

The interest coefficient of the money deposited at the end of m'' month will be

$$r''' \left(1 - \frac{30\, m''}{30\, m''' + d'''} \right)$$

The final instalment will be deposited in the Sanchaya Kosh at the end of (m''+1) months and will constitute the total contribution for d'' day of (m''+1)th month.

The interest coefficient of money deposited at the end of (m''+1)th month will be

$$r''' \left[1 - \frac{30\,(m'' + 1)}{30\, m''' + d'''} \right]$$

The money deposited at the end of each of the first m'' months is [Ykp + (n-1) Gkp] each but the money deposited at the'end of (m''+1) month is equal to

$$\frac{d''}{30} [Ykp + (n-1)\, Gkp].$$

So, the average interest coefficient of the money deposited in the n th year is

$$r_{av''} = \frac{[Ykp + (n-1)\, Gkp] \left(1 - \frac{30}{30\, m''' + d'''}\right) r''' + [Ykp + (n-1)\, Gkp] \left(1 - \frac{30 \times 2}{30\, m''' + d'''}\right) r''' + \ldots\ldots + [Ykp + (n-1)\, Gkp] \left(1 - \frac{30\, m''}{30 m''' + d'''}\right) r''' + \frac{d''}{30} [Ykp + (n-1)\, Gkp] \left[1 - \frac{30\,(m''+1)}{30\, m''' + d'''}\right] r'''}{m'' [Ykp + (n-1)\, Gkp] + \frac{d''}{30} [Ykp + (n-1)\, Gkp]}$$

Substitution of r''' by $\frac{(30\, m''' + d''')r}{360}$ from formula [3.2.1] and simplification will give:

$$r_{av}'' = \frac{r}{12} \left[\left(m''' + \frac{d'''}{30} \right) - (m'' + 1) + \frac{m''\,(m'' + 1)}{2\left(m'' + \frac{d''}{30}\right)} \right] \quad \ldots \ldots \ldots \ldots \ldots [3.2.2]$$

As may be expected, in case of withdrawal of money from Sanchaya Kosh at the end of thirteenth month of the n th year and termination of employment at the last day of the n th year, m'' = 12, d'' = 0, m''' = 13 and d''' = 0 and substitution of these figures in formula [3.2.2] will give $r_{av}'' = \frac{13r}{24} = r_{av}$. The amount withdrawable from Sanchaya Kosh will be geven by

$$A_{nil} = A_{(n-1)} (1+r''') + (m'' + \frac{d''}{30}) [Ykp + (n-1) Gkp] (1+r_{av}'') \quad \ldots \ldots [3.2.3]$$

where A_{nil} is the amount of n years in which the last year is incomplete.

r''' is the interest coefficient for $A_{(n-1)}$ and is calculated from the formula [3.2.1]

r_{av}'' is the average interest coefficient for the variable sum in the n th year and is given by formula in [3.2.2]

$A_{(n-1)}$ can be calculated applying [1.3]

The third section deals with the a situation in which grade is obtainable only after some years.

Let n_4 be the number of years after which the grade is possible. One condition, however, will be reserved that it will be less than n_2 which was assumed to be the number of years after which the grade was limited.

That is, $n_2 > n_4$ ... ... [3.3.0.1]

If the expression for the amount is analysed, it can be found that case being considered in this section affects only the grade component.

From [1.2], $G_{An} = 12 \, Gkpr_{av1} \, [(n-1) + (n-2) \, r_1 + (n-3) \, r_1^2 + \ldots + r_1^{(n-2)}]$

Where G_{An} is attributed to the grade obtainable from the end of the 1st year. If the grade is obtainable after n_4 years i,e from the beginning of $(n_4 + 1)$ years, then

$$G_{AnM} = 12\ Gkpr_{avl}\ [\ (n-n_4) + (n-n_4-1)\ r_1 + (n-n_4-2)\ r_1^2 + \ldots\ldots + r_1^{(n-n_4-1)}\ ]$$

If the process of calculating the sum of the series $(n-n_4) + (n-n_4-1)\ r_1 + \ldots + r_1^{(n-n_4-1)}$ applied in part one is carred out here, the grade component of A_n will be equal to

$$G_{AnM} = 12\ \frac{Gkpr_{avl}}{r^2}\ [\ r_1^{(n-n_4+1)} - r_1 - (n-n_4)\ r\ ] \quad \ldots \quad .. \quad \ldots \quad [3.3.1]$$

Here G_{AnM} is the grade component of A_n with the grade starting after n_4 years.

In case of limitation of grade, there are two components of grade i,e

$$G_{An\ GL} = 12\ Gkpr_{avl}\ \frac{[\ r_1^{n_2} - n_2\ r - 1\ ]}{r^2}\ r_1^{(n-n_2)} + 12\ (n_2 - 1)\ Gkpr_{avl}$$

$$[\ 1 + r_1 + r_1^2 + \ldots\ldots\ldots\ldots\ldots + r_1^{(n-n_2-1)}\ ] \quad \text{as is seen from [2.2.1]}$$

Here, G_{AnGL} is the grade component of A_{nGL} when the grade is limited after n_2 years and the grade is obtainable after the 1st year.

If the grade is obtainable after n_4 years and limited after n_2 years the grade component will be

$$G_{AnGLM} = 12\ Gkpr_{avl}\ \frac{[\ r_1^{(n_2-n_4+1)} - r_1 - (n_2 - n_4)\ r]}{r^2}\ r_1^{(n-n_2)}$$

$$+ 12\ (n_2 - n_4)\ Gkpr_{avl}\ [\ 1 + r_1 + r_1^2 + \ldots\ldots\ldots + r_1^{(n-n_2-1)}\ ]$$

$$= 12\ Gkpr_{av1} \frac{[r_1^{(n_2 - n_4 + 1)} - r_1 - (n_2 - n_4)\,r]}{r^2}\, r_1^{(n-n_2)}$$

$$+ \ 12\,(n_2 - n_4)\,Gkpr_{av1} \frac{[r_1^{(n-n_2)} - 1]}{r} \quad \ldots \ \ldots \ \ldots \ \ldots \ \ldots \ \ldots \ \ldots \ [3.2.2]$$

Here G_{AnGLM} is the grade component of A_{nGL} when the grade is obtainable after n_4 years and limited after n_2 years.

Part V

The part four of this article gives two examples and analyses some of the important practical aspects.

In the present situation, 10% of the salary is contributed by an employee, the same amount of money is added by HMG, and the interest rate per annum is 10% . Hence

$p = 0.1$. $k = 2$. $r = 0.1$, from where $r_1 = 1 + r = 1.1$

$r_{av} = 13/24r = \frac{13}{24} \times 0.1 = 0.05417$ rounding upto 5 places after decimal.

$r_{av1} = 1 + r_{av} = 1 + 0.05417 = 1.05417$

Example 1

An Assistant Engineer begins the gazetted service from the first days of the first year, 2000 rupees is supposedly deposited in Sanchaya Kosh from his non–gazetted service, serves 35 years in the gazetted third class with no promotion, no increase in salary, no increase in grade,no dividend, gets his first grade in the first month of the second year, the grade is limited after 22 years. The amount withdrawable from the Sanchaya Kosh at the end of month of 36 th year is to be calculated.

The amount at the end of first month of 36 the year is given by the formula in [2.2.1].

$$A_{nGL} = Xr_1^n + 12\ Ykpr_{avl} \frac{(r_1^n - 1)}{r} + 12\ Gkpr_{avl} \frac{[r_1^{n_2} - n_2\ r - 1]}{r^2} r_1^{(n-n_2)} + 12$$

$$(n_2 - 1)\ Gkpr_{avl} \frac{[r_1^{(n-n_2)} - 1]}{r} \quad \ldots \quad [4.1]$$

where, $n = 35$ yrs, $n_2 = 22$ yrs, $X =$ Rs 2000

For an Assistant Engineer the salary, $Y =$ Rs 700, the grade, $G =$ Rs 15.

The formula in [4.1] takes the form

$$A_{35GL} = 2000\ (1.1)^{35} + 12 \times 700 \times 2 \times 0.1 \times 1.05417 \frac{(1.1^{35} - 1)}{0.1}$$

$$+ 12 \times 15 \times 2 \times 0.1 \times 1.05417 \frac{[1.1^{22} - 22 \times 0.1 - 1]}{(0.1)^2} 1.1^{(35-12)}$$

$$+ 12\ (22-1) \times 15 \times 2 \times 0.1 \times 1.05417 \frac{[1.1^{(35-22)} - 1]}{0.1}$$

Before attempting to calculate the amount it will be useful to prepare a table of different powers of 1.1.

Differcut powers of 1.1

n	0	1	2	3	4	5	6
1.1^n	1.00000	1.10000	1.21000	1.33100	1.46410	1.61051	1.77156
n	7	8	9	10	11	12	13
1.1^n	1.94872	2.14359	2.35795	2.59374	2.85312	3.13843	3.45227

n	14	15	16	7	18	19	20
1.1^n	3.79750	4.17725	4.59497	5.05447	5.55992	6.11591	6.72750
n	21	22	23	24	25	26	27
1.1^n	7.40025	8.14028	8 95430	9.84973	10.83471	11.91818	13.11000
n	28	29	30	31	32	33	34
1.1^n	14.42100	15.86309	17.44940	19.19434	21.11378	23.22515	25.54767
n	35	36	37	38	39	40	41
1.1^n	28.10244	30.91268	34.00395	37.40434	41.14478	45.25926	

The different powers of 1.1 are calculated by producing an 11–digit result which is rounded off to 8 digit display from which only 5 figures after decimal are taken.

With the value of the corresponding powers of 1.1, the value of A_{35GL} will be equal to

$$
\begin{aligned}
A_{35GL} &= 2{,}000 \times 28.10244 + 17{,}710.06\,(28.10244 - 1) \\
&+ 3{,}795.01\,[8.14028 - 22 \times 0.1 - 1]\;3.45227 \\
&+ 7969.52\,(3.45227 - 1) \\
&= 56{,}204.88 + 4{,}79{,}985.73 + 6.4724.58 + 19{,}543.41 \\
&= 56{,}204.88 + 5{,}64{,}253.72 \\
&= \text{Rs } 6{,}20{,}458.60
\end{aligned}
$$

The value of average interest coefficient reduced to one year was obtained in part one and is equal $r_{av} = \frac{13r}{24} = 0.5417r$ taking 4 significant figures after decimal.

The necessary number of significant figures after decimal in the coefficient 0.5417r will be fund out such that the error caused by ignoring the remaining figures in the coefficient will be within the permissible limit. If the error causes difference in the calculated amount of more than or equal to 0.1 % of the amount obtained by accepting the coefficient equal to 0.5417, the error shall be deemed to be impermissible.

Going back to the general formula in [1.3], it is seen that if no money is deposited in the SanchayaKosh before the employee starts his service with salary Y and grade G obtainable from the second year onwards $A_n = r_{av1} S_n$

here, $$S_n = 12\,Ykp\,\frac{(r_1^n - 1)}{r} + 12\,Gkp\,\frac{[r_1^n - n\,r - 1]}{r^2}$$

or, $$A_n = (1 + 0.5417r)\,S_n$$

If E_b be the percent of the error, it is given by

$$E_6 = \frac{[A_n(r_{av} = 0.5417r) - A_n(r_{av} = br)] \times 100}{A_n(r_{av} = 0.5417r)}\,\%$$

here b = 0.5, 0.54, and 0.541

$$E_{0.5} = \frac{4.17}{\frac{1}{r} + 0.5417}\,\%,\quad E_{0.54} = \frac{0.17}{\frac{1}{r} + 0.5417}\,\%,\quad E_{0.541} = \frac{0.07}{\frac{1}{r} + 0.5417}\,\%.$$

It can be seen that the percentage of the error depends on the value of interert rate r, and depending upon the different value of r, the percent of the error can be estimated.

In the present context, where r = 0.1, the values of $E_{0.5}$, $E_{0.54}$ and $E_{0.541}$ shall be 0.396 %, 0.016 %, 0.007 % respectively. The error $E_{0.5}$ exceeds the permissible limit set above i,e 0.1% and $E_{0.54}$ is less than the permissible limit. So, it can be concluded that the value of $r_{av} = 0.05$ is not acceptable and it is suggested that r_{av} be accepted as 0.054. For other values of r, however, the number of significant figures after decimal differs from what is inferred above.

In some cases the value of r_{av} can be taken as 0.5r and value of r_{av}' can also be taken as 0.5r. If the employee starts service in the last months of the first year and if he is provided interest for the money deposited in the first year at the rate of $r_{av}' = 0.5r'$ he would certainly be benefitted. If the number of months increases in the first year, the average interest coefficient r_{av}' also increases. From the other side, when m' increases. the money liable to interest also increases. So, there should be some optimum value of m' and d' for which the value of the interest difference on the deposited money at the rate of 0.5r and r_{av}' is maximum. If I_d be the interest difference on the money of m' months and d' days, then

$$I_d = \left(m' + \frac{d'}{30}\right) Ykp \times 0.5r - \left(m' + \frac{d'}{30}\right) Ykpr_{av}' \quad \ldots \quad \ldots \quad - \quad \ldots \quad [4.3.1]$$

$$= 0.5 \left(m' + \frac{d'}{30}\right) Ykpr - \left(m' + \frac{d'}{30}\right) Ykp \frac{(m'+1)(15m'+d')}{12(30m'+d')} r$$

here r_{av}' is substituted by its value from [3.1.1]

$$\text{or,} \quad \frac{360Id}{Ykpr} = 6(30m'+d') - (m'+1)(15m'+d') \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad [4.3.2]$$

When both sides are differentiated with respect to m', the value of optimum number of months shall be obtained as

$$m'_{opt} = \frac{165 - d'}{30} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad [4.3.2.1]$$

The value of m'_{opt} from [4.3.2.1] substituted in the expression for r'_{av} gives

$$r'_{av\,opt} = \frac{[\,32400 - (d' + 15)^2\,]\,r}{118800} \quad \ldots \quad [4.3.2.2]$$

For the value of $m'_{opt} = \frac{165 - d'}{30}$, the value of I_d is maximum.

From expression [4.3.2], m' when substituted by the value of m'_{opt} from [4.3.2.1] and simplification of the whole equation will give

$$I_{dmax} = \frac{Ykpr}{21600}\,[(d' - 15)^2 + 27000] \quad \ldots \quad [4.3.2.3]$$

It is clear that the value of $I_{dmax} = \frac{Ykpr}{21600}\,[(d' - 15)^2 + 27000]$ will be maximum for d' = 0 or 30 from among the possible values of d' which is supposed to take the maximum values of 30, though, practically, it is not quite so. However, for the discussions d'_{max} shall be taken as 30. Both cases of d' = 0 and d' = 30 shall be considered separately.

$$\text{wnen } d' = 0,\ I_{dmax} = \frac{Ykpr}{21600}\,(15^2 + 2700) = Ykpr\left(\frac{5}{4} + \frac{1}{96}\right) \quad \ldots \quad [4.3.2.4]$$

$$\text{When } d' = 30,\ I_{dmax} = \frac{Ykpr}{21600}\,(15^2 + 27000) = Ykpr\left(\frac{5}{4} + \frac{1}{96}\right)$$

It is not surprising that the value of I_{dmax} is the same for both d' = 0 and d'=30, because when d' = 0, the number of whole months m'_{opt} from formula in [4.3.2.1] is 5.5 and the incomplete month contains not a single day. When d' = 30, the number of whole months m'_{opt} from the same formula is 4.5 and there is one more month containing 30 days and that is equivalent to one additional month which makes the total period equal to 5.5 months

One interesting thing in the whole above analysis should be pointed out here. m' was assumed to be a whole number of months and yet m'_{opt} was obtained as a fractional number. It shows that all the above results could be obtained by subsitution of d' by zero in the expression for $r_{av}' = \frac{(m'+1)(15m'+d')}{12(30m'+d')} r$ i,e by taking $r'_{av} = \frac{(m'+1)r}{24}$

The process will not be repeated here. One important feature of the method adopted in the present analysis is that by use of r'_{av} equal to $\frac{(m'+1)(15m'+d')}{12(30m'+d')} r$ instead of $r'_{av} = \frac{(m'+1)r}{24}$ the result shows at a glance how much is attributed to each of the quantities m' and d'. In the above case, if d' = 15, $I_{dmax} = \frac{27000 \text{ Ykpr}}{21600}$, but then m'_{opt} = 5 months. So, clearly, the sum attributed to the remaining half month is $\frac{225 \text{ Ykpr}}{21600} = \frac{\text{Ykpr}}{96}$.

It is interesting to evaluate whether an Assistant Engineer starting service 5.5 months before the end of first year and serving 35 years and 5.5 months is benefitted or not if both the values of r_{av} and r'_{av} are taken as 0.5r instead of the suggested value 0.54r for r_{av} and $\frac{(m'+1)(15m'+d')r}{12(30m'+d')}$ r'_{av}. As was shown before, the Engineer gains when he is given r'_{av} = 0.5r as interest of the first year and loses in the remaining 35 years due to the lesser value of r_{av} than suggested.

The maximum interest difference at the end of first year, as was shown,

$$I_{dmax} = \text{Ykpr} \left(\frac{5}{4} + \frac{1}{96}\right)$$

Here, Y = Rs 700, k = 2, p = 0.1, r = 0.1.

And so, $I_{dmax} = 700 \times 2 \times 0.1 \times 0.1 \left(\frac{5}{4} + \frac{1}{96}\right)$ = Rs 17.65

The interest difference compounded annually for the remaining 35 years will give

$I_{dmax\ 35} = 17.65 \times 1.1^{35} = 17.65 \times 28.10244$ = Rs 496.01.

This is the maximum profit to the Enginineer due to $r_{av}' = 0.5r$ insetad of r'_{avopt} .

The loss to the Engineer due to the lesser value of $r_{av} = 0.5r$ instead of suggested $r_{av} = 0.54r$ is given by

$$L = \frac{(E_{0.5} - E_{0.54})}{100} \times A_{35}$$

Where A_{35} is the amount deposited in the Sanchaya Kosh in the complete 35 years at the rate of 0.5417r which is equal to

$A_{35} = 5,64,253.72$ ... ... (See example 1 above)

$$L = \frac{(0.396 - 0.016)}{100} \times 5,64,253.72 = \text{Rs } 2144.11$$

So, the Engineer will lose and overall loss will be equal to

$$L_0 = L - I_{dmax\ 35} = 2144.11 - 496.01 = \text{Rs } 1648.10$$

Of course, the value of L_0 depends upon the number of service years which affects L and the date of start of employment which affects I_{d35}.

Now a few words of caution regarding the application of the formula presented in the first three parts of the article. Formulas in the first two parts can be used when employment conditions conform to the conditions laid down at the beginning of the first part. But in practice this rarely happens. Hence use of formula in [3.1.1] for r_{av}' in case of non–conformity of start of employment and respective formulas in [3.2.1.1] and [3.2.2] for the values of r''' and r_{av}'' in case of termination of employment and withdrawal of money from Sanchaya Kosh will provide the solutions. Cumulative tables can be prepared by the Sanchaya Kosh for the values of r_{av}', r_{av}'' and r''' for quick reference. But the value of r''' needs to be calculated very carefully and accurately because as can be seen from [3.2.3], for a large number of service years n, the value of the amount A_{nil} is highly sensible to the value of r'''.As the formula in [3.1.2] shows, the value of $r'_{av} = (r'_{av1} - 1)$ is also important. The valus of r''_{av} is not very important and can be neglected admitting not more than permissible error. This is more true for greater number of service years.

As far as the formulas in section three of part III are concerned, the formulas in [3.3.2.1], [3.3.3.1] and [3.3.4] may be practically useful.

The methodology adopted for the solution of a complex problem is of great importance. What is meant by a complex problem? Till now, it has been assumed that increase in grade and increase in salary, provision of dividend etc. happen only once in the whole service period. These do not, however, reflect the reality. The increase in grade and salary may take place more than once; it may be in regular, or irregular intervals, the increase in grade and salary may be equal at different times or may be different at different times. All these possibilities apply to the dividend as well. No considerations have been made till now regarding the promotion of an employee. All these considerations would require a method for solving a complex problem. A unified general formula for solving a complex problem is very difficult to deduce. However, some method will be suggested here.

First of all, the formula in [3.1 2] shows how the non-conformity of the start of employment with the conditions laid down in the first part of the article should be taken care of. The formula shows that the X component of the amount is affected by the non-conformity and a new component attributed to the incomplete year appears in the formula.

Formula [2.1.4] shows that the increase in salary causes an additional component to be added to the amount and the increase in grade causes additional component of grade to be added to the final amount. So, if the increases occur more than once, the necessary corrections can be inserted by adding one more component for each increase, be it in grade or in salary. The additional component can be calculated as shown in section one of part two. Section two of part two shows how the limitation of grade can be taken account of. It shows that the grade component needs to be split into two components. Formulas in [2.3.1] or [2.3.2] depending on the case, show that the dividend provided by the Sanchaya Kosh causes one additional component attributed to the dividend to be added to the amount.

If an employee gets promoted, the whole amount upto the day of his first promotion may be taken as X_1, and X_1 may be calculated by applying one or more of the above formulas. After promotion the salary and the grade of the employee will be new additions and until the second promotion the amount may be calculated by applying the above-mentioned methods with the new values of salary and grade. When the employee gets a second promotion, the whole amount upto the promotion' may be taken as X_2 and the calculations carried out accordingly and so on.

To clarify the method described above, an example of a complex problem is illustrated here.

Example 2

A fellow starts service in HMG as a Section Officer on the 16th day of the 10th month with the initial salary of Rs 600 per month, with a grade of Rs 20 per year. The grade is obtainable from the beginning of the year. At the beginning of the fourth year, the salary increases by Rs 100 per month. After exactly six years of service, he is promoted to Under Secretary with the-than salary of Rs 1000 per month and grade of Rs 30 per year, the grade being obtainable from the beginning of 8th year. The grade increases to Rs 40 per year at the beginning of 11th year. The fellow gets second promotion on the first day of 15th year to the post of Joint Secretary with the salary of Rs 1500 per month and grade of Rs 60 per year which is obtainable from the beginning of the 17th year. The salary increases to Rs 2000 as from the beginning of 20th year. The grade is limited from the beginning of 27th year The fellow retires from the post of Joint Secretary on the first day of 30th year and withdraws money from the Sanchaya Kosh one month later. The dividend added to the fellow's fund is 5% of money deposited at the end of 9th year and is added to the fund on the 45th day of the 10th year. A second dividend of 10% of money deposited at the end of 18th year is added to the fund on the 75th day of 19th year. The interest rate per annum is 10%. The money withdrawable from the Sanchaya Kosh can be calculated as follows:

Solution:-

The average interest coefficient for the first year,

$$r'_{av} = \frac{(m'+1)\,(15\,m'+d')\,r}{12\,(30\,m'+d')}$$, where m' = 1 month, d' = 1 5days, [r = 0.1

$$\text{so, } r'_{av} = \frac{(1+1)\,(15\times 1+15)\times 0.1}{12\,(30\times 1+15)} = 0.01$$

$$r'_{av1} = 1 + r'_{av} = 1 + 0.01 = 1.01$$

Since, the fellow gets first promotion after 6 years of his service, he gets promotion on the 16th day of the tenth month of the 7th year. The amount at the end of 6th year will be

$$A_6 = (m' + \frac{d'}{30})\,Ykpr'_{av1}\ r_1^{(n-1)} + 12\,Ykpr_{av1}\frac{[r_1^{(n-1)} - 1]}{r} + 12\,Gkpr_{av1}$$

$$\frac{[r_1^{(n-1)} - (n-1)\,r - 1]}{r^2} + 12\,\Delta Ykpr_{av1}\frac{[r_1^{(n-n_1)} - 1]}{r} \quad \ldots \ldots \ldots \ldots [4.4.4.1]$$

Where $n = 6$ years, $n_1 = 3$ years, $m' = 1$ month, $d' = 15$ days, $Y =$ Rs 600, $\Delta Y =$ Rs 100, $k = 2$, $p = 0.1$, $r'_{av1} = 1.01$, $r_{av1} = 1.054$, $r = 0.1$, $r_1 = 1.1$

Substitution of these values in the expression for A_6 will give

$$A_6 = (1 + \frac{15}{30}) \times 600 \times 2 \times 0.1 \times 1.01 \times 1.1^{(6-1)} + 12 \times 600 \times 2 \times 0.1$$

$$\times\ 1.054\,\frac{[1.1^{(6-1)} - 1]}{0.1} + 12 \times 20 \times 2 \times 0.1 \times 1.054\,\frac{[1.1^{(6-1)} - (6-1) \times 0.1 - 1]}{(0.1)^2} + 12$$

$$\times\ 100 \times 2 \times 0.1 \times 1.054\,\frac{[1.1^{(6-3)} - 1]}{0.1}$$

$$= 292{,}79 + 9266.08 + 559.09 + 837.30 = \text{Rs } 10{,}955.26$$

If X_1 be the money withdrawable at the end of the first month of the 8th year, it is given by

$$X_1 = A_6\,(1 + r''') + (m'' + \frac{d''}{30})\,[Y'pk + (6-2)\,Gpk]\,(1 + r''_{av})$$

$$\text{Where } r''' = \frac{dl}{365} r = \frac{(365-45)}{365} \times 0.1 = 0.088, \quad m'' = 10 \text{ months}, \quad d'' = 15 \text{ days},$$

Y' is the increased salary and is given by Y' = 600 + 100 = Rs 700

$$r''_{av} = \frac{r}{12} \left[(m''' + \frac{d'''}{30}) - (m'' + 1) + \frac{m''(m''+1)}{2(m'' + \frac{d''}{30})} \right]$$

Here, clearly $m''' = 13$ months, $d''' = 0$ days and so $r''_{av} = \frac{0.1}{12} [13 - (10 + 1)$

$$+ \frac{10(10+1)}{2(10 + \frac{15}{30})}] = 0.06$$

$$X_1 = 10{,}955.26\,(1+0.088) + (10 + \frac{15}{30})\,[700\times2\times0.1 + 4\times20\times2\times0.1]\,(1+0.06)$$

$$= \text{Rs } 13{,}655.60$$

This is the amount deposited in the account of the fellow as Section Officer at the end of first month of 8th year. This will be treated as constant sum for the remaining years. At the same time, his service as Under Secretary starts after 6 years, 10 months and 16 days of his service. Since he gets a second promotion on the first day of the 15th year, the second major step of the calculation will be to find the amount at the end of 14th year. During his service period as Under Secretary, he gets one increase in grade and a dividend. The amount at the end of 14th year is given by

$$A_{14} = X_1 r_1^{(n'-1)} + (m' + \frac{d'_1}{30}) Y_1 \, pkr'_{av1} \, r_1^{(n'-1)} + 12 Y_1 \, pkr_{av1}$$

$$\frac{[r_1^{(n'-1)} - 1]}{r} + 12 G_1 \, pkr_{av1} \frac{[r_1^{(n'-1)} - (n'-1) r - 1]}{r^2} + 12 \triangle G_1 \, pkr_{av1}$$

$$\frac{[(n'_1 - 1)r_1^{n'-n'_1+1} - (n'_1 - 2)\, r_1^{(n'-n'_1)} - (n'-1)\, r - 1]}{r^2} + D_1\, A_9\, r_{d_1}\, r_1^{(n'-n'_3-1)} \quad \ldots \ldots \ldots \ldots [4.4.4.2]$$

Where, $n' = 14-6 = 8$ years, $m'_1 = 1$ month, $d'_1 = 15$ days, $Y_1 = $ Rs 1000

$r'_{av1} = r'_{av1}$ (since $m'_1 = m'$ and $d'_1 = d'$) $= 1.01$, $G_1 = $ Rs 30

$\triangle G_1 = $ Rs 10, $n'_1 = 10-6 = 4$ years, $D_1 = 0.05$, $r_d = \dfrac{365-45}{365}\, r$

$= 0.088$, $r_{d_1} = 1 + r_d = 1 + 0.088 = 1.088$, $n'_3 = 9-6 = 3$ years

Note that r'_{av1} does not appear in the component of X_1 since X_1 already includes interest of m' months and d' days.

A_9 is the amount at the end of 9 years and is given by

$$A_9 = X_1\, r_1^{(n''-1)} + (m'_1 + \frac{d'_1}{30})\, Y_1\, pkr'_{av1}\, r_1^{(n''-1)}$$

$$+ 12\, Y_1\, pkr_{av1} \frac{[\,(r_1^{(n''-1)} - 1)\,]}{r} + 12\, G_1\, pkr_{av1} \frac{[r_1^{(n''-1)} - (n''-1)\, r - 1]}{r^2} \quad \ldots \ldots \ldots \ldots \ldots [4.4.4.3]$$

Here, $n'' = 9-6 = 3$ years

Since the increase in grade occurs after 9th year, so the component of grade increase does not enter in the above formula for A_9 .

Substitution of corresponding values in the formula for A_9 will give

$$A_9 = 13{,}655.60 \times 1.1^{(3-1)} + (1 + \frac{15}{30}) \times 1000 \times 0.1 \times 2 \times 1.01 \times 1.1^{(3-1)}$$

$$+ 12\times 1000\times 2\times 0.1\times 1.054 \frac{[1.1^{(3-1)}-1]}{0.1} + 12\times 30\times 2\times 0.1\times 1.054 \frac{[1.1^{(3-1)} - (3-1)^{0.1} - 1]}{(0.1)^2}$$

$$= 16523.28 + 366.63 + 5312.16 + 75.88$$

$$= \text{Rs } 22{,}277.95$$

Substitution of corresponding values in [4.4.4.2] will give

$$A_{14} = 13{,}655.60 \times 1.1^{(8-1)} + (1 + \frac{15}{30}) \times 1000 \times 0.1 \times 2 \times 1.01 \times 1.1^{(8-1)}$$

$$+ 12\times 1000\times 2\times 0.1\times 1.054 \frac{[1.1^{(8-1)}-1]}{0.1} + 12\times 30\times 2\times 0.1\times 1.054 \frac{[1.1^{(8-1)}-(8-1)\times 0.1-1]}{(0.1)^2}$$

$$+ 12\times 10\times 0.1\times 2\times 1.054 \frac{[(4-1)\, 1.1^{(8-4+1)} - (4-2)1.1^{(8-4)} - (8-1)\times 0.1-1]}{(0.1)^2} + 0.05$$

$$\times 22{,}277.95\times 1.088\times 1.1^{(8-3-1)} = 26{,}610.94 + 590.46 + 23{,}998.82 + 1887.48 + 514.34 + 1774.37$$

$$= \text{Rs } 55376.41$$

On the basis of discussions presented before, $X_2 = A_{14} = \text{Rs } 55376.41$

Since the fellow retires at the beginning of 30th year and withdraws money one month later, the money withdrawable from the Sanchaya Kosh will be A_{29} which can be calculated as:

$$A_{29} = X_2\, r_1^t + 12\, Y_2\, kpr_{av1} \frac{(r_1^t - 1)}{r} + 12\, G_2\, kpr_{av1} \frac{[r_1^{(t_2 - 1)} - (t_2-1)r-1]}{r^2}$$

$$\times r_1^{(t-t_2)} + 12\,(t_2 - 2)\, G_2\, kpr_{av1} \frac{[r_1^{(t-t_2)} - 1]}{r} + 12\, \Delta Y_2\, kpr_{av1} \frac{[r_1^{(t-t_1)} - 1]}{r} + D_2$$

$A_{18}\, r_{d'_3}\, r_1^{(t-t_3\ -1)}$ ... . – ... ... [4.4.4.4]

Here, $t = (29\text{–}14) = 15$ years, $t_2 = (26\text{–}14) = 12$ years, $t_1 = 19\text{–}14 = 5$ years

$t_3 = (18\text{–}14) = 4$ years, $Y_2 =$ Rs 1500, $G_2 =$ Rs 60, $\triangle Y_2 =$ Rs 500

$$A_{18} = X_2\, r_1^{t'} + 12\, Y_2\, kpr_{av1} \frac{(r_1^{t'} - 1)}{r} + 12\, G_2\, kpr_{av1} \frac{[r_1^{(t'-1)} - (t'-1)\, r - 1]}{r^2}$$

Where $t' = 18\text{–}14 = 4$ years and so $A_{18} = 55{,}376.41 \times 1.1^4 + 12 \times 1500 \times 2 \times 0.1$

c

$$\times 1.054 \frac{(1.1^4 - 1)}{0.1} + 12 \times 60 \times 2 \times 0.1 \times 1.054 \frac{[1.1^{(4-1)} - (4-1)\, 0.1 - 1)}{(0.1)^2} = 81{,}076.60 + 17{,}609.81$$

$+ 470.50 =$ Rs 99,156.91

$$r'_d = \frac{365-75}{365}\, r = 0.077 \text{ and so } r'_{d_3} = 1 + r'_d = 1 + 0.077 = 1.077$$

Taking account of all these,

$$A_{29} = 55{,}376.41 \times 1.1^{15} + 12 \times 1500 \times 2 \times 0.1 \times 1.054 \frac{(1.1^{15} - 1)}{0.1} + 12 \times 60 \times 2 \times 0.1 \times 1.054$$

$$\frac{[1.1^{(12-1)} - (12-1) \times 0.1 - 1]}{(0.1)^2} \times 1.1^{\,(15-12)} + 12\,(12-2) \times 60 \times 2 \times 0.1 \times 1.054 \frac{[1.1^{(15-12)} - 1]}{0.1}$$

$$+ 12 \times 500 \times 2 \times 0.1 \times 1.054 \frac{[1.1^{(15-5)} - 1]}{0.1} + 0.1 \times 99{,}156.91 \times 1.077 \times 1.1^{(15-4-1)}$$

$= 2{,}31{,}321.11 + 1{,}20.557.57 + 15{,}214.07 + 5023.78 + 20{,}157.62 + 27{,}699.07$

$=$ Rs 4,19'973.22

As can be seen in the example, calculation of the sum attributable to the dividend is a very cumbersome job. In this example unnecessary explanations are avoided. This should not create any difficulty in understanding the underlying technique.

Part VI

Conclusion

In conclusion, I like to sum up the main ideas put in the article They are as follows:

1) The traditional method of calculation presented in the initial part shows the following shortcomings:

(a) The process of calculation is very lengthy.

(b) The precision of calculation if not being adhered to due to the arbitrary choice of average interest coefficient as equal to half of the maximum interest coefficient. Actually this is not so as the fourth part of the article shows.

(c) There is no consistency in the conditions of deposit of money in Sanchaya Kosh.

(d) There is no coincidence of the year of which the interest is calculated by Sanchaya Kosh with the Government fiscal year.

2) To eliminate the above shortcomings the following method is suggested:

(a) The calculation made according to the formula will be shorter. It may help in saving valuable time of the employees of the Sanchaya Kosh.

(b) The calculation will be more precise with application of values of r_{av}, r'_{av}, r''_{av} and r'''. The average interest coefficient reduced to a year should be taken as 0.54 of the maximum interesr coefficient.

(c) There should be a strict unformity in the conditions of diposit of money in Sanchaya Kosh.

(d) The year of which the interest is calculated by Sanchaya Kosh should coincide with the government fiscale year.

3) To meet the applicability of the formulas derived, the conditions should conform to the assumptions made in the article. It should be stressed that the assumptions made here are real ones.

4) The sum of money at Sanchaya Kosh is always composed of the constant sum and the variable sum.

5) The amount of money attributed to the salary or the grade can be separately calculated. The increase in salary causes an additional component to be added to the amount and the increase in grade causes additional component of the grade to be added to the final amount. If the increases occur more than once, the necessary corrections can be inserted by adding one more component for each increase, be it in grade or in salary.

6) The divident provided by the Sanchaya Kosh causes one additional component attributed to the dividend to be added to the amount.

7) It is desirable that cumulative tables be prepared for r'_{av}, r''_{av} and r''' for easy reference.

8) Although the examples of calculations are carried out in the context of Employees' Provident Fund of Nepal, the idea holds good for any other similar Provident Fund Systems.

Glossary of Terms used in the article

1. **Total contribution–** It is the sum of contribution to the Sanchaya Kosh from employees side and the percent from HMG's side.

2. **Interest coefficient–** It shows the ratio of the inferest on money for any member of days of deposit of money in Sanchaya Kosh and the money itself.

3. **Maximum interest coefficient–** It shows the ratio of the interest on money that is being deposited in Sauchaya Kosh for a period of one year and the money itself.

It the interest rate per annum is R, then R/100 is the maximum interest coefficent.

4. **Average interest coefficent–** It shows the ratio of the average interest on

money deposited within a certain period of year in Sanchaya Kosh and the money itself.

5. **Constant sum**- It is the money in deposit at Sanchaya Kosh at the end of the previous year which is liable to interest at the rate of maximum interest coefficient.

6. **Variable sum**– It is the money which is deposited in Sanchaya Kosh during one year and which is liable to interest at the rate of average interest coefficient.