BIBECHANA
A Multidisciplinary Journal of Science, Technology and Mathematics
ISSN 2091-0762 (print), 2382-5340 (online)
Journal homepage: http://nepjol.info/index.php/BIBECHANA

Energy of Sitnikov's restricted three body problem if the primaries are source of radiation and triaxial rigid bodies

R.R.Thapa[1,2*]
[1]University Department of Mathematics., T.M. Bhagalpur University, Bhagalpur, India
[2]P.G. Department of Mathematics., P.G. Campus, Biratnagar, T.U., Nepal.
*E-mail: thaparajuram@yahoo.com
Accepted for publication: December 13, 2014

Abstract

In this paper the joint effect of source of radiation and triaxial rigid body has been studied. The energy of Sitnikov's restricted three body problem when primaries are sources of radiation and energy of Sitnikov's restricted problem of three bodies when primaries are triaxial rigid bodies have been studied to calculate the joint effect. Equation of motion of the third body of infitesimal mass, if primaries are sources of radiation and triaxial rigid bodies, are calculated.

DOI: http://dx.doi.org/10.3126/bibechana.v12i0.11707

Keywords: Dimensional distance; oscillatory motion; periodicity; radiation parameter.

1. Introduction

Sitnikov [1] problem can be integrated in case of restricted problem if primaries are taken to be equal masses. If primaries are triaxial rigid bodies then distance between them will be changed. Then third body can move along z axis. But if primaries are sources of radiation then the third body moves along the line perpendicular through the centre of mass of the primaries and is perpendicular to the plane of motion.

Thapa et al. [2] obtained the periodicity of Sitnikov's restricted three body problem when the primaries are sources of radiation. They found the equations of motion of the third body and total energy of the system. R.R. Thapa et al. [3] also obtained the equations of motion of the Sitnikov's restricted problem of three bodies when Primaries are triaxial rigid bodies. They got the equations of motion in dimensionless variables and Cartesian form as

$$\ddot{x} - 2n\dot{y} = \Omega_x$$

$$\ddot{y} + 2n\dot{x} = \Omega_y$$

$$\ddot{z} = \Omega_z$$

where $\Omega = \frac{n^2}{2}(x^2+y^2) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2} + \frac{(1-\mu)(2\sigma_1-\sigma_2)}{2r_1^3}$

$+\frac{\mu(2\sigma_1'-\sigma_2')}{2r_2^3} - \frac{3(1-\mu)(\sigma_1-\sigma_2)y^2}{2r_1^5}$

$-\frac{3\mu(\sigma_1'-\sigma_2')y^2}{2r_2^5} - \frac{3(1-\mu)\sigma_1 z^2}{2r_1^5} - \frac{3\mu\sigma_1' z^2}{2r_2^5}$

In present work we have proposed to get the energy of restricted three body problem if the primaries are source of radiation and triaxial rigid bodies.

2. Equation of motion

We will adopt the notation and terminology of szehebely [4].

We have the equation of motion of the third body of infinitesimal mass is

$$\ddot{z} = \frac{\partial\Omega}{\partial z} = \frac{-(1-p)z}{\left(z^2+\frac{a^2}{4}\right)^{\frac{3}{2}}} - \frac{3(2\sigma_1-\sigma_2+2\sigma_1'-\sigma_2')z}{4\left(z^2+\frac{a^2}{4}\right)^{\frac{5}{2}}} + \frac{15(\sigma_1+\sigma_1')z^3}{4\left(z^2+\frac{a^2}{4}\right)^{\frac{7}{2}}}$$

$$\frac{d^2z}{dt^2} = \frac{-(1-p)z}{\left(z^2+\frac{a^2}{4}\right)^{\frac{3}{2}}} - \frac{3\alpha z}{4\left(z^2+\frac{a^2}{4}\right)^{\frac{5}{2}}} + \frac{15}{4}\frac{(\sigma_1+\sigma_1')z^3}{\left(z^2+\frac{a^2}{4}\right)^{\frac{7}{2}}} \tag{1}$$

where $\alpha = 2\sigma_1 - \sigma_2 + 2\sigma_1' - \sigma_2'$

Equation (1) is non-linear equation. It can be integrated by linearizing it.

3. Energy of the third body

If we multiply (1) by $2\frac{dz}{dt}$ we get,

$$2\frac{dz}{dt}\frac{d^2z}{dt^2} = -\frac{2(1-p)z}{\left(z^2+\frac{a^2}{4}\right)^{\frac{3}{2}}}\frac{dz}{dt} - \frac{3\alpha}{4\left(z^2+\frac{a^2}{4}\right)^{\frac{5}{2}}} \times 2z\frac{dz}{dt} + \frac{15(\sigma_1+\sigma_1')z^3}{4\left(z^2+\frac{a^2}{4}\right)^{\frac{7}{2}}}\frac{2dz}{dt}$$

Or, $$\frac{1}{2}\frac{d}{dt}\left(\frac{dz}{dt}\right)^2 = -\frac{(1-p)z}{\left(z^2+\frac{a^2}{4}\right)^{\frac{3}{2}}}\frac{dz}{dt} - \frac{3\alpha z}{4\left(z^2+\frac{a^2}{4}\right)^{\frac{5}{2}}} \times \frac{dz}{dt} + \frac{15(\sigma_1+\sigma_1')}{4}\frac{\left(z^2+\frac{a^2}{4}-\frac{a^2}{4}\right)z}{\left(z^2+\frac{a^2}{4}\right)^{\frac{7}{2}}}\frac{dz}{dt}$$

$$-\frac{(1-p)z}{\left(z^2+\frac{a^2}{4}\right)^{\frac{3}{2}}}\frac{dz}{dt}-\frac{3\alpha z}{4\left(z^2+\frac{a^2}{4}\right)^{\frac{5}{2}}}\frac{dz}{dt}+\frac{15(\sigma_1+\sigma_1')}{4}\left[\frac{z}{\left(z^2+\frac{a^2}{4}\right)^{\frac{5}{2}}}\frac{dz}{dt}-\frac{a^2}{4}\frac{z}{\left(z^2+\frac{a^2}{4}\right)^{\frac{7}{2}}}\frac{dz}{dt}\right] \qquad (2)$$

If we substitute $u\frac{du}{dt}=z\frac{dz}{dt}$ and $u^2=z^2+\frac{a^2}{4}$ we get,

$$\frac{1}{2}\frac{d}{dt}\left(\frac{dz}{dt}\right)^2=-\frac{(1-p)}{\left(u^2\right)^{\frac{3}{2}}}u\frac{du}{dt}-\frac{3\alpha}{4\left(u^2\right)^{\frac{5}{2}}}u\frac{du}{dt}+\frac{15(\sigma_1+\sigma_1')}{4}\left[\frac{1}{\left(u^2\right)^{\frac{5}{2}}}u\frac{du}{dt}-\frac{a^2}{4}\frac{1}{\left(u^2\right)^{\frac{7}{2}}}u\frac{du}{dt}\right]$$

$$=-\frac{(1-p)}{u^2}\frac{du}{dt}-\frac{3\alpha}{4u^4}\frac{du}{dt}+\frac{15(\sigma_1+\sigma_1')}{4}\left[\frac{1}{u^4}\frac{du}{dt}-\frac{1}{4}a^2\frac{1}{u^6}\frac{du}{dt}\right]$$

$$=-\frac{1}{u^2}\frac{du}{dt}-\frac{3\alpha}{4u^4}\frac{du}{dt}+\frac{15}{4}(\sigma_1+\sigma_1')\frac{1}{u^4}\frac{du}{dt}-\frac{15}{4}(\sigma_1+\sigma_1')\frac{a^2}{4}\cdot\frac{1}{u^6}\frac{du}{dt}$$

$$=\frac{(1-p)du^{-1}}{dt}+\frac{\alpha}{4}\frac{du^{-3}}{dt}-\frac{5(\sigma_1+\sigma_1')}{4}\frac{du^{-3}}{dt}+\frac{3(\sigma_1+\sigma_1')a^2}{16}\frac{d}{dt}\left(u^{-5}\right)$$

$$\frac{1}{2}\int\frac{d}{dt}\left(\frac{dz}{dt}\right)^2dt=(1-p)\int\frac{du^{-1}}{dt}dt+\frac{\alpha}{4}\int\frac{du^{-3}}{dt}dt-\frac{5(\sigma_1+\sigma_1')}{4}\int\frac{du^{-3}}{dt}dt\,\frac{3(\sigma_1+\sigma_1')a^2}{16}\int\frac{du^{-5}}{dt}dt$$

$$\Rightarrow\frac{1}{2}\left(\frac{dz}{dt}\right)^2=(1-p)u^{-1}+\frac{\alpha}{4}u^{-3}-\frac{5}{4}(\sigma_1+\sigma_1')u^{-3}+\frac{3(\sigma_1+\sigma_1')a^2}{16}u^{-5}+c$$

$$\Rightarrow\frac{1}{2}\left(\frac{dz}{dt}\right)^2=\frac{(1-p)}{u}+\frac{\alpha}{4u^3}-\frac{5(\sigma_1+\sigma_1')}{4u^3}+\frac{3(\sigma_1+\sigma_1')a^2}{16}\frac{1}{u^5}+c$$

$$\Rightarrow\frac{1}{2}\left(\frac{dz}{dt}\right)^2=\frac{(1-p)}{\sqrt{z^2+\frac{a^2}{4}}}+\frac{\alpha}{4\left(z^2+\frac{a^2}{4}\right)^{\frac{3}{2}}}-\frac{5(\sigma_1+\sigma_1')}{4\left(z^2+\frac{a^2}{4}\right)^{\frac{3}{2}}}+\frac{3(\sigma_1+\sigma_1')a^2}{16\left(z^2+\frac{a^2}{4}\right)^{\frac{5}{2}}}+c$$

$$\Rightarrow\frac{1}{2}\left(\frac{dz}{dt}\right)^2-\frac{(1-p)}{\sqrt{z^2+\frac{a^2}{4}}}-\frac{\alpha}{4\left(z^2+\frac{a^2}{4}\right)^{\frac{3}{2}}}+\frac{5(\sigma_1+\sigma_1')}{4\left(z^2+\frac{a^2}{4}\right)^{\frac{3}{2}}}-\frac{3(\sigma_1+\sigma_1')a^2}{16\left(z^2+\frac{a^2}{4}\right)^{\frac{5}{2}}}=c$$

$$\Rightarrow\frac{1}{2}\left(\frac{dz}{dt}\right)^2-\frac{(1-p)}{\sqrt{z^2+\frac{a^2}{4}}}+\frac{\left(5\sigma_1+5\sigma_1'-2\sigma_1+\sigma_2-2\sigma_1'+\sigma_2'\right)}{4\left(z^2+\frac{a^2}{4}\right)^{\frac{3}{2}}}-\frac{3(\sigma_1+\sigma_1')a^2}{16\left(z^2+\frac{a^2}{4}\right)^{\frac{5}{2}}}=c$$

$$\Rightarrow \frac{1}{2}\left(\frac{dz}{dt}\right)^2 - \frac{(1-p)}{\sqrt{z^2+\frac{a^2}{4}}} + \frac{\left(3\sigma_1+\sigma_2+3\sigma_1'+\sigma_2'\right)}{4\left(z^2+\frac{a^2}{4}\right)^{\frac{3}{2}}} - \frac{3(\sigma_1+\sigma_1')a^2}{16\left(z^2+\frac{a^2}{4}\right)^{\frac{5}{2}}} = c$$

$$\Rightarrow \frac{1}{2}\left(\frac{dz}{dt}\right)^2 - \frac{(1-p)}{\left(z^2+\frac{a^2}{4}\right)^{\frac{1}{2}}} + \frac{p}{4\left(z^2+\frac{a^2}{4}\right)^{\frac{3}{2}}} - \frac{3(\sigma_1+\sigma_1')a^2}{16\left(z^2+\frac{a^2}{4}\right)^{\frac{5}{2}}} = \text{ constant}$$

$$E = \frac{1}{2}\left(\frac{dz}{dt}\right)^2 - \frac{(1-p)}{\left(z^2+\frac{a^2}{4}\right)^{\frac{1}{2}}} + \frac{p}{4\left(z^2+\frac{a^2}{4}\right)^{\frac{3}{2}}} - \frac{3(\sigma_1+\sigma_1')a^2}{16\left(z^2+\frac{a^2}{4}\right)^{\frac{5}{2}}} = \text{constant.}$$

$$E = \frac{1}{2}\left(\frac{dz}{dt}\right)^2 - \frac{(1-p)}{\sqrt{z^2+\frac{a^2}{4}}} + \frac{p}{4\left(z^2+\frac{a^2}{4}\right)^{\frac{3}{2}}} - \frac{3(\sigma_1+\sigma_1')}{16\left(z^2+\frac{a^2}{4}\right)^{\frac{5}{2}}} = \text{constant.}$$

Now from equation (1)

$$\frac{d^2z}{dt^2} + \frac{(1-p)z}{\left(z^2+\frac{a^2}{4}\right)^{\frac{3}{2}}} + \frac{3\alpha z}{4\left(z^2+\frac{a^2}{4}\right)^{\frac{5}{2}}} - \frac{15(\sigma_1+\sigma_1')}{4\left(z^2+\frac{a^2}{4}\right)^{\frac{7}{2}}} z^3 = 0$$

$$\Rightarrow \frac{d^2z}{dt^2} + \frac{(1-p)z}{\left(\frac{a}{2}\right)^3\left(1+\frac{4z^2}{a^2}\right)^{\frac{3}{2}}} + \frac{3\alpha z}{4\left(\frac{a}{2}\right)^5\left(1+\frac{4z^2}{a^2}\right)^{\frac{5}{2}}} - \frac{15(\sigma_1+\sigma_1')}{4\left(\frac{a}{2}\right)^7\left(1+\frac{4z^2}{a^2}\right)^{\frac{7}{2}}} z^3 = 0$$

$$\Rightarrow \frac{d^2z}{dt^2} + \frac{(1-p)8z}{a^3}\left(1+\frac{4z^2}{a^2}\right)^{-\frac{3}{2}} + \frac{24\alpha z}{a^5}\left(1+\frac{4z^2}{a^2}\right)^{-\frac{5}{2}}$$
$$- \frac{15(\sigma_1+\sigma_1')\times 32z^3}{a^7}\left(1+\frac{4z^2}{a^2}\right)^{-\frac{7}{2}} = 0$$

(as $z << a$ so $\left(\frac{z}{a}\right) <<< 1$)

$$\Rightarrow \frac{d^2z}{dt^2} + \frac{(1-p)8z}{a^3}\left(1-\frac{3}{2}\left(\frac{2z}{a}\right)^2+\ldots\right) + \frac{24\alpha z}{a^5}\left(1-\frac{5}{2}\left(\frac{2z}{a}\right)^2+\ldots\right) - \frac{15\times 32}{a^7}(\sigma_1+\sigma_1')z^3\left(1-\frac{7}{2}\left(\frac{2z}{a}\right)^2+\ldots\right) = 0$$

Neglecting higher order term of (z/a) above the third order, we get,

$$\frac{d^2z}{dt^2}+\frac{(1-p)8z}{a^3}-\frac{8z}{a^3}\cdot\frac{12}{2}\left(\frac{z}{a}\right)^2+\frac{24\alpha z}{a^5}-\frac{24\alpha z}{a^5}\times\frac{5\times 4z^2}{2a^2}-\frac{480(\sigma_1+\sigma_1')}{a^7}z^3$$

$$+\frac{480(\sigma_1+\sigma_1')}{a^7}z^3\frac{7}{2}\times\frac{4z^2}{a^2}=0$$

$$\frac{d^2z}{dt^2}+\left(\frac{(1-p)8}{a^3}+\frac{24\alpha}{a^5}\right)z-\left(\frac{48}{a^5}+\frac{240\alpha}{a^7}+\frac{480(\sigma_1+\sigma_1')}{a^7}\right)z^3+\ldots=0$$

$$\frac{d^2z}{dt^2}+\frac{8(1-p)}{a^5}\left(a^2+3\alpha\right)z-\frac{48}{a^7}\left(a^2+5\alpha+10(\sigma_1+\sigma_1')\right)z^3=0 \tag{3}$$

$$\frac{d^2z}{dt}+\eta_0^2 z-\varepsilon z^3=0 \tag{4}$$

where $\eta_0^2=\frac{8(1-p)}{a^5}\left(a^2+3\alpha\right)=\frac{8q(a^2+3\alpha)}{a^5}$

and $\varepsilon=\frac{48}{a^7}\left[a^2+5\alpha+10(\sigma_1+\sigma_1')\right]$

The equations (3), (4) represent energy of Sitnikov's restricted three body problem if the primaries are source of radiation and triaxial rigid bodies.

4. Result and Discussion

The energy of third body depends on variable z, radiation parameter p, (parameter) $\alpha =2\sigma_1-\sigma_2+2\sigma_1^1-\sigma_2^1$, distance between the primaries a; where $\sigma_1=\frac{a_1^2-a_3^2}{5R^2}$, $\sigma_2=\frac{a_2^2-a_3^2}{5R^2}$, $\sigma_1^2=\frac{b_1^2-b_3^2}{5R^2}$, $\sigma_2^1=\frac{b_2^2-b_3^2}{5R^2}$; R is dimensional distance between the primaries and a_1,a_2,a_3; b_1,b_2,b_3 are lengths of semi-axes of two primaries parallel to axes respectively. The kinetic energy of the third body depends on variable z but potential energy depends on η_0 and ε.

where $\eta_0^2=8q\frac{(a^2+3\alpha)}{a^5}$ and $\varepsilon=\frac{48}{a^7}[(a^2+5\alpha+10(\sigma+\sigma^1)]$.

Hence the third body will perform oscillatory motion.

Acknowledgement

I want to acknowledge with thanks my supervisor Dr. M.R. Hassan, Department of Mathematics, S.M. College, Bhagalpur, T.M.B.U. Bghagalpur, 812007, India for his meticulous guidance during the work.

References

[1] K.A. Sitnikov, Dokl. Akad, Nauk, USSR **132(2)** (1960) 303.

[2] R.R. Thapa, and M.R. Hassan, Journal of the Bihar Mathematical society **27**(2013) 99-105.

[3] R.R. Thapa and M.R. Hassan, A research journal in mathematics "M_0" **1**(2014) 37-39.

[4] V. Szebehely, Theory of orbits, Academic Press, New York (1967).